Compression of Beam Energy Via Off-Axis Traversal of an RF Cavity

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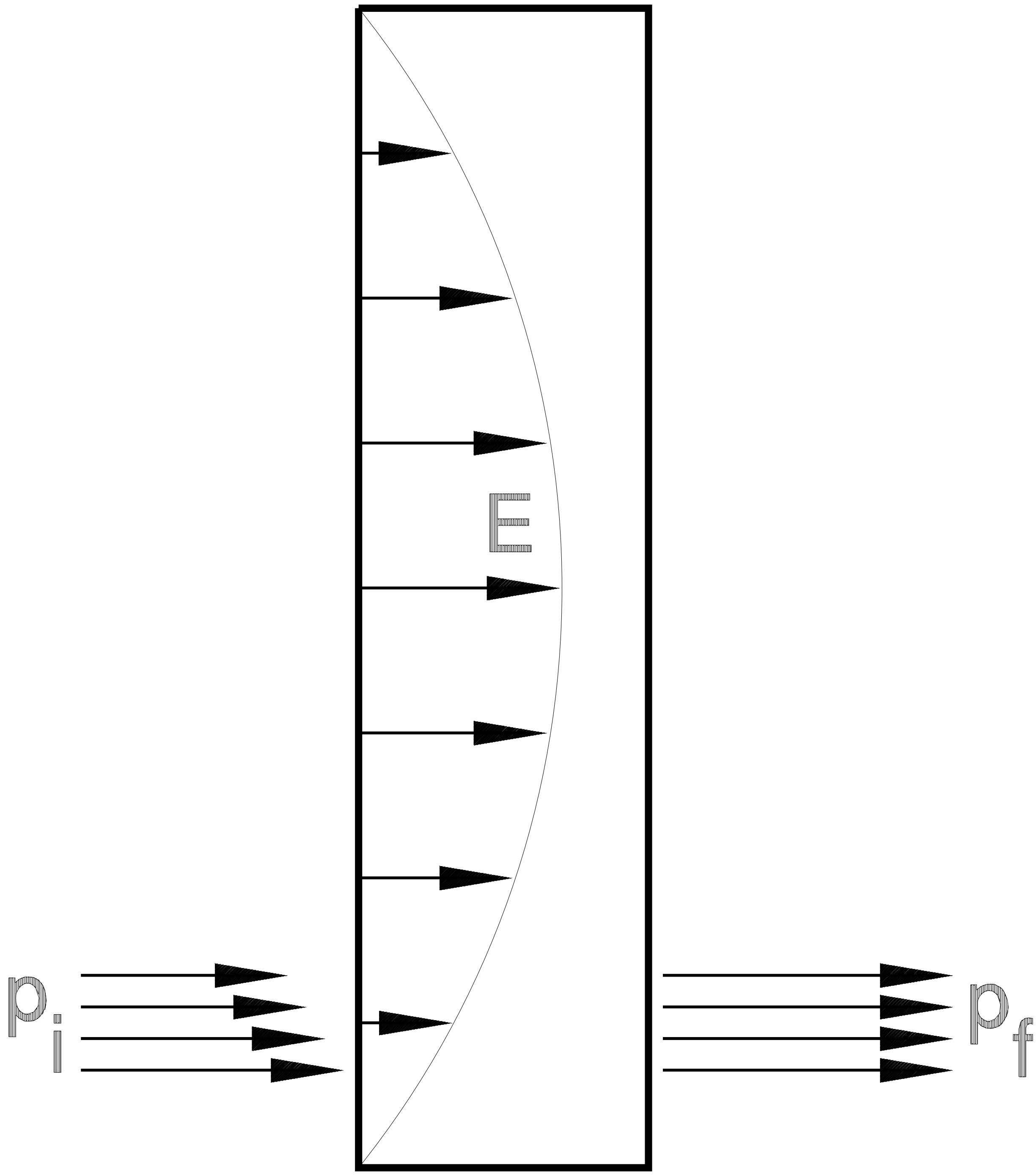
The Basic Idea



The muon pass through the cavity wall at a momentum dispersion point well off the cavity axis.

Details

Momentum dispersion at $z = 0$ along beam:

$$p = p_0 + k(x - \bar{x}),$$

Cavity of size (a,a,b) centered at (0,0,0).

