C. Lu
Princeton University
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Stress Analysis of the PEP-4 Solenoid Coil

The TPC solenoid magnet coil, which was used for PEP-4 experiment at the Stanford Linear Accelerator Center, is considered to be used in the Muon Collider targetry experiment. The solenoid coil is supposed to operate without the return yoke. This is rather thin composite structure which consists of following two major components: a 1.41cm thick of aluminum cylinder as the bore tube and a 1.74mm thick copper-based Nb-Ti superconductor coil running at high current to create about 1 ~ 1.9Tesla *B* field at the center. The composite structure will suffer the stress due to the magnetic force. If we put this coil in an experimental hall, the bare coil also will encounter the magnetic forces exerted from the steal walls and roof. Careful stress analysis is certainly needed. In this note we present the analytical estimate and detailed FEA simulation of a simplified model of this solenoid coil.

Model description

The solenoid coil is modeled by two layers of 3m long cylinders: the inner layer is an aluminum cylinder with 1m in radius and 1.4cm in thickness, the outer layer is 1.74mm thick copper-based Ni-Ti superconductor cylinder. The Young's moduli of aluminum and Ni-Ti are 69Gpa and 110Gpa, respectively, and their Poisson ratios are the same as 0.32.

Analytical calculation

1. Magnetic flux density *B*

Along the central line of the solenoid the *B* field can be readily calculated by the analytical formula

$$B = \frac{\mu_0}{2}\frac{I}{L}(\cos\alpha - \cos\beta),$$

where I is the total current in the solenoid, L is the length of the solenoid, α and β are illustrated in the figure 1. *B* distribution along the solenoid axis is plotted in the figure 2.

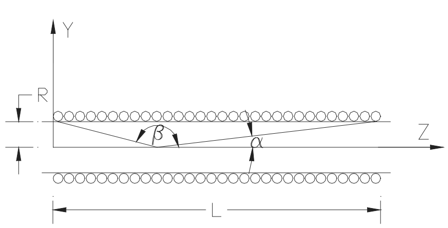

Figure 1. Solenoid magnet

To calculate the B distribution at any point in the space we can use the following basic formula

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^2},$$

where I is the current in a small element of the coil dl, r is the distance between dl and the space

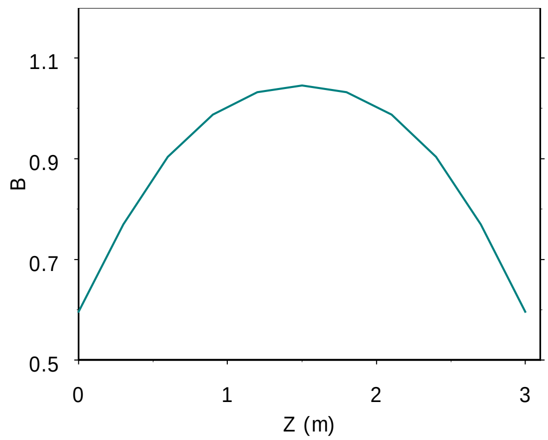

Figure 2. *B* field distribution along the axis of the solenoid

point. Using the numerical integration[1] we can carry out the calculation over the whole space. The B and B_z distribution along four lines, which are parallel to the axis with different distances, are shown in figure 3. For the distance = 0m case the curve in the figure 3 is identical to figure 2.

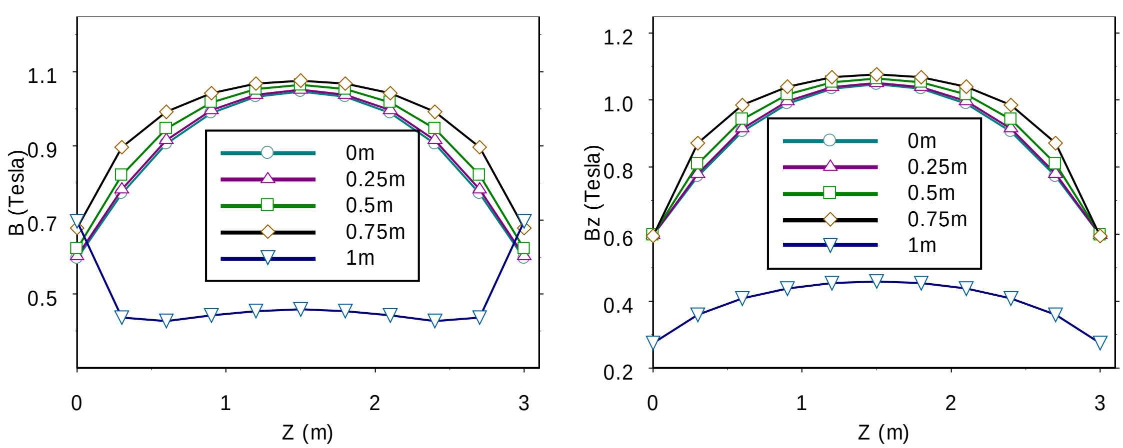

Figure 3. B and Bz distributions along with five lines, which are parallel to the axis with different distances to the axis

[1] We used Mathcad 2000 to do the numerical integration. Mathcad 2000, MathSoft, Inc.

