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A Study of the Feasibility of a Multi-Bev Circular Electron Accelerator

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Most of the consideration given to the application of strong focusing to particle accelerators has been limited to proton machines. While protons are in general the most versatile tools for the study of very high energy interactions, if several multi-Bev accelerators are to be built it might be desirable to have one capable of accelerating electrons and producing gamma rays. Among the reasons for such a machine might be the relative theoretical simplicity of analyzing photo meson experiments, the possibility of exploring electron scattering from nucleons, and the possible photoproduction of nucleon pairs. Although Panofsky is considering the possibility of extending the linear accelerator principle into the multi-Bev range, there has been general pessimism about the practicality of a multi-Bev circular electron machine due to the central acceleration radiation loss.

The purpose of the present preliminary study is to demonstrate that such a circular electron accelerator appears in fact practical up to an electron energy of 10.5 Bev, using existing R.F. tubes at conservative power ratings and convenient frequencies. The only differences between an electron machine and proton machine of about the same energy are the R. F. and injection systems. It has been pointed out that by modifying or adding some R. F. cavities and using a different injector, an electron machine such as herein discussed could be used for protons or vice versa.

Before considering a machine in detail, a few general facts should be reviewed. The most general use of any very high energy machine is to produce particles, therefore the energy available in the center of mass system (considering a proton target) and not in the lab is the figure of interest.

For protons on protons,

$$E_{c.m.} = 2(1+\tfrac{1}{2}E_{LAB})^{\frac{1}{2}} - 2$$

where $E_{c.m.}$ and E_{lab} are the kinetic energies available in the center of mass system and in the lab in units of proton rest masses respectively.

For photons on protons,

$$E_{c.m.} = (2E_\gamma + 1)^{\frac{1}{2}} - 1$$

Where E_γ is the lab energy of the incident quanta in units of proton rest masses.

These two expressions are plotted figure 1 for lab energies to 100 Bev.

The two-nucleon threshold in the center of mass system is 1.86 Bev. For protons or protons, this requires 5.6 Bev in the lab, and for photons on protons 3.72 Bev. A 10.5 Bev electron machine is equivalent to a 14.3 Bev proton machine in the center of mass. Both give a center of mass energy of 3.6 Bev. A 50 Bev proton machine gives 8 Bev center of mass and a 100 Bev machine gives 11.8 Bev center of mass.

The radiation loss per turn of an electron in a circular machine determines the voltage the R.F. accelerating cavities must supply. In volts per turn, this is given as:

$$V = 716\, B^4 \rho^3$$

where B is the guide field in webers per square meter and ρ is the radius of curvature in meters. Some typical values for a 10,000 gauss field are given below:

E_{max} (Bev)	V($\frac{\text{Megavolts}}{\text{turn}}$)
1	.026
3	.716
6	5.75
8	13.5
10	6.5
13	58.4

For the machine discussed below where B_{max} = 6,800 gauss and E_{max} = 10.5 Bev, the energy loss per turn is 21 megavolts at the end of acceleration.

To illustrate the feasibility of a 10.5 Bev circular electron accelerator, the following numbers have been worked out. The determining parameters are the frequency and power limits of the R.F. tubes, and a number of R.F. stations comparable to the number considered in various proton accelerator designs.

PEAK R.F. REQUIREMENTS:

electron energy E_{max} = 10.5 Bev
maximum guide field B = 6.8 Kgauss
orbit radius of curvature ρ = $\begin{cases} 51.4 \text{ meters} \\ 169 \text{ feet} \end{cases}$
number of straight sections N = 64
number of R.F. accelerators N/2 = 32
length of straight sections ℓ = $\begin{cases} .915 \text{ meters} \\ 3 \text{ feet} \end{cases}$

frequency of rotation $= \dfrac{3 \times 10^{10}}{2\pi \times 5.14 \times 10^3 + 64 \times 91.5}$ = .785 Mc/sec.

peak radiation loss (volts per turn)	$\mathcal{E}_r = 21$ MV/turn
Rise time of magnetic field	T = 15 milliseconds
energy gain per turn at injection $\left(\frac{E_{MAX}}{15\times10^{-3} f_0}\right)$	$\mathcal{E}_a = .892$ MV/turn
Maximum volts per turn supplied to electrons	$\mathcal{E}_r + \mathcal{E}_a = \mathcal{E} = 21.9$ MV/turn
Peak R.F. cavity volts (assume phase angle of 45° near end of acceleration)	$\mathcal{E}_p = \frac{\mathcal{E}\sqrt{2}}{32} = .97$ MV/CAVITY
R.M.S. cavity volts	$\mathcal{E}_c = .684$ MV/CAVITY

R.F. CAVITY

harmonic order of R. F.	$h = 64$
R. F. frequency	$f = 50.3$ Mc/SEC.
R. F. wavelength	$\lambda = 596$ cm.

cavity dimensions: Re-entrant, or Rhumbatron" cavity.

from Terman (Handbook) § 3, P 34

gap spacing	d = 7.0 cm
peak voltage gradient in gap	V/d = 139 KV/CM
cavity outside radius	b = 107.5 cm
gap radius	a = 38 cm
cavity length	$2Z_0 = 84$ cm
Cavity Q	Q = 37,000
cavity shunt impedance	Z = 2.04 megohms
R.F. power dissapated per cavity $P = \frac{V_{RMS}^2}{Z}$	P = 0.23 M.W.
R. F. amplifier tube	RCA 2332
Total R.F. peak power $P_t = 32P$	$P_t = 7.34$ M.W.

