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September 8, 1989

Will 'Ordinary' Electron-Laser Interactions Preclude Observation of Nonlinear Strong-Field Effects?

Abstract

We consider whether electrons might never penetrate to the high-field core of a laser pulse either because of deflection by the ponderomotive force or attenuation by Compton scattering.

1 Deflection of Electrons by the Ponderomotive Force

I first recall the argument given in section 2-1d of my review paper, DOE/ER/3072-38 (Sept. 2, 1986).

The effective mass, $\bar{m}$, of an electron inside a wave field can be thought of as associated with an effective potential:

$$U_{\text{eff}} = \bar{m}c^2 = mc^2\sqrt{1+\eta^2},$$

where $\eta^2 = e^2\langle E^2\rangle/(m\omega c)^2$ is the classical, dimensionless measure of the intensity of the electric field E.

For a field that is nonuniform, the intensity gradient can be associated with a force,

$$F = -\nabla U = -\frac{mc^2}{2\sqrt{1+\eta^2}}\nabla\eta^2.$$

If the wave field is that of a focused laser beam, intensity gradients occur because of the laws of diffraction. This is usefully expressed by the shape of a gaussian laser beam,

$$\eta^2(r,z) = \frac{\eta_0^2}{1+z^2/z_0^2}\exp\left(\frac{-r^2}{2\sigma_r^2(1+z^2/z_0^2)}\right),$$

where σ_r describes the transverse gaussian profile of the beam at its waist, and

$$z_0 = \frac{2\pi\sigma_r^2}{\lambda}$$

is called the Rayleigh range and measures the length over which the intensity falls by 2 along z (assuming a sinusoidal time dependance).

The laser beam may also be pulsed. We suppose it to have a gaussian profile in time, and let σ_t be the corresponding variance.

Note that there are now two reasons for longitudinal intensity gradients: the Rayleigh range of diffraction, and the temporal pulse width. For a tightly focused laser beam, the Rayleigh range is likely to be shorter than the temporal pulse width, and hence dominates the longitudinal intensity gradient.

A focus is achieved with a lens (or mirror). If the laser beam is 'matched' to the lens in the sense that $\sigma_r(\text{lens}) = \sqrt{2}D/4$ where D is the diameter of the lens, then the σ of the *intensity* profile at the lens is $D/4$ and about 86% of the beam passes through the lens to be focused. In this case, the diffraction-limited spot size, σ_r, at the focus is given by

$$\sigma_r = \frac{\sqrt{2}}{\pi}\frac{f}{D}\lambda,$$

where f is the focal length of the lens. A good lens might have $f/D = \pi/\sqrt{2}$, leading to $\sigma_r = \lambda$ and $z_0 = 2\pi\lambda$. For σ_t to be less than z_0 would require a pulse of duration less than 6 cycles r.m.s, or 14 cycles full-width at half-maximum. This condition has been met only in very special low-power lasers.

Given that the laws of diffraction determine the intensity profile, we see that the transverse gradient is always larger than the longitudinal for any lens with $f/D > 1/4$. [I missed this point in the 1986 paper, where I remarked only on the longitudinal gradient.] Thus the relevant form for $\nabla\eta^2$ on p. 1 is

$$\frac{d\eta^2}{dr} \text{ whose peak value is approximately } \frac{\eta_0^2}{\sigma_r}.$$

The corresponding transverse force lasts for a time approximately $z_0/c = 2\pi\sigma_r^2/\lambda c$, so the electron experiences a transverse momentum kick of

$$\Delta P_T \approx \pi mc\frac{\eta^2}{\sqrt{1+\eta^2}}\frac{\sigma_r}{\lambda}$$

on its way into the laser pulse. The average transverse velocity of the electron due to the gradient force is

$$\langle v\rangle = \frac{\Delta P_T}{2\gamma m},$$

on recalling a result of section 2-2d of my review paper that the energy of the electron inside the wave field is approximately γmc^2 so long as $\eta \ll \gamma$.

The electron moves transversely by $\langle v\rangle z_0/c$ on its way into the pulse. We require this to be much less than σ_r, otherwise the electron will be deflected out of the core of the beam. Thus we arrive at the condition

$$\frac{\eta^2}{\sqrt{1+\eta^2}} \ll \frac{\gamma\lambda^2}{\pi^2\sigma_r^2} = \frac{\gamma}{2(f/D)^2} = \frac{4\gamma}{(f/\sigma_r(\text{lens}))^2}.$$

In an initial experiment to demonstrate the nonlinear effects of large η on Compton scattering, a value of $\eta \approx 0.3$ is perhaps optimal. For smaller η the effects are very small, and for larger η the various multiphoton contributions blur into a continuum. Hence we need

$$\gamma \gg 0.2(f/D)^2 = 1.8,$$

if we use a lens with $f/D = 3$.

At Brookhaven Lab, an experiment is in preparation to study the nonlinear Compton effect with 50-MeV electrons, and a CO_2 laser with $\lambda = 10$ μm and $\eta \approx 0.1$-0.3. In this example, $\gamma = 100$ and there should be little problem of the electrons being diverted from the core of the laser pulse.

