Floating-Wire Simulation of the Trajectory of a Charged Particle in a Magnetic Field

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1 Problem

Show that the trajectory of a charged particle in a magnetic field can be duplicated by that of a current-carrying wire held at rest under constant tension (by fixtures outside the field region. Deduce the current I required in a wire of tension T to match the trajectory of an proton of momentum P.

2 Solution

The equation of motion of a (relativistic) particle of charge e, mass m and velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is (in MKSA units),

$$\frac{d\mathbf{P}}{dt} = e\mathbf{v} \times \mathbf{B}. \tag{1}$$

An increment $d\mathbf{s}$ of arc length along the particle's trajectory can be written as,

$$d\mathbf{s} = \mathbf{v}dt. \tag{2}$$

Using this to replace $\mathbf{v}$ in eq. (1), the particle's trajectory can be described as,

$$d\mathbf{P} = e\ d\mathbf{s} \times \mathbf{B}. \tag{3}$$

The momentum $\mathbf{P}$ is along the trajectory, so we can write,

$$\mathbf{P} = P\hat{\mathbf{s}}, \qquad \text{and} \qquad d\mathbf{P} = Pd\hat{\mathbf{s}}, \tag{4}$$

noting that the magnetic field changes the direction, but not the magnitude, of the momentum. Hence, the equation of the trajectory is,

$$d\hat{\mathbf{s}} = \frac{e}{P}d\mathbf{s} \times \mathbf{B}. \tag{5}$$

The equation for static equilibrium of a current-carrying wire under tension T in the same magnetic field is,

$$\sum \mathbf{F} = 0 = T(\mathbf{s} + d\hat{\mathbf{s}}) - T\hat{\mathbf{s}} + Id\mathbf{s} \times \mathbf{B}, \tag{6}$$

or,

$$d\hat{\mathbf{s}} = -\frac{I}{T}d\mathbf{s} \times \mathbf{B}. \tag{7}$$

The trajectory of the "floating" wire can be the same as that of the charged particle when,

$$I[\mathrm{A}] = -\frac{eT[\mathrm{N}]}{P[\mathrm{kg\text{-}m/s}]}\,. \tag{8}$$

We express this relation in practical units by noting that,

$$T[\mathrm{N}] = 0.0098\ T[\mathrm{gm}], \tag{9}$$

when the tension is maintained by a weight T in grams attached to one end of the wire over a pulley, and that the momentum of the proton is,

$$P[\mathrm{kg\text{-}m/s}] = \frac{10^6 e}{c} P[\mathrm{MeV}/c] = \frac{e}{300} P[\mathrm{MeV}/c]. \tag{10}$$

Hence,

$$I[\mathrm{A}] = -\frac{2.94\ T[\mathrm{gm}]}{P[\mathrm{MeV}/c]}\,. \tag{11}$$

An application of this principle is described in C.Y. Prescott, S.U. Cheng and K.T. McDonald, *Wire Orbit Ray Tracing of Magnets Using Magnetostrictive Wire Chamber Techniques*, Nucl. Instr. and Meth. **76**, 173 (1969),
http://kirkmcd.princeton.edu/examples/detectors/prescott_nim_76_173_69.pdf