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Muon Rate in the μBooNE TPC

Abstract

This note describes a calculation of the muon rate at sea-level. The muon rate in the μBooNE TPC with 5 meters of dirt overburden is estimated to be $4652\,\mathrm{s}^{-1}$.

1 Muon Flux at Sea Level

Muons are the most numerous charged particles at sea level. The muon flux at ground-level has been measured many times in the past 50 years [1, 2, 3, 4, 5]. When muon decay is negligible ($E_\mu > 100/\cos\theta\,\mathrm{GeV}$, where θ is the polar angle of the incoming muon), and the curvature of the Earth can be neglected ($\theta < 70°$), the differential flux across a horizontal surface can be well described by the Gaisser's parameterization [6],

$$\frac{dI}{dEd\Omega} = \frac{0.14E^{-2.7}}{\mathrm{cm}^2\cdot\mathrm{s}\cdot\mathrm{sr}\cdot\mathrm{GeV}}\left(\frac{1}{1+\frac{1.1E\cos\theta}{115\,\mathrm{GeV}}} + \frac{0.054}{1+\frac{1.1E\cos\theta}{850\,\mathrm{GeV}}}\right). \tag{1}$$

In order to describe the full range of zenith angles, $0° \le \theta \le 90°$, a parameterization in which the $\cos\theta \to \cos\theta^*$ was introduced by Dmitry [7], where,

$$\cos\theta^* = \sqrt{\frac{(\cos\theta)^2 + P_1^2 + P_2(\cos\theta)^{P_3} + P_4(\cos\theta)^{P_5}}{1+P_1^2+P_2+P_4}}. \tag{2}$$

The best-fit coefficients $P_1, \ldots, P_5$ are shown in Table 1.

Table 1: Best-fit parameters.

Parameter	Value
P_1	0.102573
P_2	-0.068287
P_3	0.958633
P_4	0.0407253
P_5	0.817285

Guan *et al.* [8] augmented Eqs. (1)-(2) with an additional term to describe low-energy muons better,

$$\frac{dI}{dEd\Omega} = \frac{0.14}{\mathrm{cm}^2\cdot\mathrm{s}\cdot\mathrm{sr}\cdot\mathrm{GeV}}\left[E\left(1+\frac{3.64\,\mathrm{GeV}}{E(\cos\theta^*)^{1.29}}\right)\right]^{-2.7}\left(\frac{1}{1+\frac{1.1E\cos\theta^*}{115\,\mathrm{GeV}}} + \frac{0.054}{1+\frac{1.1E\cos\theta^*}{850\,\mathrm{GeV}}}\right). \tag{3}$$

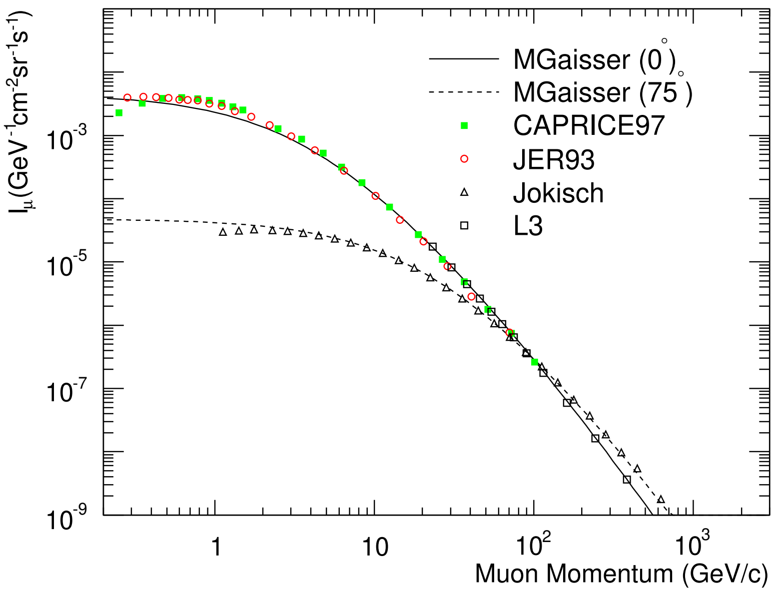

Figure 1: Comparison between best fit (lines, Eq. (3)) and experimental data (points) for the differential flux $I(E) = dI/dEd\Omega$ at sea level. The solid line shows the differential flux at $\theta = 0°$; the dashed line shows the differential flux at $\theta = 75°$.

