Princeton/$\mu\mu$/99-20
MUCOOL-39
Aug. 10, 1999

An Emittance Diagnostic Channel for R&D on the Front End of a Muon Collider/Neutrino Factory

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Abstract

Using the analytic estimates developed for the emittance diagnostic channel for the final-stage cooling experiment, we conclude that diagnostics for both an initial-stage cooling experiment, and for the targetry experiment could be performed with a bent solenoid of 1.25 T field strength. The solenoid bore would need to be at least 40 cm for either study The targetry study is less demanding in accuracy of momentum analysis, and could be accomplished with a smaller bend angle to the solenoid, 0.1 rad, compared to 0.25 rad as appears more appropriate to an initial cooling study. The small bean angle permits a relatively large bend radius, which in turn implies that a relatively weak guiding dipole field will suffice. That guiding dipole field could be provided by tilting the coils in the bend region, which would commit to a single central momentum. The cost of such magnet systems is estimated to be $1M, including the straight sections needed for field uniformity over the TPC's. The cost of the magnets is about half that for a diagnostic channel of final-stage cooling. Some brief remarks about instrumentation are given.

1 Introduction

A bent solenoid channel with low-pressure time projection chambers and particle identification and timing via Čerenkov radiation has been proposed both for characterization of the pion yield in the muon collider targetry R&D program [1], and for the determination of the 6-d emittance in the muon collider cooling R&D program [2, 3]. The two bent solenoids, however, differed in magnetic field strength and bore, in view of the different requirements of the two programs as initially conceived. The targetry program is concerned with the early history of the π/μ beam during which the phase volume is large, favoring a large, low-field solenoid channel. The cooling R&D program presently emphasizes the final stages of cooling for a muon collider at which the phase volume is smaller, so a smaller, but higher-field solenoid channel is appropriate.

The recently increased interest in a Neutrino Factory based on a muon storage ring [4, 5], both as a neutrino-physics tool and as a step towards a Muon Collider, suggests that the cooling R&D program should also study the initial stage of cooling, which is relevant to both a Neutrino Factory and a Muon Collider. In this case, the muon phase volume is still essentially that of the π/μ beam captured from the primary target, and the diagnostic channels for the targetry R&D program and a front-end cooling R&D program could be nearly identical.

While the concept of the initial cooling stage for a neutrino factory is still undergoing rapid development [6, 7], the parameters of the muon beam at the entrance to the cooling channel (after the phase rotation) are approximately as given in Table 1. The normalized transverse emittance, $\epsilon_{x,N}$, would be $15,000\pi$ were it not for "mini-cooling" by 3 m of liquid hydrogen in the phase-rotation channel.

Table 1: Phase-space parameters of the muon beam at the beginning of the cooling channel at a Neutrino Factory.

Parameter	Value
P_0 (MeV/c)	185
E_0 (MeV)	198
γ	2.02
β	0.87
$\gamma\beta$	1.76
$\epsilon_{x,N} = \epsilon_{y,N}$ (π mm-mrad)	9,000
$\epsilon_x = \epsilon_y$ (π mm-mrad)	5,100
$\beta^\star$ (cm) [typical]	63
$\sigma_x = \sigma_y$ (mm)	57
$\sigma_{x'} = \sigma_{y'}$ (mrad)	90
σ_P/P	0.10
$\sigma_E/E = \beta^2\sigma_P/P$	0.076
σ_z (cm)	10
$\sigma_t = \sigma_z/\beta c$ (ps)	340

In the targetry R&D program, we will study the pion beam prior to the phase rotation, so the momentum spread is much larger than that given in Table 1. Indeed, we would like to characterize the pion yield over the momentum interval $50 < P < 400$ MeV/c. We define the central momentum to be $P_0 = 185$ MeV/c for consistency with the cooling-study parameters. Then, we wish to analyze the momentum bite around this with $\Delta P/P_0 \approx 1$.

In the rest of this note, we use the analytic formulae developed in [3] to determine the parameters and performance of a bent-solenoid diagnostic channel suitable for both targetry and a front-end cooling R&D programs.

2 Parameters of the Bent Solenoid Channel

2.1 Magnetic Field Strength

The magnetic field strength of the solenoid channel is chosen to be $B_s = 1.25$ T, so the bent solenoid channel is matched to the 1.25-T solenoid that contains the 70-MHz rf cavity in the targetry R&D program.

