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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

μ^+–μ^- colliders: possibilities and challenges ☆

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The current status of the $\mu^+-\mu^-$ collider concept is reviewed and discussed. In a reference scenario, a high-intensity pulsed proton accelerator (of K-factory class) produces large numbers of secondary π's in a nuclear target, which produce muons by decay. The muons are collected and cooled (by "ionization cooling") to form high-intensity bunches that are accelerated to high-energy collisions. High-luminosity $\mu^+-\mu^-$ and μ^-–p colliders at TeV or higher energy scales may be possible. Challenges in implementing the scenario are described. Possible variations in muon production, accumulation, and collisions are discussed; further innovations and improvements are encouraged.

1. Introduction

Current and planned highest-energy colliders are hadronic proton–(anti)proton (p–p or p–$\bar{\text{p}}$) colliders or electron–positron (e^+–e^-) colliders, and both approaches have significant difficulties in extension to higher energies. Hadrons are composite objects; so only a small fraction of the total energy participates in a collision, and this fraction decreases with increasing energy. Also, production of new particle states is masked by a large background of nonresonant hadronic events; identification of new physics becomes increasingly difficult with increasing energy. Leptonic (e^+–e^-) colliders have had the advantage of providing simple, single-particle interactions with little background. However, extension to higher energies is limited in energy, luminosity, and resolution by radiative effects (synchrotron radiation in circular colliders, beam–beam radiation and pair creation in linear colliders). At very high energies, the collisions are no longer point-like, because of the radiative background.

However, this radiation scales inversely as the fourth power of the lepton mass. Thus, we can extend the high-quality features of e^+–e^- colliders to much higher energies by colliding higher-mass leptons such as muons. The μ^+–μ^- collider concept has been suggested, and is described in some detail in Refs. [1–4], and an example is displayed graphically in Fig. 1. In those initial concepts, a high-intensity multi-GeV hadron accelerator beam produces pions from a hadronic target, and muons are obtained from π-decay. The μ^+'s and μ^-'s are accumulated and cooled by ionization cooling, and then accelerated (in linacs and/or synchrotrons) to high energies for high-energy collisions in a storage ring. The process is repeated at a rate matched to the high-energy muon lifetime to obtain potentially high luminosities.

Since the initiation of the muon collider concept, some subsequent developments have increased interest in the possibility of muon colliders, and recent progress in related fields may increase their potential capabilities.

In high-energy physics (HEP), plans for the current generation of high-energy facilities are now reasonably well established. The next HEP devices are to be high-energy (8–20 TeV) p–p colliders (SSC and/or LHC), to be followed by an e^+–e^- linear collider (up to 0.5 TeV per beam). It is now possible to begin serious consideration of projects to follow these, such as a μ^+–μ^- collider. The current generation of HEP devices includes large components (injectors, tunnels, storage rings, linacs) and technology (high-gradient linacs, low-β^* optics, large-bandwidth stochastic cooling, etc.) which may be incorporated into a muon collider.

Development of high-intensity accelerator concepts (for kaon factories, or accelerator transmutation of waste, or tritium production, or for μ-catalyzed fusion) can also provide methods and facilities for improved μ production, collection and cooling. Some of the possible extensions are suggested below.

Recent studies of e^+–e^- linear colliders show that that approach appears to be limited in energy or luminosity at the TeV scale by beamstrahlung radiation effects. This calls for a "new paradigm" in extension to higher energies [5], possibly including a μ^+–μ^- collider option. Also, the next frontier in HEP appears to be in understanding the "Higgs sector", the generation of masses. The greater direct coupling of muons to the Higgs sector (by $(m_\mu/m_e)^2$) may provide an important incentive for devel-

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oping μ^+–μ^- colliders, possibly in the energy-specific form of a Higgs-resonance "factory".

In this paper we present an overview of possible high-energy μ^+–μ^- colliders for the μ^+–μ^- workshop (Napa, CA, 12/1992), following the concepts previously presented in the Refs. [1–4]. In this overview, we will identify some of the critical problems in implementing the scenario, suggest some possible variations and improvements, and, hopefully, inspire some further directions for innovation and invention from the participants and other readers.

2. μ production

A critical problem in μ^+–μ^- colliders is the production and collection of adequate numbers of muons. The critical difficulties are the large initial phase-space volume of the muons (since they are produced as secondary or tertiary products of high-energy collisions) and the fact that muons decay, with a lifetime of 2.2×10^{-6} γ decay time. (γ is the usual kinetic factor (E_μ/m_μ).) The problem is to obtain, compress, and use sufficient muons before decay.

We will first consider as a baseline reference a high-energy hadronic (HEH) muon source of the type described in Refs. [1,2,4]: a medium-energy high-intensity hadron accelerator beam is transported onto a high-density target to produce GeV-energy π's, and the π's are confined in a transport channel where they decay to produce μ's, which are collected, compressed, and accelerated. In this section, the baseline HEH source is described, and guidelines for optimization and improvement are suggested. Variations and alternative approaches are then mentioned; these include a low-energy (GeV-scale) "π-factory" beam, possibly producing surface muon beams, or an e^- beam producing μ^+–μ^- pairs by photoproduction.