At high energies, the additional term is negligible. The parameters 3.64 in the numerator and 1.29 in the power of $\cos\theta^*$ of Eq. (3) were obtained by fitting experiment data [1, 2, 3, 4, 5]. Comparisons of the differential parameterization (3) and data are shown in Figs. 1 and 2.

The total rate $R_H(E_{\min})$ of muons above an energy threshold $E_{\min}$ across a horizontal surface is obtained by integrating the differential flux (3), as shown in Fig. 3. Some selected results are listed in Table 2, which agrees with the well-known approximation for the total rate of muons across a horizontal surface, $I \approx 1\,\mathrm{cm}^{-2}\mathrm{min}^{-1}$.

Table 2: Total muon rates R_H across a horizontal surface at sea level for various energy thresholds.

Threshold	0.2 GeV	0.3 GeV	0.4 GeV	0.6 GeV	0.8 GeV	1 GeV
R_H ($\mathrm{m}^{-2}\mathrm{s}^{-1}$)	172.2	166.7	161.5	151.8	143.1	135.2

2 Total Muon Rate across a Vertical Surface

To convert the total rate R_H of muons crossing a horizontal surface to the total rate R_V of muons crossing a vertical surface, one first multiplies the differential rate $dI/dEd\Omega$ across a horizontal surface by the factor $\tan\theta\sin\phi$ [9], and then integrates over energy and solid angle.

Total muon rates R_V from integration of Eq. (3) are listed in Table 3 for various energy thresholds. Note that R_H/R_V is very close to π, as predicted by the approximation that the

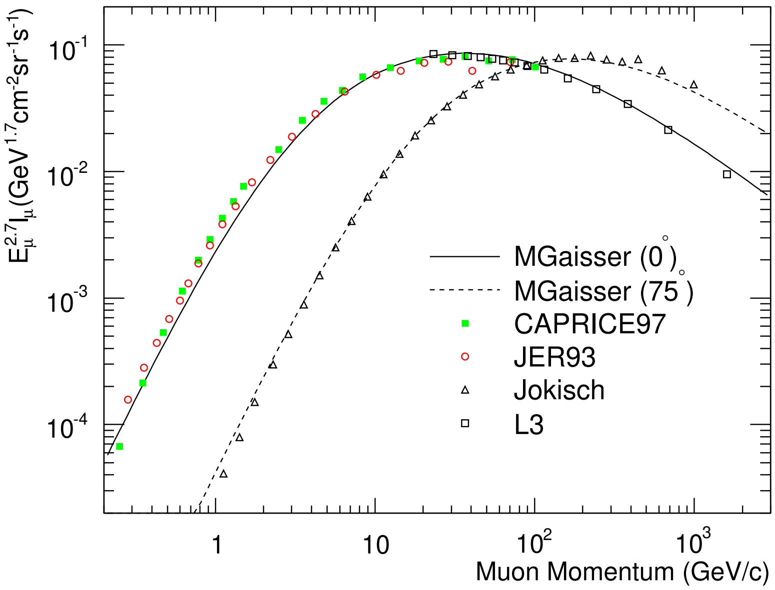

Figure 2: Comparison between best fit (lines) and experimental data (points) for $E^{2.7}I(E)$ at sea level. The solid line shows the differential flux times $E^{2.7}$ at $\theta = 0°$; the dashed line shows the differential flux times $E^{2.7}$ at $\theta = 75°$.

angular distribution of the muons is proportional to $\cos^2\theta$ [9].

Table 3: Total muon rate R_V across a vertical surface ar sea level (from one side only).

