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COLLIDING MUON BEAMS AT 90 GeV

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I. Introduction

Colliding beams ($p - p$, $e^+ - e^-$ beams) have been constructed in order to provide large center of mass energies for the observation of high energy reactions and for the production of new high energy particles. Since the colliding beam particle bunches must be circulated and recollided against each other an extremely large number of times ($10^6 - 10^{10}$) in order to obtain reaction rates comparable to fixed target experiments, only stable particles (p, $\bar{p}$, e^+, e^-) have usually been considered practical for colliding beams experiments. However, it is possible that some particularly significant reactions may best be observed using unstable particles in a colliding bunch. In this note we estimate the rate at which the neutral vector boson Z_o may be produced in the reaction:

$$\mu^+ + \mu^- \to Z_o \qquad (1)$$

using muon beams obtainable from existing or planned high energy proton accelerator facilities. The Z_o boson is predicted to have a mass of ~ 90 GeV by the Weinberg-Salam model ($\theta_W = 28°$).[1] Z_o production requires colliding muon beams of ~ 45 GeV/c momentum, and these can be obtained as tertiary beams from existing proton machines.

The cross section for reaction (1) has been estimated to be about 2×10^{-31} cm^2 with a width of 1.5 to 3 GeV.[2,3] In this note we use a width of 2 GeV and the above cross section to estimate the reaction rate. This reaction is physically equivalent to the electron-positron reaction

$$e^+ + e^- \to Z_o$$

to be observed in future very large storage rings (LEP). However, muons are not greatly affected by synchrotron radiation, which is the most significant design constraint for high energy electron storage rings.

The colliding beams design considered in this note is sketched in Figure 1. For 45 GeV/c muons, $\gamma = 426.$ and the laboratory muon lifetime is $\gamma\tau_0 = 0.94 \times 10^{-3}$ seconds. The muon beams can then be stored for about a millisecond and a storage ring is used in this design. A high energy high intensity proton beam is extracted from some proton accelerator and collided into a stationary target at A, producing secondary particles. A bending magnet system separates positive from negative particles for insertion into two beam lines, labeled π^+ and π^-. The insertion is designed so that the straight π^+, π^- beam lines receive 60 GeV/c particles, with as large a momentum acceptance width and transverse momentum acceptance as reasonable. The length of the beam lines is matched to the decay length of 60 GeV/c pions (2.5 km) so that most of the pions have decayed into muons before reaching the storage ring. The muons are inserted in the storage ring, which are set to circulate 45 GeV/c particles, and collisions between muons are observed at low-beta function insertions labeled B and B'.

The luminosity is given by the usual formula:

$$L = \frac{fN_+N_-}{A} \text{ cm}^{-2} \text{ sec}^{-1} \tag{2}$$

where f is the number of collisions per second, N^+ and N^- are the number of positive and negative muons in the colliding bunches and A is the beam transverse area in the colliding areas B and B'.

II. Muon Production

The number of muons in the colliding bunches can be estimated by the following formula:

$$N = a \left[\int_{o}^{\theta_{max}} P \, \frac{d^2N}{dpd\Omega} \, d\Omega \right]_{\substack{60 \text{ GeV/c} \\ \pi}} \cdot \left(\frac{\delta P}{P}\right)_{\substack{45 \text{ GeV/c} \\ \mu}} \tag{3}$$

$\frac{d^2N}{dpd\Omega}$ is the inclusive differential π-production, the number of pions produced per interacting proton per unit momentum per solid angle. We use the empirical formulae of C. L. Wang[4] to estimate this. These formulae are based on inclusive pion production data from 20 GeV/c to 1500 GeV/c incident proton momenta and are expected to be accurate within a factor of 2. The Wang formula is:

$$P_\pi \frac{d^2N}{dpd\Omega} = a \; P_\pi \; P_p \; X(1 - X) \; \exp(- \; BX^C - D \; P_\pi \, \theta) \tag{4}$$

where P_p and P_π are the incident proton and produced pion momenta, $X = P_\pi / P_p$, and $P_\pi \theta$ is the transverse pion momentum. For $\pi(\pi^+)$, $A = 1.572 \; (2.285)$, $B = 5.732 \; (3.558)$, $C = 1.333 \; (1.333)$, $D = 4.247 \; (4.727)$ with all momenta in terms of GeV/c.