Use the numerical integration we also can get B_y distribution along the bottom axial line of the coil, which is shown in the figure 4. B_x along this line is zero due to the axisymmetry.

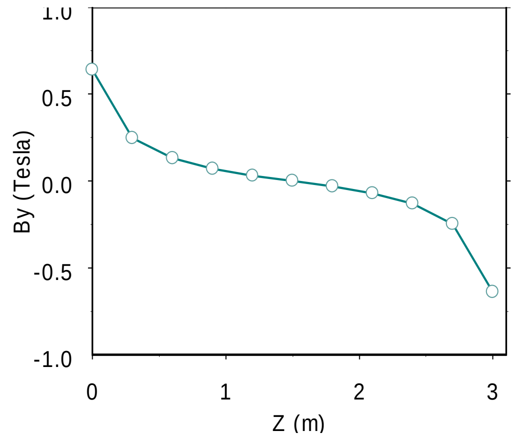

Figure 4. ***B_y* distribution along with the bottom line of the coil**

2. Magnetic force

As long as we know the B distribution at the coil and the current density carried by the coil, it is readily to get the magnetic force exerted on the coil:

$$f_R = iB_z,$$

where f_R is the radial magnetic force for unit area of the coil, i is the total current in a unit axial length of the coil. f_R is shown in figure 5. For a 30° section of the coil summing up the radial force along with axial direction, the total F_R for the 3m long panel is 680000 Newton, or 69 Tons. The magnetic field on the perpendicular direction to the axis of the coil will produce the magnetic force on axial direction. For the unit axial length and a unit circumference length of the coil this force will be

$$f_z = iB_y,$$

which is also shown in figure 5.

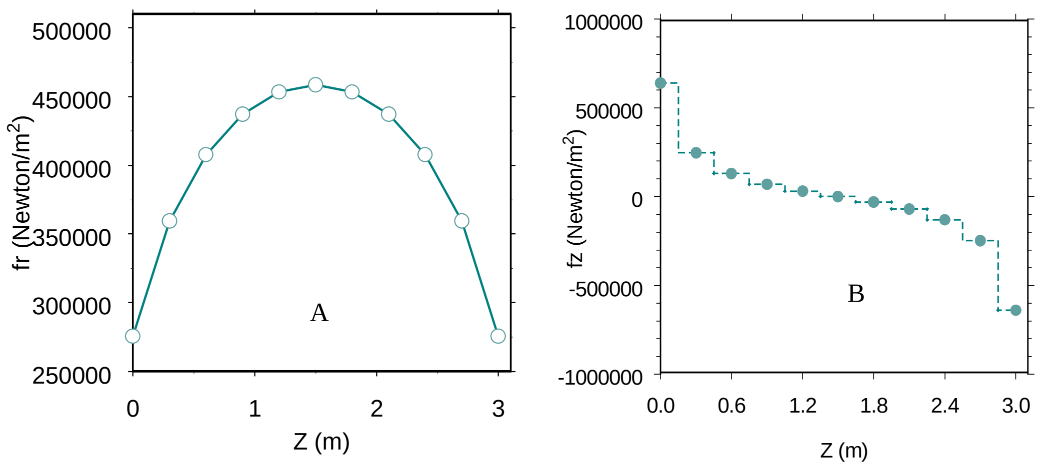

Figure 5. Distributions of (A) the radial and (B) axial force on a unit area of the coil. The dashed curve in (B) is a simplified *f_z* distribution curve.

3. Stress estimation

Knowing the distribution of the radial force on the unit area coil and assuming the averaged force f_R uniformly exerted on the inner surface of the cylinder, this is the case of a thin-walled pressure vessels. From the calculated force above we obtain the pressure to be 0.4Mpa. Using Roark and Young's formulas[2] (Table 28 – 1B) and TK Solver on Roark & Young[3], we can get the radial displacement and axial height change as shown in Table 1. In the table we used an equivalent aluminum thickness 16.67mm, which corresponds to 14.1mm aluminum (E = 73Gpa) pluses 1.71mm copper-based Nb-Ti superconductor (E = 110Gpa).

