Accuracy of Measurements in the Muon-Collider Cooling Experiment

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$$\sigma_i = \text{ rms width.}$$

$$\text{Uncertainty in } \sigma_i \equiv \delta_{\sigma_i}.$$

$$\epsilon = \prod_{i=1}^{6} \sigma_i, \qquad \text{so} \qquad \frac{\delta_\epsilon}{\epsilon} = \sqrt{\sum_{i=1}^{6} \left(\frac{\delta_{\sigma_i}}{\sigma_i}\right)^2} \approx \sqrt{6}\frac{\delta_\sigma}{\sigma}.$$

$$\text{Proposed goal:} \qquad \frac{\delta_\sigma}{\sigma} = 0.01, \qquad \text{so} \qquad \frac{\delta_\epsilon}{\epsilon} = 0.03.$$

Effect of Detector Resolution

$\sigma_D =$ detector resolution (in parameter i).

$\delta_{\sigma_D} =$ uncertainty in detector resolution.

$\sigma_O =$ observed rms width, $\qquad \sigma_O^2 = \sigma_i^2 + \sigma_D^2$.

$$\Rightarrow \sigma_i^2 = \sigma_O^2 - \sigma_D^2.$$

$$\delta^2_{\sigma_i^2} = \delta^2_{\sigma_O^2} + \delta^2_{\sigma_D^2}.$$

$$\delta_{\sigma_O^2} = \sqrt{\frac{2}{N}}\sigma_O^2 = \sqrt{\frac{2}{N}}\left(\sigma_i^2 + \sigma_D^2\right).$$

$$\left(\frac{\delta_{\sigma_i}}{\sigma_i}\right)^2 = \frac{1}{2N}\left(1 + \frac{\sigma_D^2}{\sigma_i^2}\right)^2 + \left(\frac{\sigma_D}{\sigma_i}\right)^4 \left(\frac{\delta_{\sigma_D}}{\sigma_D}\right)^2.$$

Perfectly Known Resolution

$$\delta_{\sigma_D} = 0 \Rightarrow \frac{\delta_{\sigma_i}}{\sigma_i} = \sqrt{\frac{1}{2N}}\left(1 + \frac{\sigma_D^2}{\sigma_i^2}\right).$$

$$\text{If } \sigma_D > \sigma_i, \qquad \text{then} \qquad N \propto \left(\frac{\sigma_D}{\sigma_i}\right)^4.$$

$$\text{If } \sigma_D < \sigma_i, \qquad \text{then} \qquad \frac{\delta_{\sigma_i}}{\sigma_i} \approx \sqrt{\frac{1}{2N}}.$$

$\Rightarrow$ Require $\sigma_D < \sigma_i$.

Large-N Limit

$$\frac{\delta_{\sigma_i}}{\sigma_i} = \left(\frac{\sigma_D}{\sigma_i}\right)^2 \frac{\delta_{\sigma_D}}{\sigma_D} = \frac{\sigma_D}{\sigma_i}\frac{\delta_{\sigma_D}}{\sigma_i}.$$

Good results can only be obtained if σ_D/σ_i is less than one.

If this ratio is much less than one very good results are possible.

Maximum Acceptable Detector Resolution

$$\text{Suppose} \;\; \frac{\delta_{\sigma_D}}{\sigma_D} < 0.2.$$

$$\text{Then} \;\; \sigma_D < \sqrt{\frac{\delta_{\sigma_i}/\sigma_i}{\delta_{\sigma_D}/\sigma_D}}\sigma_i = 0.19\sigma_i.$$

If $\dfrac{\delta_{\sigma_D}}{\sigma_D} < 0.01,$ then we can have $\sigma_D \approx \sigma_i$ and $\dfrac{\delta_{\sigma_i}}{\sigma_i} = 0.01.$

Phase-space parameters of the FOFO-channel cooling experiment.

Table 1:

Parameter	Input Value	Output Value
P (MeV/c)	165	165
E (MeV)	198	198
γ	1.85	1.85
β	0.84	0.84
$\gamma\beta$	1.56	1.56
$\epsilon_{x,N} = \epsilon_{y,N}$ (π mm-mrad)	1200	600
$\epsilon_x = \epsilon_y$ (π mm-mrad)	769	385
$\sigma_x = \sigma_y$ (mm)	10	10
$\sigma_{x'} = \sigma_{y'}$ (mrad)	77	39
σ_P/P	0.03	0.04
$\sigma_E/E = \beta^2\sigma_P/P$	0.021	0.028
σ_t (cm)	1	1.2
$\sigma_t = \sigma_t/\beta c$ (ps)	40	48

Required detector resolution to achieve measurement accuracy of 1% on the rms widths σ_i, assuming the detector resolution function is known to 20%, *i.e.*, $\delta_{\sigma_D}/\sigma_D = 0.2$.

Table 2:

Parameter	Value
$\sigma_{x,D} = \sigma_{y,D}$	2 mm
$\sigma_{x',D} = \sigma_{y',D}$	8 mrad
$\sigma_{P,D}/P$	0.006
[$\Rightarrow \sigma_{x',D}$	6 mrad]
$\sigma_{z,D}$	2 mm
$\sigma_{t,D}$	8 ps

The required detector resolution σ_D varies as the reciprocal of the square root of the uncertainty δ_{σ_D} in the resolution.

The requirement on the momentum resolution $\sigma_{P,D}/\sigma_P$ leads to a second requirement on the angular resolution $\sigma_{x',D}$.