The Larmor period of a muon or pion in the solenoid is given by eq. (21) of [3],

$$\lambda_B \text{ [m]} = \frac{2\pi\beta_z c}{\Omega_B} = \frac{2\pi P_z}{eB_s} \approx \frac{2\pi P}{eB_s} = \frac{2\pi \times 10^6}{3 \times 10^8} \frac{P \text{ [MeV/}c\text{]}}{B_s \text{ [T]}} = 2\pi \frac{P \text{ [MeV/}c\text{]}}{300 B_s \text{ [T]}}, \tag{1}$$

and the radius of curvature of the helical trajectory is given by eq. (22),

$$R_{\text{curv}} \text{ [m]} = \frac{P_\perp}{eB_s} = \frac{P}{eB_s}\sin\theta = \frac{P \text{ [MeV/}c\text{]}}{300 B_s \text{ [T]}}\sin\theta. \tag{2}$$

For example, 185-MeV/c muons in a 1.25-T field have Larmor period $\lambda_B = 3.1$ m. The helix radius of curvature is $R_{\text{curv}} = 49\sin\theta$ cm. Then, $R_{\text{curv}} = 15$ cm for muons with $\theta_{\max} \approx 0.3$,

2.2 Solenoid Radius, Bend Angle, and Bend Radius

2.2.1 Constraints from the Targetry Program

The pions are produced in a target at radii close to the axis of capture solenoid (which implies that their initial canonical angular momentum is near zero [9]). Our goal is to capture all pions with transverse momentum $P_\perp < 225$ MeV/c, for longitudinal momenta where the rate is large, roughly $50 < P_z < 400$ MeV/c.

The field around the target is 20 T, which is reduced adiabatically by a factor of 16 to 1.25 T at the entrance to the first rf cavity in the decay/phase-rotation channel. The adiabatic invariant is the flux $B_s R_\perp^2$, where $R_\perp = P_\perp/eB_s$, so the invariant can also be written as $P_\perp^2/B_s$. Hence, as B_s decrease by a factor of 16, $P_\perp$ decreases by a factor of 4.

Thus, our goal of capturing all pions with $P_\perp < 225$ MeV/c at the target can be realized by transporting all pions with $P_\perp < 56$ MeV/c through the first rf cavity. The radius of the largest helix of such pions is $R_\perp = P_\perp/eB_s = 56/(300)(125) = 0.15$ m. The canonical angular momentum is still low, so the helices are still nearly tangent to the magnetic axis, and the maximum excursion of the pion from the magnetic axis is roughly $2R_\perp = 30$ cm.

The iris of the 70-MHz rf cavity in the targetry program will have 30 cm radius, so the radius of the bent solenoid channel must be at least this large.

The bent solenoid will disperse the beam "vertically," according to eq. (29) of [3], by amount,

$$\Delta y_G \approx \frac{P_0}{eB_s}\frac{\Delta P}{P_0}\theta_{\text{bend}}, \tag{3}$$

where subscript G refers to the guiding ray of the helical trajectory. For $P_0 = 185$ MeV/c, $B_s = 1.25$ T, and $\Delta P/P_0 = 1$, we have $\Delta y_G = 49\theta_{\text{bend}}$ cm.

We chose $\theta_{\text{bend}} = 1$ rad in the final-stage cooling diagnostic channel, but that choice would lead to extremely large vertical dispersion of the pion beam near the target.

To keep the vertical dispersion to, say, only 5 cm, we would need $\theta_{\rm bend} = 0.1$ rad.

We anticipate that the TPC readout cards occupy 5 cm radially beyond the active region. Hence, for an active radius of 45 cm, the solenoid need an actual radius of $R_s = 50$ cm.

Can we perform adequate momentum analysis with such small dispersion?

We don't need precision momentum analysis in the targetry study (unlike the case of a cooling study). Suppose we require only 10% accuracy, *i.e.*, $\sigma_{P,D}/P = 0.1$ for $50 < P < 400$ MeV/c.

Multiple scattering limits the resolution at low momentum. Following eq. (41) of [3], the number of radiation lengths in the TPC gas must obey,

$$N_X = \frac{L_{\rm tracking}}{X_0} < \left(\frac{(\sigma_{P,D}/P)P\beta\theta_{\rm bend}}{13.6 \text{ MeV}/c}\right)^2 . \tag{4}$$

For a 50-MeV/c muon, $\beta = v/c = 0.43$, so for $(\sigma_{P,D}/P = 0.1$, we need $N_X < 0.025$ (compared to 0.0002 for the final-stage cooling study). Even a heavy gas such as isobutane has on 0.006 radiation lengths per meter at atmospheric pressure. Hence, multiple scattering in the chamber gas is not an issue in the targetry experiment – if we are content with 10% momentum resolution.