Equation 4 must be integrated over the solid angle of acceptance for the separate beam lines; we approximate the acceptance as azimuthally symmetric and characterized by a maximum acceptance angle θ_{max}. With $\theta_{max} \cong 6$ milliradians about half of the produced pions are accepted and this is the acceptance used in this note. With this acceptance the integrated pion production is:

$$P \frac{dN}{dp} \approx .11 \; \pi^- \text{ and } .18 \; \pi^+ \text{ pions/interacting proton}$$

for incident 300 GeV/c protons and .23 π^- and .29 π^+ for 1 TeV/c protons.

The factor a in equation 2 is the product of several dilution factors appearing in the production of muon beams from decaying pions from interacting protons. One is a target efficiency which we set at .4, the "maximum target efficiency for an external target"[4] of one interaction length. By using this single interaction efficiency, we have ignored the possibility, previously noted by Frisch,[5] that the production of low momentum (60 GeV/c) π's can be greatly enhanced by hadron showers in targets of several interaction lengths, perhaps by factors of 3 to 10. Exploitation of this possibility requires a more careful target design not included in this note.

There is a second dilution factor due to the limited momentum acceptance of the π beam lines. Pions which produce 45 GeV/c muons will have initial momenta between 45 GeV/c and 85 GeV/c and only some fraction of these will be accepted by the beam line. If the momentum acceptance is ~ ± .10 only about .4 of the pions which would produce 45 GeV/c muons are accepted.

There is another dilution from undecayed pions. If the π-beam lines are ~ 1 mean decay length or ~ 2.5 km, only .6 of them will produce muons. Combining these three dilutions we find a $\cong$ 0.1.

For the momentum acceptance in equation 2, we will only use the momentum acceptance useable for production of Z_0 from colliding muons. Thus, for one of the separate beams we may use the momentum acceptance $\frac{\delta P}{P}$ of the storage ring, for which we will use $(\frac{\delta P}{P})_{ring} \cong .06$. For the other beam we must use the width of the Z_0 resonance (2 GeV/c) or $(\frac{\delta P}{P}) \cong .04$. The mean of these is ~ .05.

Combining the above numbers with an incident proton flux of 5×10^{13} particles, we produce 2.8×10^{10} μ^- and 4.5×10^{10} μ^+ with 300 GeV/c protons and 5.8×10^{10} μ^-, $7.3 \times 10^{10} \mu^+$ with 3 TeV/c protons.

The transverse emittance of the muon beams is set by the target acceptance and by the pion decay. The produced pion beam emittance is given by

$\varepsilon_\pi = r_T \, \theta_T$ with $\theta_T \cong 6.5 \times 10^{-3}$.

Pi decay produces muons with transverse momenta of ~ 20 MeV or $\frac{\delta P}{P} \cong 5 \times 10^{-4}$. The emittance contribution is

$$\varepsilon_{\mu} \cong r_b \cdot \frac{\delta P}{P}$$

where r_b is the beam radius at decay. If the target radius is less than about 1 mm and the pion beam radius through the transport is less than about 1 cm, then the produced muon beam will have an emittance of 5×10^{-6} meter radians. (The above radii are moderately conservative numbers.) We will use 5×10^{-6} m - R as a reference value for the magnitude of the transverse emittance.

III. Storage Ring Parameters

As noted above, 45 GeV/c muons can be stored for ~ 1 millisecond or ~ 300 km, and in the design conception outlined in this note the muons produced by pion decay in the π beam lines are inserted into a storage ring, which is adjusted to accept 45 GeV/c particles, which have a rigidity $B\rho$ of 150 T-m. If the storage ring has conventional 2 T magnets with a packing factor of 0.5, the storage ring radius is 150 m, and 4 T magnets with the same packing factor provide a 75 m radius. For this note we assume the total storage ring circumference is ~ 500 m which implies that each set of produced muon beam bunches can be stored to provide ~ 600 collisions. To calculate the collision rate f in equation 2, we multiply this by the rate of primary proton beam production (.17/sec for FNAL), then multiply by two for the use of two equivalent interaction regions (B and B' in Figure 1), and divide by two because of the reduced intensity due to muon decay in both colliding beams. Our resulting value for f is 10^2.