Production of large numbers of muons in the baseline scenario is not difficult. Hadronic interactions of the beam with the target produce large numbers of π's, and almost all of these π's decay by producing a muon plus a neutrino. The difficult problem of optimizing production and collection for maximal μ intensity is not yet solved; however, some guidelines may be obtained from approximate calculations. A first estimate of $\pi^\pm$ production in proton–hadron collisions may be obtained using known and calculated particle spectra, such as the empirical formulae of Wang [6]:

$$\frac{d^2N}{dP\,d\Omega} \cong AP_p X(1-X)e^{-BX^C - DP_t} \frac{\text{pions}}{\text{sr-GeV}/c} \quad /\text{interacting proton} \qquad (1)$$

where P_p is the incident proton momentum, $X = P_\pi/P_p$ is the pion/proton momentum ratio, P_t is the pion transverse momentum and $A = 2.385$ (1.572), $B = 3.558$ (5.732), $C = 1.333$ (1.333), and $D = 4.727$ (4.247) for positive (negative) pions. In this formula, pions are produced with a mean transverse momentum of $\sim D^{-1}$ or ~ 0.2 GeV. Also, if $P_\pi/P_p \ll 1$, pion production is nearly independent of proton energy, and pion production within a given momentum bite $(\Delta P_\pi/P_\pi)$ is nearly constant. If the Wang model is accurate, an optimal π-source may be obtained from a medium-energy proton beam (20–50 GeV) which collides into a nuclear target, followed by strong-focusing optics which collects secondaries (P_t acceptance ~ 0.3 GeV) and a strong-focusing transport line for $\pi \to \mu\nu$ decay. Small spot sizes on the production target and small beam sizes in the transport line are desired to minimize π and μ emittance. High momentum acceptance ($\delta P/P >$ $\pm 10\%$) is desired for maximal production. Economy would favor lower energies.

Fig. 2 displays a possible configuration for μ^+ or μ^-

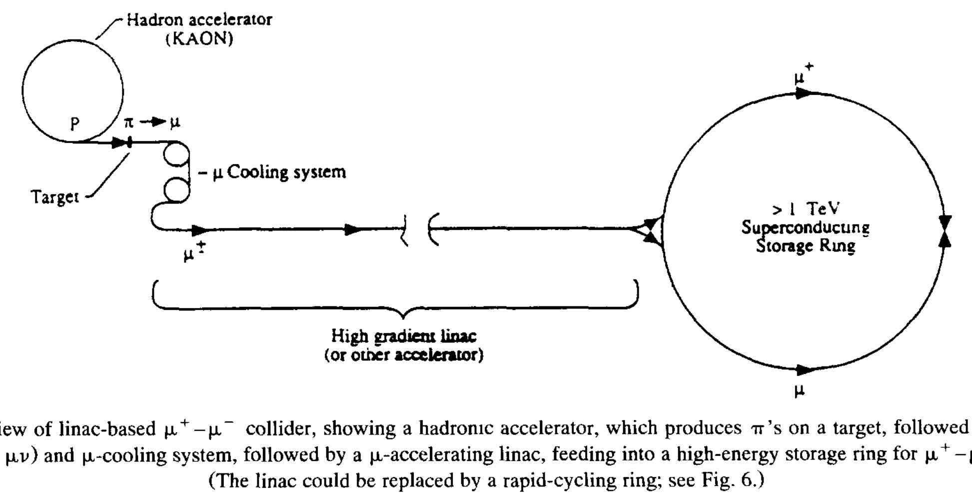

Fig. 1. Overview of linac-based μ^+–μ^- collider, showing a hadronic accelerator, which produces π's on a target, followed by a π-decay channel ($\pi \to \mu\nu$) and μ-cooling system, followed by a μ-accelerating linac, feeding into a high-energy storage ring for μ^+–μ^- collisions. (The linac could be replaced by a rapid-cycling ring; see Fig. 6.)

production. A 40 GeV proton beam is focused (possibly with a Li lens) onto a ~ 5 cm W target to a mm-scale spot, producing π's, which are captured by the following optics, which is designed to accept 2 GeV π's at angles up to $\Theta_T = 150$ mrad ($P_t < 0.3$ GeV/c). This optics is approximated by a 2.0 cm radius Li lens of 10 cm length centered 15 cm downstream from the target center. The capture optics is to be followed by a strong-focusing transport of ~ 1 π decay length (~ 110 m at 2 GeV), which maintains a mean betatron function of 1 m. The resulting muons are inserted into a muon accumulator system for cooling and acceleration.

According to the Wang model, this outline system would produce and accept 0.12 (0.08) π's /(GeV/c)/interacting proton. This must be multiplied by a target efficiency factor $\eta_T \approx 0.4$ (the probability that an incident proton produces an exiting π), and by the momentum acceptance width (0.4 GeV/c for $\pm 10\%$ acceptance), to obtain the number of π's accepted in the decay channel per primary proton (0.019/0.012 $\pi^+\pi^-$).

In the decay channel, π-decay ($\pi \to \mu\nu$) produces μ's within a uniform energy distribution (between 0.57 and $1.0E_\pi$) and with a maximum transverse momentum of 29.8 MeV/c. With 63%(1/e) of the π's decaying and $\sim 40\%$ of the product μ's within the transport acceptance, we find ~ 0.005 (0.0033) $\mu^+(\mu^-)$ per primary proton are delivered to the μ-collector.