Equation (35) of [3] gave an estimate of the momentum resolution that could be achieved using a pair of low-pressure (0.01 atm.) TPC's of length L surrounding the bent solenoid. This can be rewritten as,

$$L_{\rm tracking} = \left(\frac{1}{\theta_{\rm bend}}\frac{P}{eB_s}\frac{\sigma_{x,D}}{\sigma_{P,D}/P}\sqrt{\frac{720}{n}}\right)^{0.4} . \tag{5}$$

In [3] we considered a low-pressure TPC such that the number of ionization clusters was 33/m. Here, we normalize to that pressure, and consider $P = 400$ MeV/s as the most demanding case, to find that,

$$L_{\rm tracking} = \left(\frac{(400)(0.0002)}{(0.1)(300)(1.25)(0.1}\sqrt{\frac{720}{33}}\right)^{0.4}\left(\frac{33}{n}\right)^{0.2} = 0.40 \text{ m}\left(\frac{33}{n}\right)^{0.2}, \tag{6}$$

assuming that the transverse spatial resolution is $\sigma_{x,D} = 200$ μm. To reduce the tracking length from 40 to 20 cm requires increasing the ionization density, and hence the gas pressure, by a factor of $2^5 = 32$. For methane gas, the pressure would then be about 1/3 atmosphere. The chamber voltage would increase by a factor of 16 to about 10 kV, which may still be acceptable.

In sum, the requirements of the targetry R& D program could be met with a bent-solenoid channel with $R_s = 40$ cm and $\theta_{\rm bend} = 0.1$ rad.

2.2.2 Constraints from the Cooling Program

As was shown in [3], a demanding constraint on the emittance-diagnostic system is the timing measurement. We adopted the goal that the measurement resolution be 0.2 of the rms size of each of the 6 projections of the 6-d phase volume. Then, according to Table 1 for the initial cooling stage, we desire $\sigma_t, D \approx 70$ ps (compared to 8 ps for a diagnostic of the final cooling stage).

It was also noted that the timing requirement induces a requirement on the momentum measurement in that the time of the nonrelativistic muons must be extrapolated from the position of the timing device to the entrance or exit of the cooling apparatus. In eq. (16) of [3] we deduced that the timing uncertainty δt induced by a momentum uncertainty δP over a path length L is,

$$\delta t \approx 1000 \text{ [ps]} \left[\frac{L}{1 \text{ m}}\right] \frac{\delta P}{P}. \tag{7}$$

For example, if we desire that δt be only 40 ps over a 4-m path, then we must have $\sigma_{P,D}/P = 0.01$. This is about 7 times less demanding than the corresponding requirement for the final-stage cooling study.

If we accept that $\theta_{\text{bend}} = 0.25$ rad as suggested above, and take the transverse position resolution of the TPC to be 200 μm when operated at a pressure such that $n = 33$ clusters/m, we find that,

$$L_{\text{tracking}} = \left(\frac{1}{0.25}\frac{185}{(300)(1.25)}\frac{0.0002}{0.01}\sqrt{\frac{720}{33}}\right)^{0.4} = 0.51 \text{ m}. \tag{8}$$

This is very similar to the length of 45 cm as appears appropriate for the final-stage cooling diagnostic.

The radius needed for the bent solenoid channel of a study of initial cooling will then be that needed to contain the initial beam. If the beam has been transported from the first rf cavity of the decay/phase-rotation channel continuously inside a 1.25 T solenoid, the transverse size of the beam should be unchanged. That is, the beam should still fit within an aperture of 30 cm radius, as discuss in the previous section.

This conclusion is similar to that obtained when considering the transport of a beam of geometrical transverse emittance $\epsilon_x = \epsilon_y = 5,100\pi$ mm-mrad in a 1.25 T solenoid, but ignoring the issue of canonical angular momentum. As discussed in sec. 2.6.2 of [3], an estimate of the rms value of the largest radius on each helical trajectory is,

$$\sigma_{R,\max} = 2\sqrt{\epsilon_x \beta^\star} = 2\sqrt{\frac{\epsilon_x P_0}{eB_s}}, \tag{9}$$

where $\beta^\star = P_0/eB_s$ is the betatron function for a solenoid (sec. 5 of [9]). Using the values in Table 1, this method of estimation yields $\sigma_{R,\max} = 2\sqrt{(.0051)(185)/(300)(1.25)} = 0.10$ m. The 3-σ aperture would then be 30 cm.