To achieve maximum luminosity, the muon beam bunch length L_{μ} must be less than the observable part of the interaction region, that is, less than about 10 m.

In fact, as noted below, a much shorter beam bunch would be desirable. For 45 GeV/c muons β = .999997 and individual particle motion $(\frac{\delta\beta}{\beta})$ with respect to the center of the bunch is limited to less than a few centimeters over the life time of the bunch. The muon beam bunch length remains that of the primary proton bunch. This implies that the protons in the main accelerator must be bunched to the length L_μ. This can be accomplished either by placing all the synchrotron protons in a single bunch of length L_μ or by setting the storage ring circumference at an integer fraction $1/(N_B \cdot n)$ of the proton synchrotron circumference, where N_B and n are integers, forming N_B evenly spaced bunches, each with length L_μ, and extracting them within a single turn. This second procedure may be more closely matched to synchrotron performance, but it will dilute the transverse emittance of the muon beam by a factor $\sqrt{N_B}$, if injection is Liouvillian.

The factor A in equation 2 is a function of the muon beam emittance ε_μ and the average value $\bar{\beta}$ of the storage ring β-function in the interaction region. This average value $\bar{\beta}$ is further constrained by the bunch length L_μ by $\bar{\beta} > \frac{L_\mu}{2}$. We have

$$A = \pi \overline{R_B^2} = \pi \varepsilon_\mu \bar{\beta} \geq \frac{\pi}{2} \varepsilon_\mu L_\mu$$

where r_B is the mean radius of the beam in the interaction region. In this note, we do not estimate an allowable value for the proton bunch length, but we simply assume that it can be made sufficiently small. That may be difficult, however.

To estimate an allowable value for $\bar{\beta}$, we assume that the low $\bar{\beta}$ region is produced by quadrupole doublets placed a few meters upstream and downstream of the interaction point, and we use the parameterized tables of A. A. Garren[7] to determine the allowable focus. With 4 T quadrupoles and $\varepsilon_\mu = 10^{-5}$ m-R, we find a value $\bar{\beta} = 0.2$ m is reasonable. This corresponds to a minimum radius of $r_B \cong 0.15$ cm or $A \cong 0.07$ cm^2. This particular combination is also reasonably well matched to the chromatic and geometric aberrations requirements of the beams without large correction elements. We use this for our reference value of A.

An important advantage of this decay line plus storage ring concept is that the colliding muon bunches will be relatively free of contamination by other particle species. The bending magnet system near A favors acceptance of of 60 GeV/c particles, and the storage ring accepts 45 GeV/c particles, so only particles whose momentum changes from ~ 60 GeV/c to ~ 45 GeV/c in passing from A to B are accepted; that is π's decaying to μ's. In the storage ring itself, any surviving π's and K's will be removed by particle decay within a few bunch turns; also heavier particles $(p,\bar{p})$ will be separated in time of flight from the muon bunches. (The velocity of 45 GeV/c protons (.9998 c) is significantly slower than the muon velocity.)

IV. Comments on Luminosity and Reaction Rate

Using the reference values of the parameters f, N_+, N_-, A obtained above, we obtain a luminosity of 1.9×10^{24} cm^{-2} sec^{-1} with 300 GeV/c protons and $6. \times 10^{24}$ cm^{-2} sec^{-1} with 1 TeV/c protons. With a cross-section of 2×10^{-31} cm^2 for $\mu^+ + \mu^- \rightarrow Z_0$, we have a Z_0 production rate of 1.2×10^{-6} sec^{-1} with the larger luminosity (1 TeV/c protons) or 0.11 per day. Z_0 production can be observed by monitoring the total cross-section $\mu^+ + \mu^- \rightarrow$ all as a function of the tuned momentum of the storage ring. Z_0 production will appear as a pronounced peak in the total cross-section, in fact, background events will be completely unobservable. The reaction rate quoted above is, however, about a factor of 10 smaller than that necessary for a practical experiment, so some significant improvement in luminosity is necessary. This may be possible, as noted above, if the π production target can be designed to accept pions produced in hadronic showers in a long target.