The transverse emittance is determined by the π-production phase space, and the π-decay phase space. The transverse emittance from π-production from a thin target is of order $r_T\Theta_T$ or 3×10^{-4} m rad at 2 GeV, if the primary beam size on the target, r_T, is 0.002 m. The π-decay increases emittance by $\sim \beta\theta_d^2/2$, where β is the mean decay-line betatron function and θ_d is the maximum decay angle. With $\theta_d = P_t/P_\mu \approx 0.02$ and $\beta = 1$ m in our reference case, an emittance ϵ_t of $\sim 2 \times 10^{-4}$ m rad at 1.5 GeV is obtained ($\sim 3 \times 10^{-3}$ m rad normalized). (Total emittance from target size, target length, and decay effects should be less than twice this value, and could be reduced somewhat by further optimization.)

This reference case is oversimplified. The problems of separating primary p's from secondary π^+ and π^- beams, and separating the π beams from each other are not addressed; the actual optics will be more interesting. The parameters are not optimized, and the production estimates are not very accurate. The calculations do not include π's (and μ's) from secondary interactions. For $E_\pi \ll E_p$, secondary and cascade production could be large. However, μ-decay from source to collider will reduce the final number (by $\sim 2\times$). The calculations do show that an HEH source can obtain $\geq 10^{-3}$ μ's per primary proton, and that is adequate for a high-luminosity collider. This results is in agreement with an independent analysis of Noble [7]. It is possible that improvements, acceptance increases, and optimization could increase this to the 10^{-2} level, but probably not much greater.

The HEH scenario has been motivated from a high-energy physics bias, and imitates $\bar{p}$ source methods. π-production also occurs in low-energy hadronic (LEH) "π-factories", from GeV/nucleon protons or deuterons, at a level of ~ 0.5 π/p. For μ-collider use, a compressor ring would be needed to combine the proton (deuteron) linac beams into sub-μs pulses. The π's are produced at the 100 MeV energy level, where they can be stopped in an absorber to produce 29 MeV/c μ's, which will lose further energy in the absorber. (Stopping times are at ns levels; the μ-lifetime at rest is 2.2 μs.) (Nagamine has proposed extracting such slow muons in a high-intensity surface muon source [8].) An LEH source can produce large numbers of μ's with low energy and momentum spread, which may require little or no further cooling, and the LEH source could be preferable to an HEH source. The difficulty is in extracting sufficient μ's in a small phase-space volume which are suitable for acceleration in a μ^+–μ^- collider. The problem of calculating and optimizing μ production in an LEH configuration is unsolved; it is an important challenge for the reader.

Another possible μ-source can be obtained by colliding multi-GeV e^- beams into a hadronic target; bremsstrahlung would produce μ^+–μ^- pairs by photoproduction, but not as frequently as e^+–e^- pairs. Obtaining the μ's through pair production avoids the phase-space dilution of π-decay; however, μ-production does not seem to be as copious as in a hadronic source. An optimized calculated comparison has not yet been made.

The best possible μ-source is not yet identified or developed. It may follow some of the ideas suggested in this section, with added improvements and innovations, or it may be dramatically different. This is an important challenge for the workshop.

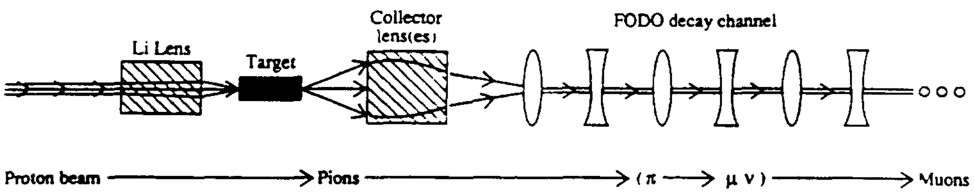

Fig. 2. Schematic view of μ-production from π-decay, with π's produced from hadrons. A high-energy hadronic beam is focused onto a target; a collector lens(es) collects the resulting π's into a strong-focusing "FODO" channel, where π-decay produces μ's for collider use.

3. μ-cooling, and combination

The μ's are produced in a relatively large phase-space volume which must be compressed to obtain high-luminosity collisions. Most of the needed compression is obtained from adiabatic damping; acceleration from GeV-scale μ collection to TeV-scale collisions reduces phase-space by $\sim 10^9$ (10^3 per dimension). Additional phase-space reduction can be obtained by "ionization cooling" of muons ("μ cooling"), which is described in some detail in Refs. [1–3], and is conceptually similar to radiation damping. In this section we first describe transverse μ cooling. Longitudinal cooling and bunch combination, and muon survival and acceleration are then discussed.

The basic mechanism of transverse μ cooling is quite simple, and is shown graphically in Fig. 3. Muons passing through a material medium lose energy (and momentum) through ionization interactions. The losses are parallel to the particle motion, and therefore include transverse and longitudinal momentum losses; the transverse energy losses reduce (normalized) emittance. Reacceleration of the beam (in rf cavities) restores only longitudinal energy. The combined process of ionization energy loss plus rf reacceleration reduces transverse momentum and hence reduces transverse emittance. However, the random process of multiple scattering in the material medium increases the emittance.