Because the momentum spread in the initial cooling stage is much smaller than at the target, the dispersion of the beam in the bent solenoid is small. To transport the beam at the 3-σ level, we consider $\Delta P/P = 0.3$, so eq. (3) gives $\Delta y_G = 3.7$ cm for $\theta_{\text{bend}} = 0.25$ rad.

Again, we must add 5 cm radial space for the TPC readout electronics, so that $R_s = 30 + 4 + 5 \approx 40$ cm will be required for the initial cooling study, as well as for the targetry experiment.

2.3 Bend Radius and Guiding Dipole Field

The guiding dipole field needed to cancel the curvature drift at momentum P_0 was given in eq. (27) of [3] as,

$$B_G \text{ [T]} = \frac{P_0}{eR_{\text{bend}}} = \frac{P_0 \text{ [MeV}/c\text{]}}{300eR_{\text{bend}} \text{ [m]}} . \qquad (10)$$

The length of the bend of the solenoid is, of course, $L_{\text{bend}} = R_{\text{bend}}\theta_{\text{bend}}$. For example, if $\theta_{\text{bend}} = 0.1$ rad, as considered for the targetry experiment, a value of $R_{\text{bend}} = 5$ m leads to ${}_{\text{bend}} = 50$ cm. Then, for $P_0 = 185$ MeV/c, the strength of the guiding dipole would be $B_G = 0.1$ T.

It might be practical to generate such a relatively small guiding dipole field by tilting the solenoid coils. In this example, a tilt of $0.1/1.25 = 0.08$ rad of the coils in the bend of the solenoid would generate the desired dipole field.

For the initial-cooling study, we have consider a bend angle of 0.25 rad, so a bend radius of 5 m would lead to a 1.25 m length of the bend of the solenoid, which may be unnecessarily costly. Instead, with, say, $R_{\text{bend}} = 3$ m, we would have $L_{\text{bend}} = 0.75$ m, and we require a guiding dipole field of $B_G = 0.164$ T. The coils would have to be tilted by 132 mrad in this case.

Recall that if the guiding dipole field is generated by tilting the coils, the central momentum of the channel cannot be varied independently of the solenoid field. This is less of a concern for the targetry study than for the initial-cooling study.

2.4 Cost Estimates

In [3], we cast Mike Green's magnet-cost estimate [8] into the form

$$\text{Cost [M\$]} \approx 0.82(B_s \text{ [T] } R_s \text{ [m]})^{1.32}(L_s \text{ [m]})^{0.66}. \qquad (11)$$

In the limit that we can ignore the effect of dispersion on the solenoid radius, we see that the quantity $B_s R_s^2$ depends only on the central momentum of the beam. Since Mike Green's cost formula is a function of $B_s R_s$, we infer that the cost of the bent solenoid channel would vary as $(1/R_s)^{1.32}$, which favors the use of low-field solenoids of large radius.

That is, raising the solenoid field to reduce the solenoid radius needed to confine the beam is not cost effective. The solenoid radius, $R_s = 65$ cm that we find above is large, but appears to be a reasonable choice.

A consequence of the large radius of the solenoid channel is that the straight sections that surround the TPC's must be rather long to achieve reasonably uniform fields. I estimate that we must add at least $2R_s$ on either side of the 0.5-m-long TPC's.

In the proposed parameters of the initial cooling study, we have $R_s = 0.4$ m, so that each straight section is 2.1 m long. The bent solenoid itself has a length $R_{\text{bend}}\theta_{\text{bend}} \approx 3{\cdot}0.25 = 0.75$ m. Thus, the total length of a bent-solenoid channel is $L = 2 \cdot 2.1 + 0.7 = 4.95$ m.

Using these values in the cost formulae (11), we estimate that,

$$\text{Cost} \approx 0.82(1.25 \cdot 0.4)^{1.32}(4.95)^{0.66} = \$0.95\text{M}. \qquad (12)$$

Table 2: Parameters for the 1.25-T bent solenoid channels.