ELECTRON LOADING OF CAVITIES

electrons per pulse	10^{11}	10^{12}	electrons
charge q	1.6×10^{-8}	1.6×10^{-7}	coulombs
current $i = qf_0$	.0126	.126	amperes
power to electrons per cavity $P = \frac{V_{tot}\, i}{32}$	.0122	.122	MW/CAVITY

Thus one could readily retain tractable control of the R.F. in the face of a 10^{12} electron load. The maximum beam pulses in existing electron machines are 10^{11} electrons per pulse.

INJECTION CONDITIONS

Injection from Stanford-type electron linear accelerator	$E_{INJ.} = 80$ Mev
Magnetic field at injection	$B_{INJ.} = 52$ gauss
field gradient $dR/R = \alpha\, \Delta P/P = (4.85/m)\frac{E_{MAX}/15\times10^{-3}}{E_{INJ.}}$	$m = 1000$ $dR/R = 5.4\times10^{-5}$; $dR = 0.28$ cm.
R.F. needed at injection with 30° phase angle,	$\mathcal{E} = 1.8$ MV/TURN
R.F. following turn-on $(t \ll \tau)$	$V = t/\tau\, V_{MAX}$
period of turn on $\tau = \frac{Q}{2\pi f}$	$\tau = 117\,\mu$ SECONDS

t to reach 1.8 MV/turn $t = V\tau$ $t = 6.8\ \mu$ seconds

number of turns in time t, $N = t f_0$ $N = 5.3$ turns

ΔR spiraling in of beam before capture $\Delta R = N\, dR$ $\Delta R = 1.5$ cm.

R.F. POWER REQUIREMENTS

R. F. energy for all cavities per cycle = $P_T\, T$ = 116 Kilojoules
D.C. energy per cycle, assuming 50% oscillator efficiency = 231 Kilojoules
Repetition rate of pulsing (continuous) = 16/second
D. C. power to oscillators = 231 x 16 = 3.7 megawatts
cost of storing 231 Kilojoules at $10 per KVA = $250,000

amplitude modulating the R.F. power to match the volts per turn required by the particles would reduce the above figure by about a factor of three.

MAGNET POWER REQUIREMENTS

magnet gap 6 cm x 10 cm

volume of air gap $V = 2\pi \times 5.14 \times 10^3 \times 60$ cm^3
stored energy in gap $E_{peak} = \frac{B^2}{8\pi} V \times 10^{-7}$ $E_{PEAK} = 0.356$ megajoules

total stored energy E = 0.713 megajoules $E_{TOTAL} = 0.713$ megajoules
(assuming an equal amount of leakage flux)

sinusoidal magnet excitation at 16 cps ω = 105 radians per second

volt amperes effective = $E\omega$ VA = 75×10^6 volt amperes
cost of condenser bank at $10 per KVA = $750,000
Q of magnet - condenser bank combination (as in present synchrotrons and betatrons): Q = 100
magnet power supplied by power source = $\frac{VA}{Q}$ P = 750 kilowatts

total power drain
R.F. = 3.7 megawatts
magnet = 0.75 "
total = 4.45 "

total cost of condenser banks to store energy
R.F. $250,000
magnet $750,000
total $1,000,000

In the tradition of existing electron machines we have assumed an A.C. magnet in a resonant circuit with a condenser bank. Since energy gain per turn due to field rate of rise is small here compared to radiation loss per turn, a faster rate of rise and high repetition rate can be had with no added R. F. cost. The higher beam rate of 10^{14} electrons per minute (assuming 10^{8} electrons per pulse) might be desirable in some experiments.