The reaction rate estimate of this report is based upon realistic estimates of accelerator parameters obtainable with existing accelerator facilities. However, this rate is dependent on a large number of parameters, none of which is precisely determined. In particular, as noted above, the inclusive 60 GeV/c pion production may be significantly underestimated. To facilitate estimates of the effects of variation of these parameters we have tabulated some of these parameters with the values used in this report in Table 1. It is possible that a more precise evaluation of these parameters may change this suggested concept from a marginal idea to a practical possibility.

In conclusion, we have investigated the possibility of the use of colliding muon beams to produce neutral vector bosons, and have found it to be difficult, but perhaps not impossible.

Table I. Parameters of Muon Colliding Beams

Parameter	Definition	Reference Values
f	Number of bunch collisions per second	100
N_p	Number of protons per machine cycle	5×10^{13}
$p \frac{dN}{dp}$	Integrated pion production: pions per unit ($\Delta p/p$) per interacting proton. (1 TeV/c protons) (300 GeV/c protons)	 $.23(\pi^-)$, $.29(\pi^+)$ $.11(\pi^-)$, $.18(\pi^+)$
$(\overline{\frac{\Delta p}{p}})$	Full width of muon momentum acceptance	.05
a	Muon production dilution factor	0.1
A	Muon bunch transverse area in interaction regions $A = \pi\varepsilon_\mu\bar{\beta}$ $\bar{\beta} \tilde{>} L_\mu/2$	$0.07\ \mathrm{cm}^2$
ε_μ	Transverse muon beam emittance	10^{-5} m-R
$\bar{\beta}$	Beta function in interaction regions	0.2 m
L_μ	Muon bunch length	< .4 m
σ	Cross-section for $\mu^+ \mu^- \to Z_o$	$2 \times 10^{-31}\ \mathrm{cm}^2$
Δ	Full width of Z_o resonance	2 GeV

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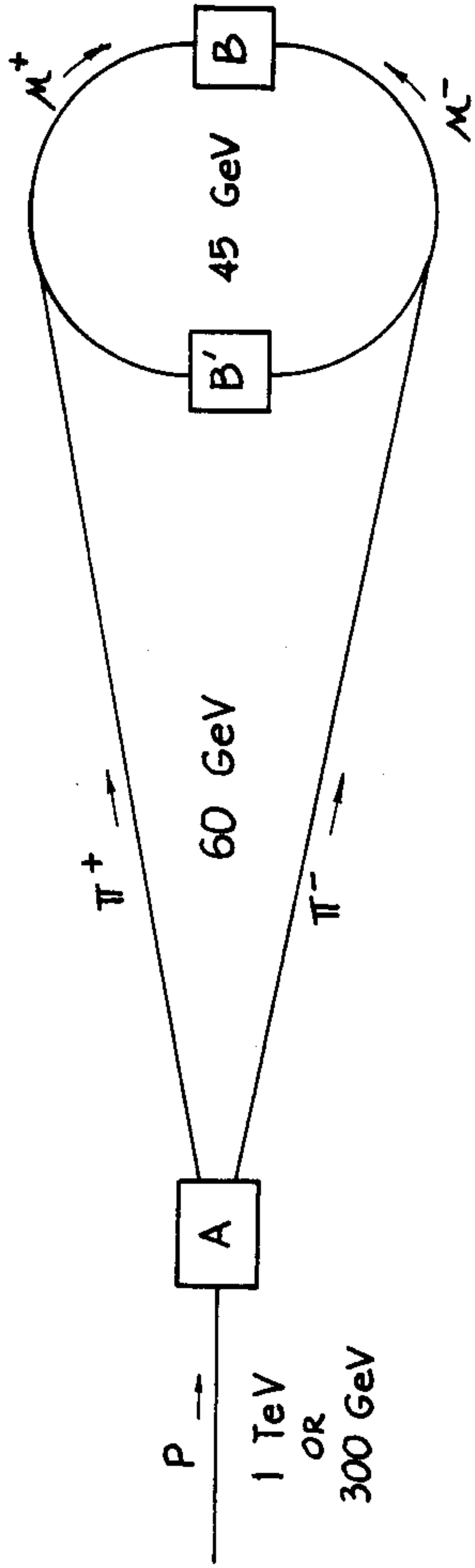
μ^+
B
μ^-
45 GeV
B'
π^+
60 GeV
π^-
A
P
1 TeV
OR
300 GeV