2 Attenuation of the Electron Beam by Compton Scattering

The electrons may suffer a Compton scatter before they reach the center of the laser pulse, and hence the signature of any nonlinear QED effects at the pulse center would be confused.

I return to an argument given in section 2-2c of my 1986 review paper.

For pulses with $\eta \lesssim 1$ we may use the Larmor formula for the rate of energy loss by an electron to Compton scattering (if the center-of-mass energy is so high that quantum corrections are important, these always reduce the rate!):

$$\frac{dU^\star}{dt^\star} = \frac{2e^4 E^{\star 2}}{3m^2c^3},$$

where the superscript * indicates quantities to be evaluated in the (average) rest frame of the initial electron. It is memorable to use one cycle of the laser field oscillation as the unit of time: $dt^\star = 2\pi/\omega^\star$. Then the energy radiated in one cycle of the wave is

$$dU^\star = \frac{4\pi e^4 E^{\star 2}}{3m^2c^3\omega^\star} \text{ per cycle.}$$

The number of photons radiated is

$$dN = \frac{dU^\star}{\hbar\omega^\star} = \frac{4\pi}{3}\frac{e^2}{\hbar c}\frac{e^2E^{\star 2}}{m^2\omega^{\star 2}c^2} = \frac{4\pi}{3}\alpha\eta^2 \text{ photons per cycle.}$$

The effective number of cycles during which this radiation occurs (as the electron enters the laser pulse) can be estimated as the Rayleigh range divided by the wavelength:

$$\frac{z_0}{\lambda} = \frac{2\pi\sigma_r^2}{\lambda^2} = \frac{4}{\pi}\left(\frac{f}{D}\right)^2,$$

supposing the laser is focused in a lens of focal length f and aperture D as described above. Again we note that this is a short time compared to the temporal pulse length of any laser that might be used for high-field studies.

Then the number of photons radiated per electron is

$$dN = \frac{16}{3}\alpha\left(\frac{f}{D}\right)^2\eta^2 \approx 0.04\left(\frac{f}{D}\right)^2\eta^2.$$

A good lens might have $f/D \approx 2$, so there is only a 16% chance of an electron undergoing a Compton scatter on the way into a laser beam with $\eta = 1$. And for $\eta < 1$, the scattering rate is quadratic in η.

Thus even when a single Compton scatter effectively removes the electron from the beam, studies of nonlinear effects should be possible for laser field with η of order 1.

For the Brookhaven experiment at $U = 50$ MeV mentioned above, the electron loses at most 3 keV in a Compton scatter, and for $\eta = 0.3$ the average number of Compton scatters per electron is 0.12.

In any case, the Compton scatter could be important only if the electron loses a significant fraction of its energy in the scatter. Now the maximum energy lost by an electron that Compton scatters in a wave field with $\eta \ll 1$ is

$$\frac{\Delta U}{U} = \frac{4\gamma\omega/m}{1 + 4\gamma\omega/m},$$

where ω is the energy of a photon of the wave field. Hence the possible attenuation of the electron will be important only for

$$\gamma \gtrsim \frac{m}{4\omega}.$$

However, this includes the interesting case that $U = 50$ GeV (SLAC or LEP) for which $\gamma = 10^5$ and a Nd:YAG laser with $\omega = 1$ eV.

3 Attenuation of Photon Beams by the Breit-Wheeler Process

My own interest in nonlinear QED with a 50-GeV electron beam involves studies of light-by-light scattering. Here the electron-laser collision serves only to produce a high-energy photon beam that is then scattered against a second piece of the laser pulse. [Both scattered and unscattered electrons are swept away before the light-by-light collision.] It is desireable for this that the probability of a Compton scatter of a 50-GeV electron be near 1, for which we need only $\eta \approx 1$ as noted above.

In principle, the high-energy photon beam might be attenuated by interaction with the leading edge of the laser pulse via the Breit-Wheeler process

$$\gamma\gamma \to e^+e^-,$$

whose cross section is similar to that for Compton scattering. However, we are actually below the kinematic threshold for this process if we use 50-GeV electrons and a YAG laser; it can only occur via multiple laser photons. Thus the high energy photons will not interact at all with the laser until they reach the core of the laser beam where $\eta \approx 1$.

4 Vacuum Čerenkov Radiation

As a final remark, I consider the possibility of vacuum Čerenkov radiation, in which an electron emits a Čerenkov photon in the vacuum as polarized by a strong wave field. The threshold electron energy for this effect is given by

$$\gamma_0 = \frac{m}{\eta\omega}\sqrt{\frac{1+\eta^2}{22\alpha/45\pi}}.$$

To keep the Čerenkov threshold 'low' it is desirable to operate the laser near $\eta = 1$, but it does not pay to go much higher as the threshold changes little once $\eta > 1$. Again, the attentuation of the electron beam by Compton scatter with the laser would be annoying but not fatal for such an experiment.