The equation for transverse cooling can be written in a differential-equation form as:

$$\frac{d\epsilon_\perp}{dz} = -\frac{\frac{dE_\mu}{dx}}{E_\mu}\epsilon_\perp + \frac{\beta_0}{2}\frac{d\langle\theta_{rms}^2\rangle}{dz}, \tag{2}$$

where $\epsilon_\perp$ is the (unnormalized) transverse emittance, dE_μ/dz is the absorber energy loss per cooler transport length z, β_0 is the betatron function in the absorber and Θ_{rms} is the mean accumulated multiple scattering angle in the absorber. Note that $dE_\mu/dz = f_A dE_\mu/ds$, where f_A is the fraction of the transport length occupied by the absorber, which has an energy absorption coefficient of dE_μ/ds. Also the multiple scattering can be estimated from:

$$\frac{d\langle\Theta_{rms}^2\rangle}{dz} \cong \frac{f_A}{L_R}\left(\frac{0.014}{E_\mu}\right)^2, \tag{3}$$

where L_R is the material radiation length and E_μ is in GeV. (The differential-equation form assumes the cooling system is formed from small alternating absorber and reaccelerator sections; a similar difference equation would be appropriate if individual sections are long.)

If the parameters are constant, Eqs. (2) and (3) may be combined to find a minimum cooled (unnormalized) emittance of

$$\epsilon_\perp \to \frac{(0.014)^2}{2E_\mu}\frac{\beta_0}{L_R\frac{dE_\mu}{dz}} \tag{4}$$

or, when normalized

$$\epsilon_N = \epsilon_\perp\gamma \Rightarrow \frac{(0.014)^2}{2m_\mu c^2}\frac{\beta_0}{L_R\frac{dE_\mu}{dz}} \tag{5}$$

(all energies are in GeV).

Avoiding longitudinal phase-space dilution implies cooling at $E_\mu \geq 0.3$ GeV, and economy implies cooling at relatively low energies (since cooling by e^{-1} requires E_μ energy loss and recovery). $E_\mu \approx 0.5$–1.5 GeV seems reasonable. Cooling can be obtained in either linacs (possibly using recirculation lines) or storage rings. Multiple stages

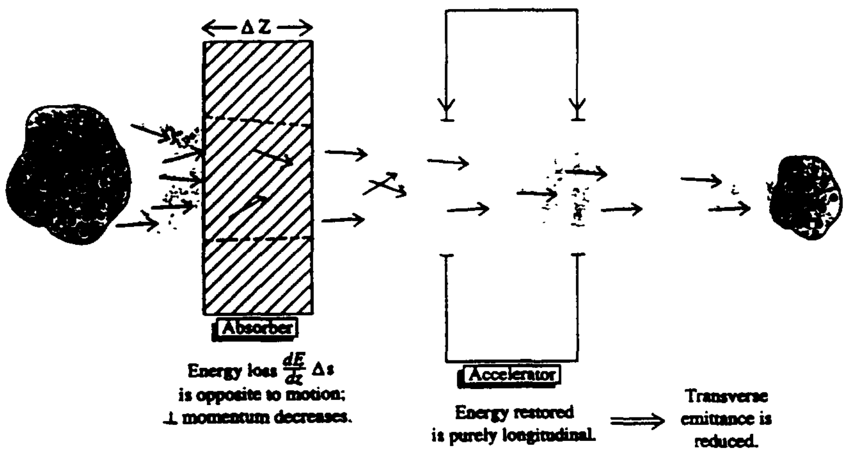

Fig. 3. Schematic view of transverse "ionization cooling". Energy loss in an absorber occurs parallel to the motion; therefore transverse momentum is lost with the longitudinal energy loss. Energy gain is longitudinal only; the net result is a decrease in transverse phase-space area.

can be used to optimize cooling scenarios. The important constraint is that cooling must be completed within a muon lifetime, which can be expressed as $\sim 300\bar{B}$ (T) turns in a storage ring, where $\bar{B}$ is the mean bending field, or as a length $L_\mu = 660\gamma$ m of path length. This constraint may be surmountable.

Some guidelines for optimal cooling may be obtained from Eqs. (2)–(5). They indicate that it is desirable to obtain small β_0 (strong focusing) at the absorber. It is also desirable to have materials with large values of the product $L_R\,\mathrm{d}E/\mathrm{d}s$. $L_R\,\mathrm{d}E/\mathrm{d}s$ is largest for light elements (0.1 GeV for Li, Be but $\sim$ 0.01 GeV for W, Pb), indicating the desirability of light absorbers. However, the need for small β_0 and the depth of focus constraint that a (non-focusing) absorber section must be less than $2\beta_0$ would favor large $\mathrm{d}E/\mathrm{d}s$ (heavy) absorbers. A conducting light-metal absorber (Li, Be, Al) can also be a continuously focusing lens, which could then be arbitrarily long while maintaining small β_0. With current technology, lenses maintaining $\beta_0 < 1$ cm appear possible.

With $\beta_0 \approx 1$ cm, and $L_R\,\mathrm{d}E/\mathrm{d}s \approx 0.1$ GeV, a normalized emittance of $\epsilon_N \approx 10^{-2}\beta_0 \approx 10^{-4}$ m rad is obtained as a reasonable goal for transverse cooling. Some improvements may be possible; the reader may develop ideas for optimal implementation.

