# The Laser-Driven Vacuum Photodiode

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## 1 Problem

A vacuum photodiode is constructed in the form of a parallel-plate capacitor with plate separation $D$. A battery maintains constant potential difference $V$ between the plates. A short laser pulse illuminates that cathode at time $t = 0$ with energy sufficient to liberate all of the surface-electron charge density. This charge moves across the capacitor gap as a sheet until it is collected at the anode at time $T$. Then, another laser pulse strikes the cathode, and the cycle repeats.

Estimate the average current density $\langle J \rangle$ that flows onto the anode from the battery, ignoring the recharging of the cathode as the charge sheet moves away. Then, calculate the current density and its time average when this effect is included.

Compare with Child's Law [1]-[5] for steady current flow.

You may suppose that the laser photon energy is equal to the work function of the cathode, so the electrons leave the cathode with zero velocity.

## 2 Solution

The initial electric field in the capacitor is $\mathbf{E} = -V/D\,\hat{\mathbf{x}}$, where the $x$ axis points from the cathode at $x = 0$ to the anode. The initial surface charge density on the cathode is (in Gaussian units),

$$\sigma = E/4\pi = -\frac{V}{4\pi D}\,. \tag{1}$$

The laser liberates this charge density at $t = 0$.

The average current density that flows onto the anode from the battery is,

$$\langle J \rangle = -\frac{\sigma}{T} = \frac{V}{4\pi DT}\,, \tag{2}$$

where $T$ is the transit time of the charge across the gap $D$.

### 2.1 Analysis with No Battery

We first estimate $T$ by ignoring the effect of the recharging of the cathode as the charge sheet moves away from it, as would be the case if no battery were present. In this approximation, the average field on the (thin) charge sheet is always $E/2 = -V/D2$, so the acceleration of an electron is $a = -eE/2m = eV/2Dm$, where $e$ and $m$ are the magnitudes of the charge and mass of the electron, respectively. The time to travel distance $D$ is $T = \sqrt{2D/a} = 2D\sqrt{m/eV}$. Hence,

$$\langle J \rangle = \frac{V}{4\pi DT} = \frac{V^{3/2}}{8\pi D^2}\sqrt{\frac{e}{m}}\,. \tag{3}$$

This is close to Child's Law for a thermionic diode,[1]

$$J_{\rm steady} = \frac{V^{3/2}}{9\pi D^2}\sqrt{\frac{2e}{m}} = \frac{V^{3/2}}{6.36\pi D^2}\sqrt{\frac{e}{m}}\,. \tag{4}$$

## 2.2 Analysis with a Battery

We now make a detailed calculation, including the effect of the recharging of the cathode by a battery, which will reduce the average current density somewhat.

At some time $t$, the charge sheet is at distance $x(t)$ from the cathode, and the anode and cathode have charge densities $\sigma_A$ and $\sigma_C$, respectively. All the field lines that leave the anode terminate on either the charge sheet or on the cathode, so,

$$\sigma + \sigma_C = -\sigma_A, \tag{5}$$

where $\sigma_A$ and $\sigma_C$ are the charge densities on the anode and cathode, respectively. The the electric field strength in the region I between the anode and the charge sheet is,

$$E_I = -4\pi\sigma_A, \tag{6}$$

and that in region II between the charge sheet and the cathode is,

$$E_{II} = 4\pi\sigma_C. \tag{7}$$

The voltage between the capacitor plates is therefore,

$$V = -E_I(D-x) - E_{II}x = 4\pi\sigma_A D - V\frac{x}{D}\,, \tag{8}$$

using eqs. (1) and (5)-(7), and taking the cathode to be at ground potential. Thus,

$$\sigma_A = \frac{V}{4\pi D}\left(1+\frac{x}{D}\right), \qquad \sigma_C = -\frac{Vx}{4\pi D^2}\,, \tag{9}$$

and the current density flowing onto the anode (from the battery) is,

$$J(t) = \dot\sigma_A = \frac{V\dot x}{4\pi D^2}. \tag{10}$$

*This instantaneous current differs from the average current density (2) of the moving charge sheet in that $\dot x/D \neq T$, since $\dot x$ varies with time.*