Threshold	0.2 GeV	0.3 GeV	0.4 GeV	0.6 GeV	0.8 GeV	1 GeV
R_V ($\mathrm{m^{-2}s^{-1}}$)	56.5	55.2	54.0	51.6	49.4	47.4
R_H/R_V	3.05	3.02	2.99	2.94	2.90	2.85

3 Matter Effects

When passing through matter, muons lose energy by ionization and radiative processes: Bremsstrahlung, e^+e^- pair production, and photonuclear reactions. In general, the muon energy-loss rate can be expressed as,

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = a + bE, \tag{4}$$

where a is the ionization loss and b is the fractional energy loss by the three radiation processes. Both parameters are slowly varying functions of energy. Table 4 lists the values of parameter a for concrete and "standard rock" as a function of muon energy [10]. Parameter b is of order $10^{-6}\,\mathrm{g^{-1}\,cm^2}$ for $E < 10\,\mathrm{GeV}$, so $bE \ll a$, and the radiation energy loss can be neglected when considering low-energy muons.

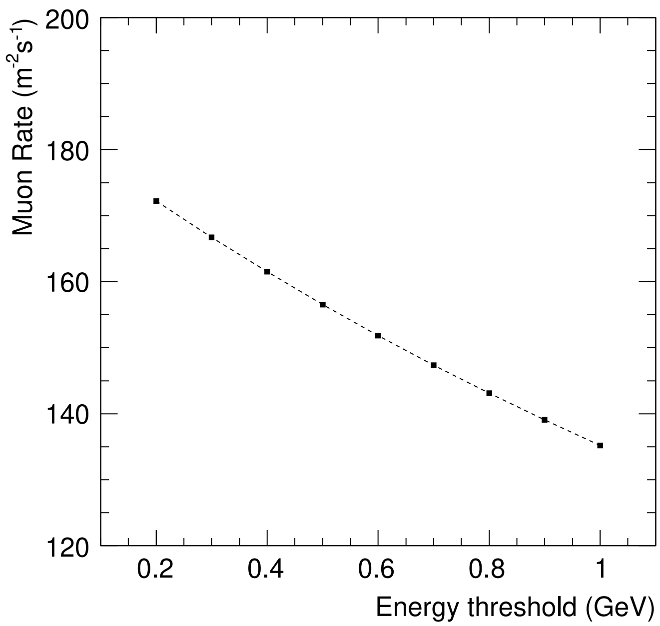

Figure 3: Muon rate across a horizontal surface at sea level *vs.* energy threshold.

Table 4: Values of parameter a in $\mathrm{MeV\,g^{-1}\,cm^2}$ for concrete and "standard rock" at various energies.

Material	0.4 GeV	0.8 GeV	1 GeV	1.4 GeV	2 GeV	3 GeV
Concrete	1.722	1.800	1.834	1.888	1.947	2.014
Standard Rock	1.698	1.774	1.808	1.861	1.920	1.986

4 Total Muon Rate in μBooNE TPC

We anticipate that 5 meters of "dirt" will be placed above the μBooNE TPC. Assuming that "dirt" has the same properties as concrete ($\rho = 2.3\,\mathrm{g/cm^3}$) and taking $a = 1.947\,\mathrm{MeV\,g^{-1}\,cm^2}$, only muons with energy larger than $2.239\,\mathrm{GeV}/\cos\theta$ can reach the detector. By integrating Eq. (3) with $E > 2.239\,\mathrm{GeV}/\cos\theta$, we find $R_H = 82.9\,\mathrm{m^{-2}s^{-1}}$, and $R_V = 27.2\,\mathrm{m^{-2}s^{-1}}$. With an active volume of $2.6 \times 2.6 \times 12\,\mathrm{m^3}$, the total muon rate in the μBooNE TPC will be $2.6 \times 12 \times R_H + 2 \times (2.6 \times 2.6 + 2.6 \times 12) \times R_V = 4652\,\mathrm{s^{-1}}$.

5 Summary

The total muon rate R_H across a horizontal surface at sea level is $172.2\,\mathrm{m^{-2}s^{-1}}$ with $E_\mu > 0.2\,\mathrm{GeV}$. The total muon rate R_V across a vertical surface (from one side) is about $R_H/3$. The total rate of muons in the μBooNE TPC with 5 meters of dirt overburden is $4652\,\mathrm{s^{-1}}$.

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