Longitudinal (energy-spread) cooling is also possible, if the energy loss increases with increasing energy. The energy loss function for muons, $\mathrm{d}E/\mathrm{d}s$, is rapidly decreasing (heating) with energy for $E_\mu < 0.3$ GeV, but is slightly increasing (cooling) for $E_\mu > 0.3$ GeV. This natural dependence can be enhanced by placing a wedge-shaped absorber at a "non-zero dispersion" region where position is energy-dependent (see Fig. 4). Longitudinal cooling is limited by statistical fluctuations in the number and energy of muon–atom interactions. An equation for energy cooling is:

$$\frac{\mathrm{d}\langle(\Delta E)^2\rangle}{\mathrm{d}z} \approx -2\frac{\partial\frac{\mathrm{d}E_\mu}{\mathrm{d}z}}{\partial E_\mu}\langle(\Delta E)^2\rangle + \frac{\mathrm{d}\Delta E_{\mathrm{rms}}^2}{\mathrm{d}s}, \qquad (6)$$

where the derivative with energy combines natural energy dependence with dispersion-enhanced dependence. An expression for this enhanced cooling derivative is:

$$\frac{\partial\frac{\mathrm{d}E_\mu}{\mathrm{d}z}}{\partial E_\mu} = f_A\left(\frac{\partial\frac{\mathrm{d}E_\mu}{\mathrm{d}s}}{\partial E_\mu}\right)_0 + f_A\frac{\mathrm{d}E_\mu}{\mathrm{d}s}\frac{\mathrm{d}\delta}{\mathrm{d}x}\frac{\eta}{E_\mu\delta_0}, \qquad (7)$$

where η is the dispersion at the absorber, and δ and $\mathrm{d}\delta/\mathrm{d}x$ are the thickness and tilt of the absorber. Note that using a wedge absorber for energy cooling will reduce transverse cooling; the sum of transverse and longitudinal cooling rates is invariant. In the long-pathlength Gaussian-distribution limit, the heating term or energy straggling term is given by [9]:

$$\frac{\mathrm{d}(\Delta E_{\mathrm{rms}})^2}{\mathrm{d}s} \cong 4\pi\left(r_e m_e c^2\right)^2 N_0 \frac{Z}{A}\rho\gamma^2\left(1-\beta^2/2\right),$$

where N_0 is Avogadro's number and ρ is the density. Since this increases as γ^2, cooling at low energies is desired. From balancing heating and cooling terms, we find that cooling of $\Delta E_\mu/E_\mu$ to ≤ 0.02 at $E_\mu = 0.25$ GeV is possible, which adiabatically damps to a 1 GeV energy spread of $\Delta E_\mu/E_\mu < 0.005$. An rf buncher plus compressor arc (or synchrotron oscillations) can use this reduced energy spread to obtain reduced bunch length.

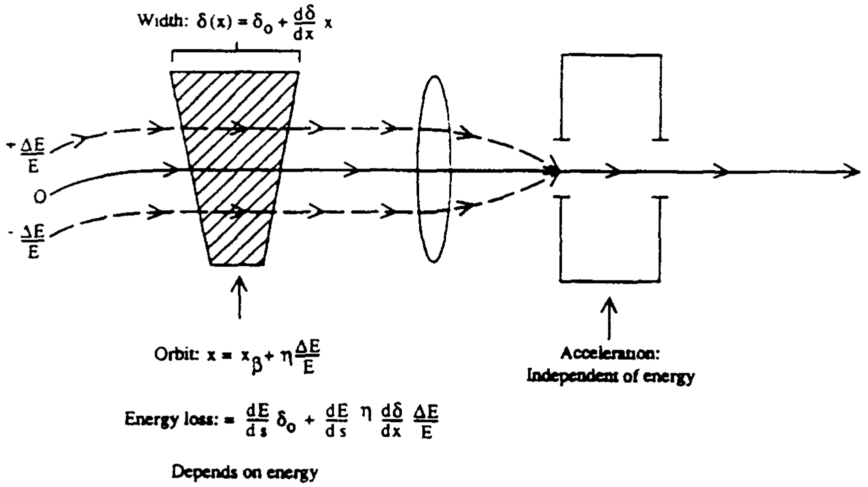

Fig. 4. Enhancement of energy cooling by using a wedge absorber placed in a non-zero dispersion region. The thickness of the absorber depends on transverse position ($\Delta = \delta_0 + (\mathrm{d}\delta/\mathrm{d}x)x$), and the position at the absorber depends on the energy ($x = \eta(\Delta E/E)$), producing an enhanced energy dependence of energy loss, decreasing energy spread. Energy recovery in the accelerator is independent of energy. (Transverse cooling decreases with enhanced energy cooling.)

The major problem in longitudinal space derives from the mismatch between the initial bunch structure of the μ-source and the desired μ-collider bunch configuration. The primary proton beam from a rapid-cycling synchrotron (RCS) would consist of ~ 100–200 bunches, while compression to one (or a few) μ-collider bunches is desired. Phase-space manipulations will be needed.

Bunch combination procedures could include:

1) Proton-bunch overlap: Before extraction from the RCS, the proton bunches can be compressed with a lower harmonic (or sideband) rf system to provide spatially overlapping bunches (with different momenta) on the target. Since target spot sizes need not be very small and π-production is not energy-dependent, a broad primary energy spread can be accepted at the target, with no degradation of π-production. Combination by at least a factor of 10 should be obtainable. A separate extraction-energy proton compressor ring for the rf bunch manipulations may be desired, and may be capable of combining the number of bunches to ~ 2–4.

2) Non-Liouvillian "stochastic injection" [10]: Bunch combination without phase-space dilution can occur during π decay, as in "stochastic injection" into a μ-storage ring, as shown in Fig. 5. In this process a train of π bunches from a hadronic target is injected into a decay channel, which is also a zero-dispersion straight-section of a μ-storage ring. The π-bunch spacing is matched to the storage ring period (or a low harmonic). The injected π's are not in the acceptance of the ring; the ring accepts lower-energy particles, in particular, those μ's from π decay in the straight section that are within that acceptance. Successive π-bunch arrivals are timed to overlap an accumulated μ bunch. The π lifetime is ≈ 1% of the μ lifetime, and is naturally matched for decay within the first-turn decay channel, while permitting multiple turn accumulation. At reasonable parameters, μ's from 10–30 π bunches can be accumulated in a single bunch, without large μ-decay losses. (The storage ring can also be used for cooling the accumulated μ's.)