To find the velocity $\dot x$ of the charge sheet, we consider the force on it, which is due to the average of the fields set up by charge densities on the anode and cathode,

$$E_{{\rm on}\ \sigma} = 2\pi(-\sigma_A + \sigma_C) = -\frac{V}{2D}\left(1+\frac{2x}{D}\right). \tag{11}$$

The equation of motion of an electron in the charge sheet is,

$$m\ddot x = -eE_{{\rm on}\ \sigma} = \frac{eV}{2D}\left(1+\frac{2x}{D}\right), \tag{12}$$

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[1] Equation (2) appears in [6]. For reviews of other recent variations, see [7, 8].

or,

$$\ddot{x} - \frac{eV}{mD^2}x = \frac{eV}{2mD}. \tag{13}$$

With the initial condition that the electrons start from rest, $x(0) = 0 = \dot{x}(0)$, we readily find that,

$$x(t) = \frac{D}{2}(\cosh kt - 1), \tag{14}$$

where

$$k = \sqrt{\frac{eV}{mD^2}}. \tag{15}$$

The charge sheet reaches the anode at time,

$$T = \frac{1}{k}\cosh^{-1}\frac{3}{2} = \frac{0.96}{k}\,, \tag{16}$$

compared to $T = 1/k$ as found in sec. 2.1 without the battery.

The average anode-current density is, using (2) and (16),

$$\langle J \rangle = \frac{V}{4\pi DT} = \frac{V^{3/2}}{4\pi\cosh^{-1}(3/2)\ D^2}\sqrt{\frac{e}{m}} = \frac{V^{3/2}}{12.09\ \pi D^2}\sqrt{\frac{e}{m}}. \tag{17}$$

The electron velocity follows from eq. (14),

$$\dot{x} = \frac{Dk}{2}\sinh kt, \tag{18}$$

so the time dependence of the anode-current density (10) is,

$$J(t) = \frac{1}{8\pi}\frac{V^{3/2}}{D^2}\sqrt{\frac{e}{m}}\sinh kt \qquad (0 < t < T). \tag{19}$$

A device that incorporates a laser-driven photocathode is the laser-triggered rf gun [9].

## 2.3 Energy Considerations

### 2.3.1 Without Battery

In the absence of the battery to recharge the capacitor, current flows between the plates only once, as the charge initially on the cathode is freed by the laser pulse, and pulled across the anode by the constant electric field. Just before the electrons arrive at the anode they have maximal kinetic energy, and the volume of electric field is essentially zero. The initial energy stored in the electric field has been transferred to the final kinetic energy of the electrons,

$$U_0 = \frac{E^2}{8\pi}AD = \frac{AV^2}{8\pi D}\,, \tag{20}$$

$$KE_f = \frac{Nmv_f^2}{2} = NmaD = NmD\frac{eV}{2Dm} = \frac{NeV}{2}\frac{\sigma AV}{2} = \frac{AV^2}{8\pi D} = U_0, \tag{21}$$

where $A$ is the area of the capacitor plates, as we recall that the acceleration of the electrons is $a = eV/2Dm$. We can say that as the charge sheet moves across the gap between the capacitor plates, it sweeps up the energy stored in the electric field.

Poynting has given a view that the flow of electromagnetic energy is described by the vector [10],

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{H}. \tag{22}$$

In the present example the fields of the accelerating charge sheet include radiation, which we ignore, as well as quasistatic "near" fields including an electric field that point away from the $x$-axis and a magnetic field that circulates about it. The correspond Poynting vector $\mathbf{S}$ in the region between the charge sheet and the anode points inwards and towards the charge sheet. We interpret this as a more detailed prescription of the process of "sweeping up" of field energy by the moving charge sheet.

### 2.3.2 With Battery

When a battery is connected to the capacitor plates the laser-induced photocurrent can be cyclic, with the battery providing energy to re-establish the electric field in the region between that cathode and the moving charge sheet. In this region the Poynting vector points inwards towards the $x$-axis and describes the flow of energy from the battery into the recharging capacitor.

## References

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