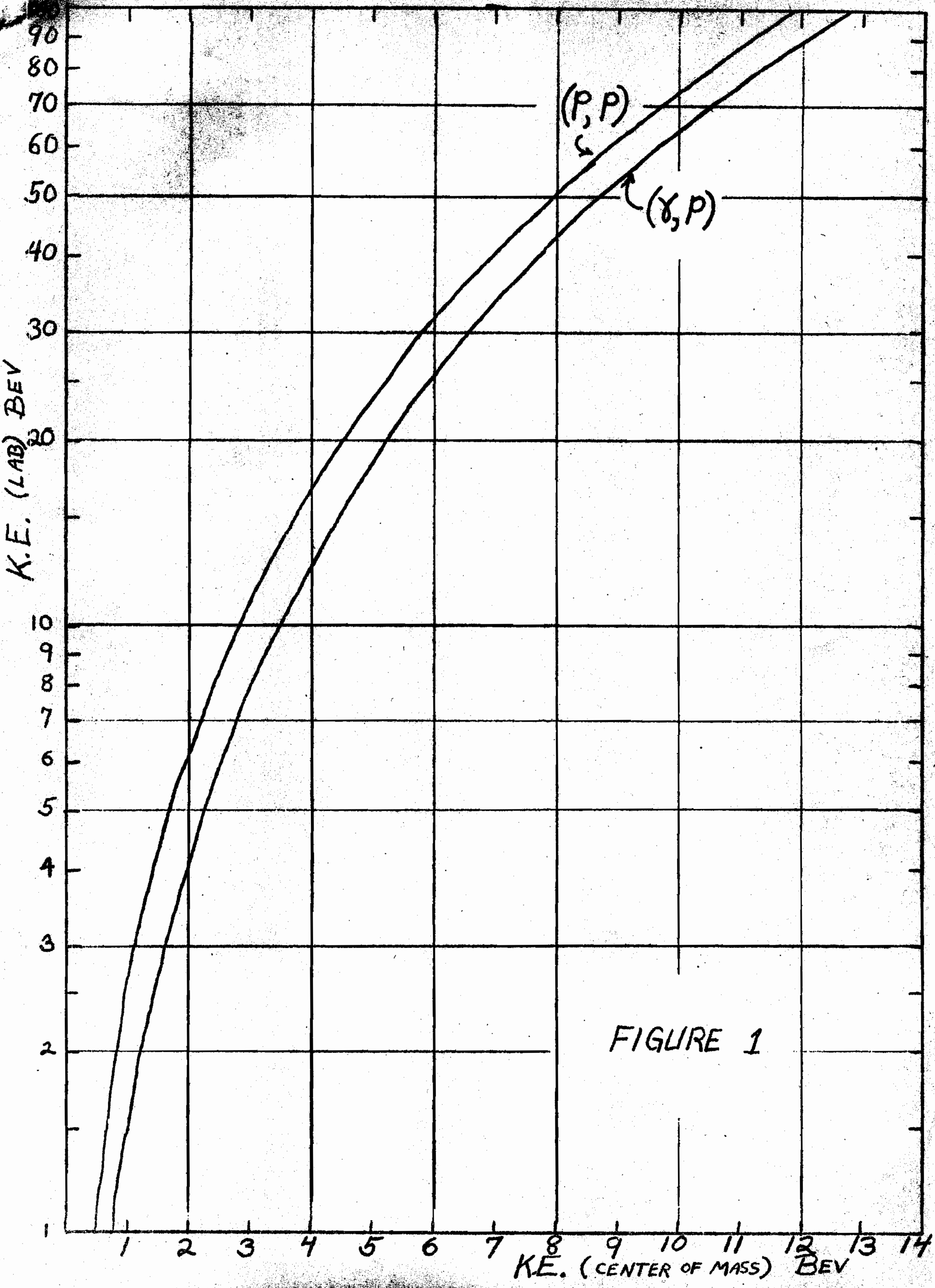
The question naturally arises as to comparison of a circular electron machine with an electron linear accelerator. Using the existing Stanford Klystrons, the stored energy for an electron linear would be 25. Kilojoules (at 18 megawatts per tube, 30% efficiency, 2 μsecond pulse length and 250 tubes to attain 10 Bev.)

With amplitude modulation, the above circular electron machine would require 75 Kilojoules stored energy for the R. F.

The problem of phasing the R. F. cavities in such a proposed machine is a serious problem to consider. The acceleration radiation is peaked at 40 kilovolts and is produced around the donut at a rate of 200kilowatts*peak. A summary of some technical advantages of an electron machine over a proton machine might be listed. No frequency modulation of the R. F. is required, the injection energy, even from a Van de Graaff, is above the transition energy, and the bremsstrahlung beam from the electrons comes out of the machine with an extremely small angle of divergence so that the experimental area could be located at a considerable distance from the machine. Pulsed electron currents of amperes are in use, so that beams of higher intensity than from proton machine are possible.

In view of the large building, overhead, and magnet costs, it seems that an electron machine of 10.5 Bev could be built for a cost comparable to that of a proton machine of the same center-of-mass energy. This energy is not a limit, technically or economically, for circular electron machines. Using more cavities, more tubes per cavity, etc., energies as high as 15 or 20 Bev might be considered.

*10^{11} electrons



100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
K.E. (LAB) BEV
1
2
3
4
5
6
7
8
9
10
11
12
13
14
K.E. (CENTER OF MASS) BEV
(P, P)
(γ, P)

FIGURE 1