3) Beam cooling with bunch combination: Transverse or energy cooling of μ bunches can compress beams to a degree where bunches can be stacked together, using conventional Liouvillian bunch-combination optics. The stacked bunch can then be further cooled to a phase-space volume ≤ the previous cooled single-bunch size. The process can continue through further stacking and cooling steps. The process adds the complications of multiple beam-combination transport lines and optics; however, combinations of many bunches (10–30) could be obtained.

Some combination of these three (plus other to-be-developed) methods can be used to reduce the number of μ bunches to a few enhanced-intensity bunches. (The first two procedures are complementary: proton bunch stacking naturally combines nearby bunches while stochastic injection more naturally combines widely spaced bunches.) Ionization cooling can then be used for further compression, in bunch length or energy spread, for optimal collider use.

4. Acceleration and collider scenarios

Cooled and compressed muon bunches can be used in high-energy high-luminosity colliders. In this section, we describe some potential scenarios. We use as a reference case a 1 TeV per beam μ^+–μ^- collider (2 TeV in the center-of-mass), as this is the energy scale where the μ^+–μ^- collider may begin to be preferable to e^+–e^- colliders. Table 1 shows a reference case, with relatively conservative choices of parameters.

1) Linac–storage ring. This scenario is displayed graphically in Figure 1, and this is probably the highest-luminosity case. μ^+–μ^- bunches from the collector/cooler are both accelerated to full energy in a high-gradient linac to 1 TeV, where both bunches are injected into a superconducting storage ring for high-energy collisions at low-β_0 interaction points. The μ beam lifetime is $\sim 300B$ turns, where B is the mean bending field in T. $B \approx 8$ T, imply-

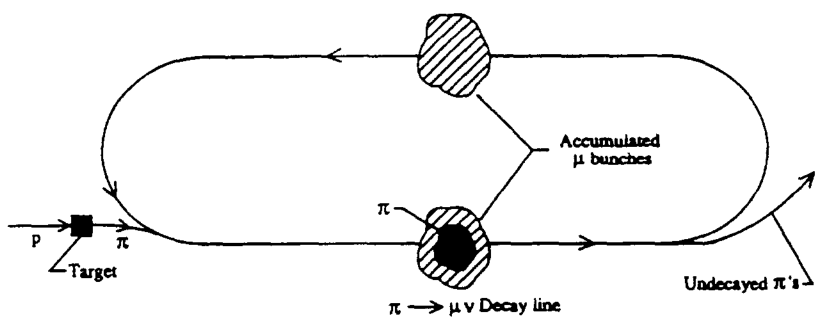

Fig. 5. Schematic view of "stochastic injection" into a storage ring. A train of p bunches produces π bunches, which are injected into a storage ring for multi-turn stacking. The initial spacing is matched to a ring harmonic ($h = 2$ in the figure), so that following π bunches overlap accumulated μ's from previous bunches. π decays in the straight section which produce μ's within the ring acceptance add to the accumulation. (Note that the short but finite π lifetime is nearly optimally matched to make this scheme practical.)

Table 1
Parameter list for TeV μ^+–μ^- collider (2 TeV collisions)

Parameter	Symbol	Value
Energy	$E_{\mu\pm}$	1 TeV
Luminosity	L	3×10^{32} cm^{-2} s^{-1}
HEH-source parameters		
Proton energy	E_p	40 GeV
P/pulse	N_p	10^{14}
Pulse rate	f_0	30 Hz
μ production efficiency	μ/p	10^{-3}
Collider parameters		
#μ^+/μ^- per bunch	$N^{\pm}$	10^{11}
#bunches	n_B	1
Storage turns	n_s	1200
μ emittance	$\epsilon_\perp = \frac{\epsilon_N}{\gamma}$	10^{-8} m rad
Interaction focus	β^*	1 mm
Beam size	σ	3 μm

ing 2400 turns, is currently achievable. The main difficulty is the relatively large cost of the full-energy TeV linac.

2) Linac–linac collider. As in e^+–e^- linear colliders, μ bunches from opposing linacs can collide. This scenario loses the luminosity magnification obtained from multiple collisions in a storage ring, which is permitted by the long μ lifetime. It also requires two full-energy linacs, and a TeV storage ring is cheaper than a TeV linac. However, an existing e^+–e^- linear collider could be modified to obtain μ^+–μ^- collisions, with the addition of a μ source.

3) Rapid-cycling synchrotron collider. At 1 TeV, the μ lifetime has increased to 0.021 s, and the lifetime increase with energy is sufficient to permit acceleration in a rapid-cycling synchrotron with acceptable losses. Fig. 6 shows the basic components: a μ source injected into a ~20–50 GeV linac followed by a rapid cycling synchrotron with 20 km circumference ($B \approx 1$ T). Acceleration from injection to full energy in ~50–100 turns could follow a 60–120 Hz waveform followed by ~0.02 s at fixed field for collisions or, for higher luminosity, transfer to a fixed-field (8 T) collider ring. Luminosity would be naively expected to be about an order of magnitude smaller than in the linac–storage ring scenario.