Parameter	Targetry Channel	Cooling Channel
P_0	185 MeV/c	185 MeV/c
σ_P/P_0	0.3	0.1
B_s	1.25 T	1.25 T
λ_B	3.1 m	3.1 m
$\theta_{\rm bend}$	0.1 rad	0.25 rad
$R_{\rm bend}$	5 m	3 m
$B_{\rm Guide}$	0.10 T	0.16 T
R_s	40 cm	40 cm
L_s	2.6 m	4.95 m
Cost (for one bend)	0.6M$	0.9M$
$\beta^\star = P_0/eB_s$	49 cm	49 cm
ϵ_x	–	$5,100\pi$ mm-mrad
$\sigma_x = \sigma_y = \sqrt{\epsilon_x \beta^\star}$	–	50 mm
$\sigma_{x'} = \sigma_{y'}$	–	102 mrad
$L_{\rm tracking}$	50 cm	50 cm
n	33 clusters/m	33 clusters/m

The corresponding estimate for the final-stage cooling study, where $B_s = 3$ T, $R_s = 0.3$ m, $\theta_{\rm bend} = 1$ rad, $R_{\rm bend} = 1$ m, and $L = 1 + 2(0.5 + 4 \cdot 0.3) = 4.4$ m is,

$$\text{Cost} \approx 0.82(3 \cdot 0.3)^{1.32}(4.4)^{0.66} = \$1.9\text{M}. \tag{13}$$

[The cost for a 3-T channel for initial cooling would be higher than (13), because the larger emittance would require larger R_s and hence larger L.]

For the target experiment, we could place the first TPC inside the PEP-4 TPC magnet coil, so we would need only one straight section, of length 2.1 m. Adding the 0.5-m length of the 0.1-rad bend, the total length the bent-solenoid channel would then be 2.6 m, and the estimated cost is,

$$\text{Cost} \approx 0.82(1.25 \cdot 0.4)^{1.32}(2.6)^{0.66} = \$0.62\text{M}. \tag{14}$$

If a initial second straight section of $R_s = 0.4$ m were required, the length would be 4.7 m, and the cost estimate is $0.9M.

2.5 Parameters of the Bent Solenoid Channel

Table 2 summarizes the parameters of the bent solenoid channels for targetry and initial-stage cooling diagnostics, based on the discussion above.

The effective $\beta^\star$ of a solenoid channel is P_z/eB_s, as discussed in sec. 5 of [9].

3 Instrumentation

3.1 Time Projection Chambers

As has been implied above, we continue to propose the use of low-pressure TPC's to track the particle trajectories before and after the bent solenoid. Although the less momentum accuracy is required in initial-stage studies, the use of a 1.25 T magnetic field leads to chamber parameters very similar to those for final-stage cooling studies: 50 cm tracking length, and 33 clusters/m ionization density.

The chamber radius is now 40 cm, compared to only 10 cm for the final-stage cooling case. If we keep the channel count at 1250 per TPC, the pad width is now 20 mm. Laboratory studies need to be performed to verify that the nominal spatial resolution of $\sigma_x = 200\ \mu$m can be achieved for such large pads. If not, the channel count will have to increase.

3.2 Timing

Timing measurements are not required in the measurement of the pion flux in the targetry experiment. However, timing remains critical to the cooling studies based on measurement of one muon at a time. The timing requirement for initial-cooling studies is $\sigma_t = 70$ ps, as discussed in sec. 2.2.2. This is well within the capability demonstrated with quartz bars viewed by fine-mesh photomultiplier tubes [10], and might well be achievable with scintillator, either fibers or bars.

3.3 Particle ID

The issues and solutions here are essentially the same for either initial- or final-stage cooling. For the targetry experiment, $\pi/\mu/K/p$ separation is not needed, π/e separation is highly desirable; also, there will be a significant number of soft protons ejected from target nuclei in the targetry study, so π/p separation is desirable in this case.

A threshold Čerenkov counter should be adequate for e/π separation, as mentioned in [1]. Low-momentum protons ionize much more heavily that do pions, so discrimination may be possible via cluster counting in the TPC. For example, at 400 MeV/c, protons ionize about 5 times as heavily as do pions, leading to an average of 75 and 15 clusters over a 50-cm track length, respectively, even at a (low) pressure where n = 33 clusters/m for minimum ionizing particles. Thus, π/p separation at greater than 5-σ will be possible in a low-pressure TPC. And, as noted in sec. 2.2.1, in the targetry study, the TPC could be operated at pressures as high as 1/3 atmosphere of methane if 10% momentum resolution is acceptable.

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