$\mathrm{TE}_{1,1,0}$ mode (cgs units):

$$E_x = E_y = 0,$$

$$E_z = E_0 \cos\frac{\pi x}{a} \cos\frac{\pi y}{a} \cos\omega t,$$

$$B_x = \frac{c}{\omega}\frac{\pi}{a} E_0 \cos\frac{\pi x}{a} \sin\frac{\pi y}{a} \sin\omega t,$$

$$B_y = \frac{c}{\omega}\frac{\pi}{a} E_0 \sin\frac{\pi x}{a} \cos\frac{\pi y}{a} \sin\omega t,$$

$$B_z = 0.$$

Energy Gain

Typical particle trajectory:

$$x = x_0 + \beta_x ct,$$

$$y = y_0 + \beta_y ct,$$

$$z = z_0 + \beta_z ct.$$

Particle is within cavity during

$$[t_{min}, t_{max}] = \left[-\frac{b}{2\beta_z c} - \frac{z_0}{\beta_z c}, \frac{b}{2\beta_z c} - \frac{z_0}{\beta_z c} \right].$$

$$\begin{aligned}
\Delta U &= e\beta_z c \int_{t_{min}}^{t_{max}} E_z dt \\
&= e\beta_z c E_0 \int_{t_{min}}^{t_{max}} \cos\frac{\pi(x_0 + \beta_x ct)}{a} \cos\frac{\pi(y_0 + \beta_y ct)}{a} \cos\omega t \, dt \\
&\approx \frac{\pi e \beta_z c E_0}{a} \int_{t_{min}}^{t_{max}} (a/2 - x_0 - \beta_x ct) \cos\omega t \, dt \\
&\approx \pi e E_0 \frac{b}{a} \left(\frac{a}{2} - x_0 + z_0 \frac{\beta_x}{\beta_z} \right),
\end{aligned}$$

supposing $y_0 \ll a$, $z_0 \ll b$ but $a/2 - x_0 \ll a/2$,

while $\omega t \ll 1$, $\beta_x \ll \beta_z$ and $\beta_y \ll \beta_z$.

Energy Compression

$$U_i = \sqrt{p^2c^2 + m^2c^4} = \sqrt{[p_0 + k(x_0 - \bar{x})]^2c^2 + m^2c^4}$$

$$\approx U_0\sqrt{1 + 2k(x_0 - \bar{x})\frac{p_0c^2}{U_0^2}} \approx U_0 + k\beta_0c(x_0 - \bar{x}),$$

where $U_0 = \sqrt{p_0^2c^2 + m^2c^4}$, $\beta_0 = p_0c/U_0$ and $\Delta p/p_0 \ll 1$.

$$U_f = U_0 + k\beta_0c(x_0 - \bar{x}) + \pi e E_0\frac{b}{a}\left(\frac{a}{2} - x_0 + z_0\frac{\beta_x}{\beta_z}\right).$$

Choose $\qquad E_0 = \frac{a}{b}\frac{k\beta_0c}{\pi e}, \qquad$ then

$$U_f = U_i + \Delta U = U_0 + \pi e E_0\frac{b}{a}\left(\frac{a}{2} - \bar{x} + z_0\frac{\beta_x}{\beta_z}\right).$$

The term $z_0\beta_x/\beta_z$ is small.

Transverse Kicks

$$\begin{aligned}\Delta p_x &= \int_{t_{min}}^{t_{max}} F_x dt = -e\beta_z c \int_{t_{min}}^{t_{max}} B_y dt \\ &= -\frac{\pi c}{a\omega} e\beta_z c \int_{t_{min}}^{t_{max}} \sin\frac{\pi(x_0+\beta_x ct)}{a} \cos\frac{\pi(y_0+\beta_y ct)}{a} \sin\omega t\, dt \\ &\approx -\frac{\pi c}{a\omega} e\beta_z c \int_{t_{min}}^{t_{max}} \sin\omega t\, dt \approx -\pi e E_0 \frac{b}{a}\frac{z_0}{\beta_z c},\end{aligned}$$

Since $\quad \Delta p_z = \dfrac{\Delta U}{\beta_z c}, \quad$ we have $\quad \dfrac{\Delta p_x}{\Delta p_z} \approx \dfrac{z_0}{a/2 - x_0}.$

Troublesome unless $z_0 \ll a/2 - x_0$.

$$\begin{aligned}\Delta p_y &= \int_{t_{min}}^{t_{max}} F_y dt = e\beta_z c \int_{t_{min}}^{t_{max}} B_x dt \\ &= \frac{\pi c}{a\omega} e\beta_z c \int_{t_{min}}^{t_{max}} \cos\frac{\pi(x_0+\beta_x ct)}{a} \sin\frac{\pi(y_0+\beta_y ct)}{a} \sin\omega t\, dt \\ &\approx 0.\end{aligned}$$