4) The μ–p collider. A μ^+–μ^- collider can also be operated as a μ^-–p collider with both μ and p beams at full energy. High luminosity would be relatively easily obtained because only one beam (μ) is unstable and diffuse. This is a probable initial and debugging operating mode for a storage ring μ^+–μ^- collider. Revolution frequencies of equal energy μ and p beams would be naturally mismatched because of unequal speeds. They can be rematched by displacing the beams in energy and using the ring nonisochronicity [2]. The required energy displacement is

$$\frac{\delta E}{E} = \gamma_T^2\left(\frac{1}{2\gamma_p^2} - \frac{1}{2\gamma_\mu^2}\right) \tag{8}$$

where γ_T^2 is the ring transition-gamma. At reference parameters ($E_\mu = 1$ TeV, $\gamma_T = 30$) $\delta E/E = 0.0005$ is required.

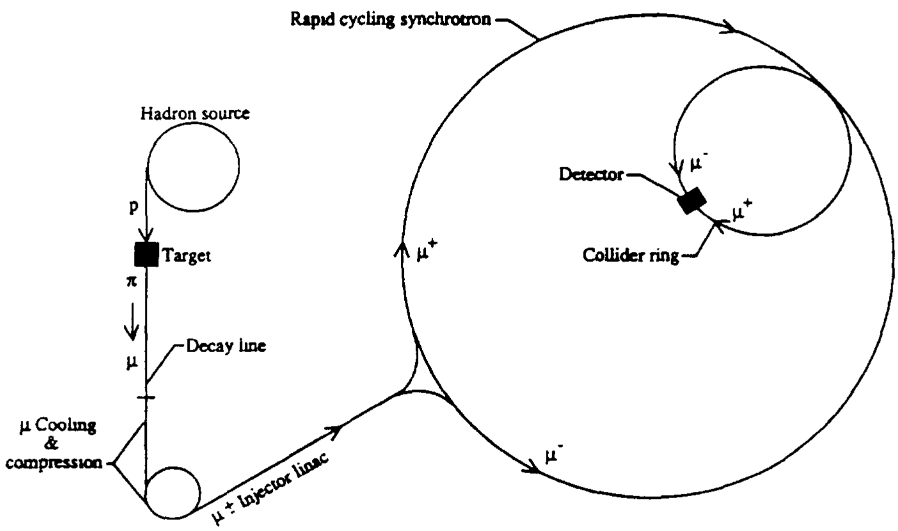

Fig. 6. Overview of a μ^+–μ^- collider, with a rapid-cycling synchrotron for the primary accelerator. The figure shows a primary proton source, producing π's on a target, which decay to μ's in a decay channel. After cooling and compression, μ^+–μ^- bunches are accelerated in a linac and in a rapid-cycling synchrotron to full energy, where they are injected into a high-field storage ring for multi-turn collisions. For example, a 20 GeV linac feeding into a 20 GeV/turn (up to 1.2 T) rapid-cycling synchrotron would produce 1 TeV μ^+–μ^- beams with acceptable decay losses.

5) Physics–opportunity colliders. A resonance, such as a new Z particle or a Higgs particle, may exist or be predicted in μ^+–μ^- collisions. At such a resonance, a lower-luminosity and/or lower-energy collider could still provide extremely important physics. Such a collider would be a simplified form of the baseline high-luminosity models, possibly with μ sources using existing accelerators, omitting μ cooling, and/or using existing storage rings for collisions. Such a facility would, of course, provide excellent training for a high-energy high-luminosity collider.

5. Luminosity possibilities and constraints

Using the previously discussed techniques, high luminosity can be obtained in a μ^+–μ^- collider. The luminosity is given by the equation:

$$L = \frac{f_C N^+ N^-}{4\pi\sigma^2} = \frac{f_C N^+ N^-}{4\pi\beta^* \epsilon_\perp}, \tag{9}$$

where N^+,N are the number of μ^+, μ^- per colliding bunch, f_C is the frequency of bunch collisions, σ^2 is the colliding beam size, β^* is the betatron function at the collision point and $\epsilon_\perp = \epsilon_N/\gamma$ is the transverse emittance. In a storage-ring collider, $f_C = f_0 n_B n_S$, where f_0 is the system cycling rate, n_B is the number of bunches, and n_S is the number of turns of storage per cycle.

This formula is applied to a reference TeV μ^+–μ^- collider (Table 1). The parameters we use include $N^+ = N^- = 10^{11}$ (which can be obtained assuming a modest production rate of 10^{-3} μ/p from 10^{14} proton high-energy pulses), $n_B = 1$, $f_0 = 30$ Hz, and $n_S = 1200$ turns storage. With $\epsilon_N = 10^{-4}$ m rad ($\epsilon_\perp = 10^{-8}$) from $\beta_0 = 1$ cm and $\beta^* = 1$ mm ($\sigma \approx 3$ μm), we obtain a respectable baseline luminosity of $L \approx 3 \times 10^{32}$.

Note that the above parameter set is relatively modest, and improvements in some of the parameters (i.e., N^+, N^-, σ) by up to an order of magnitude are conceivable. However, reliable accomplishment of high luminosity in a novel and complicated facility which uses unstable particles and has several difficult design components will still not be easy.

Luminosity in a μ^+–μ^- collider ring may be expected to be limited by the beam–beam interaction. Since long-term stability is not needed, the allowable beam–beam tune shift should be somewhat greater than the e^+–e^- storage limit of $\Delta\nu \lesssim 0.05$. The tune shift is given by

$$\Delta\nu = \frac{N r_\mu \beta^*}{4\pi\gamma\sigma^2}. \tag{10}$$

where $r_\mu = 1.363 \times 10^{-17}$ m. At the baseline parameters $\Delta\nu \approx 0.001$, luminosity would have to be increased dramatically for the beam–beam limit to be significant.

A μ^+–μ^- collider has substantial beam power requirements, particularly in the primary proton beam. In the reference case, the requirements are 10^{14} 40 GeV protons at 30 Hz, which implies 20 MW beam power. This is an order of magnitude above present facilities, but is the same magnitude as proposed K-factories. (The high-energy μ beams themselves require only 0.32 MW.) More efficient μ-production may be desirable (obtaining more μ/p, using lower-energy p's or a LEH source). However, a high-luminosity high-energy μ^+–μ^- collider is a successor to SSC- or TLC-size facilities, and on that scale, a K-factory type source is small. An even higher intensity source could be affordable.

A significant difficulty in a storage ring is that μ's decay ($\mu \to e\nu\nu$), and decay electrons at ~ 0.3 TeV will hit the walls of the storage ring. At the reference parameters, with half the μ's decaying during storage, we find 50 kW of ~ 0.3 TeV electrons will be deposited evenly within a narrow strip on the inner wall of the ring. The ring must be designed to accept this.

The obtainable luminosity L is expected to increase with increasing end point energy E_μ, as the μ lifetime increases and the emittance and energy spreads are adiabatically damped. As discussed in Ref. (4), the beam size (σ^2) at collision should decrease as E_μ^{-2}, as both β^* and $\epsilon_\perp$ can decrease. Cycle time would increase; however, the longer cycle time could permit accumulation of successive rapid-cycling proton pulses to obtain magnified-intensity μ-bunches. The net effect is that N^+ and N^- increase as E_μ and f_C is reduced by E_μ^{-1}. In all, L naturally increases as E_μ^3. (Costs increase linearly with E_μ.) This scaling would be expected to dominate until the $E_\mu \approx 100$–1000 TeV region, where μ synchrotron radiation excludes μ storage-ring colliders.

6. Summary

In this paper, we have introduced concepts which show the promise of the development of high-luminosity TeV-scale μ^+–μ^- colliders. These initial concepts need considerable practical development. While key ingredients of a future facility have been introduced, further innovations and improvements are greatly desired. These concepts plus further developments must be integrated into a fully self-consistent design for a μ^+–μ^- facility. The discussions at the first μ^+–μ^- collider workshop at Napa, California, should elucidate the possibilities and set a basis for further development. Contributions and improvements from the workshop participants and other readers are encouraged. A functional μ^+–μ^- collider will be obtained only through further innovation and development.

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Appendix

Since this report is appearing subsequent to the 1992 Napa μ^+–μ^- Collider workshop, in this section I am adding some initial impressions of the proceedings of the workshop.

D. Cline presented an interesting immediate application for a μ^+–μ^- collider [11]. There are theoretical reasons to believe that the Higgs boson might exist in the 90–180 GeV region, and that is an energy region at which only a μ^+–μ^- collider could obtain a clean observation of a Higgs boson. (μ^+–$\mu^- \rightarrow$ H is favored because of the relatively large muon mass.) The observation would require luminosity greater than $\sim 10^{29}$ cm^{-2} s^{-1}. Sample parameters with this luminosity goal are shown in Table 2. This relatively low-energy collider could be developed relatively inexpensively, possibly using existing facilities for major components, although it is unclear if any existing accelerator could deliver sufficient muon intensity. An important future goal is developing an optimal short-term path to this extremely important physics goal.

At the workshop, it was speculated that maximal muon production could be obtained from a hadronic (or leptonic) cascade source, possibly at a beam dump, rather than the single-interaction source outlined above. H. Thiessen suggested that the most efficient source could be a ~ 5 GeV hadronic source. Serious target problems were noted for any high-intensity source. Further study/optimization is needed.

Table 2
Parameter list for 100 GeV μ^+–μ^- collider

Parameter	Symbol	Value
Energy	$E_{\mu\pm}$	100 GeV
Luminosity	L	10^{29} cm^{-2} s^{-1}
Pulse rate	f_0	10 Hz
Storage turns	n_s	1000
#bunches	n_B	10
#μ^+/μ^- per bunch	$N^{\pm}$	10^{10}
μ-emittance	$\epsilon_\perp = \frac{\epsilon_N}{\gamma}$	10^{-7} m rad
Interaction β	β^*	1 cm
Beam size (at IR)	σ	30 μm

In beam cooling, the limitations on ionization cooling due to multiple scattering (described above) were discussed. It may be possible to have a muon source with initial emittance smaller than the multiple scattering limit, and therefore to avoid cooling.

The bunch combination/compression (see above) was identified as a key problem, particularly bunch length reduction to match small β^* optics. At multi-GeV energies, the muons will be relativistic and have no longitudinal motion within a linac. However, relativistic muon bunches can be compressed with an rf-induced energy tilt and transported through a bending arc (or ring). Scenario design/optimization is needed.

Although the workshop did not identify a clear path to a sufficient luminosity design, it did show important (and perhaps obtainable) physics goals, particularly the Higgs discovery opportunity. Future study goals, particularly in source design and scenario development, were identified, and the need for future workshop(s) after further development was suggested.

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