Name	Unit	Input	Output
Radius of the cylinder - R	mm	1000	
Height of the cylinder - H	mm	3000	
Thickness of the cylinder - t	mm	16.67	
Young's modulus - E	MPa	73000	
Poisson's ratio - ν		0.32	
Uniform radial pressure - q	MPa	0.4	
Circumferential stress - σ_c	MPa		28.3
Radial displacement - ΔR	mm		0.33
Change in height - ΔH	mm		-0.32

Table 1. The results of TK Solver on Roark & Young for uniform radial pressure case

We notice that the radius increases 0.33mm, and the length of the cylinder decreases 0.32mm if we only take the radial magnetic force into account. We also need to consider the effect of the axial force. In figure 5B we can see that the axial force distribution is not uniform. To estimate the stress and strain due to axial force, we can simplify the problem as follows: consider five different length of the cylinders: 0.6m, 1.2m,…, 3.0m, for each of the cylinder the corresponding forces indicated in figure 5B are applied to two ends. Enter these numbers to the Table 28 – 1A of Roark & Young's formulas, then summed up ΔR and ΔH for all five cylinders, the results are shown in table 2.

Name	Unit	Input	Output
Radius of the cylinder - R	mm	1000	
Height of the cylinder - H_i	mm	3000,2400,1800,1200,600	
Thickness of the cylinder - t	mm	16.67	
Young's modulus - E	MPa	73000	
Poisson's ratio - nu		0.32	
Uniform unit axial load - F_i	N/m	-28790,-22280,-11775, -6300,-2780	
Radial displacement - $\Sigma\Delta R_i$	mm		0.019
Change in height - $\Sigma\Delta H_i$	mm		-0.14

Table 2. The results of TK Solver on Roark & Young for uniform axial load case

[2] Warren C. Young, Roark's Formulas for Stress & Strain, Six Edition, McGraw-Hill, Inc.
[3] TK Solver 1.1, Universal Technical System, Inc.

Add these two effects together we can draw the conclusion: the maximum radial increase is about 0.35mm, and the axial decrease is about 0.46mm.

ANSYS finite element analysis

Finite element analysis can give us detailed stress and strain distribution on the solenoid coil. We use ANSYS element type SOLID62, which has the capability of modeling three-dimensional coupled magneto-structural field, to simulate the solenoid coil, and element type SOLID97 and SOLID96 to model air and wall. The INTER115 elements are used between SOLID96 and SOLID97 as the interface. INFIN111 elements are used to enclose the entire outmost surface.

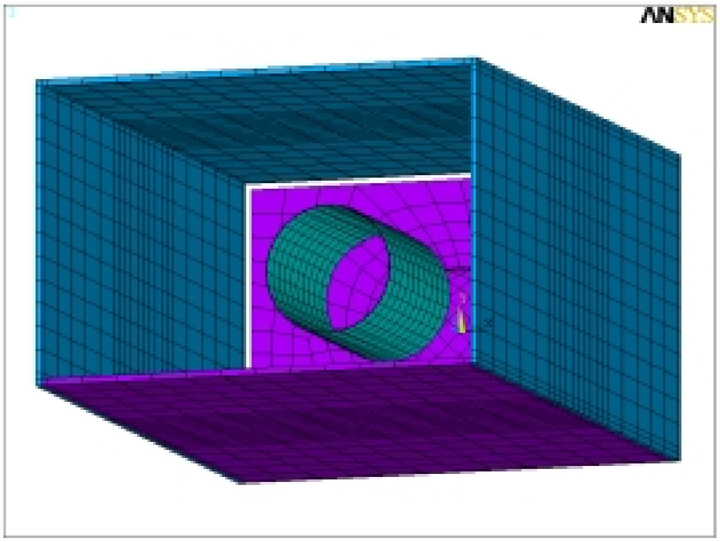

Figure 6. The geometry of FEA model

Geometry of the FEA model

The geometry of the FEA model is shown in figure 6. The green cylinder represents the solenoid coil, which consists of two components: the inner 14.1mm thick aluminum bore tube and the outer 1.74mm thick Nb-Ti superconductor coil. The purple plates are the walls with $\mu_r = 1$, the blue plates are walls with $\mu_r = 10^6$. One wall is purposely removed to reveal the position of the coil. Air elements fill up the hall and also form a layer to cover all of the outside surfaces of the walls. We set the elements of this air layer as INFIN111, and their exterior surfaces are flagged as infinite surface. The air elements are not shown in the figure.

Boundary condition

Having set the INFIN111 elements to surround the hall, we don't need to apply any other boundary condition. The whole domain of the problem is extended to the infinitive naturally. For the stress analysis part of the problem we impose the following boundary condition at two ends of the coil: at the one end we fix all of the nodes as dX, dY and dZ = 0. The nodes at the other end of the coil are set as dX, dY = 0, and they are free to move on axial direction. The current density in the coil is set at $B/(\mu \times S_{thick})$, S_{thick} is the thickness of the superconductor coil. With this current density flowing in the coil, if the coil were infinitive long, the central magnetic flux of this coil would equal to B.

Magnetic flux density *B* distribution

The *B* and *Bz* distribution along several lines, which are parallel to the axis are shown in figure 7. Compare figure 7 and figure 2 it is clear that they are very similar. The vector *B* distribution on the coil is plotted in figure 8.

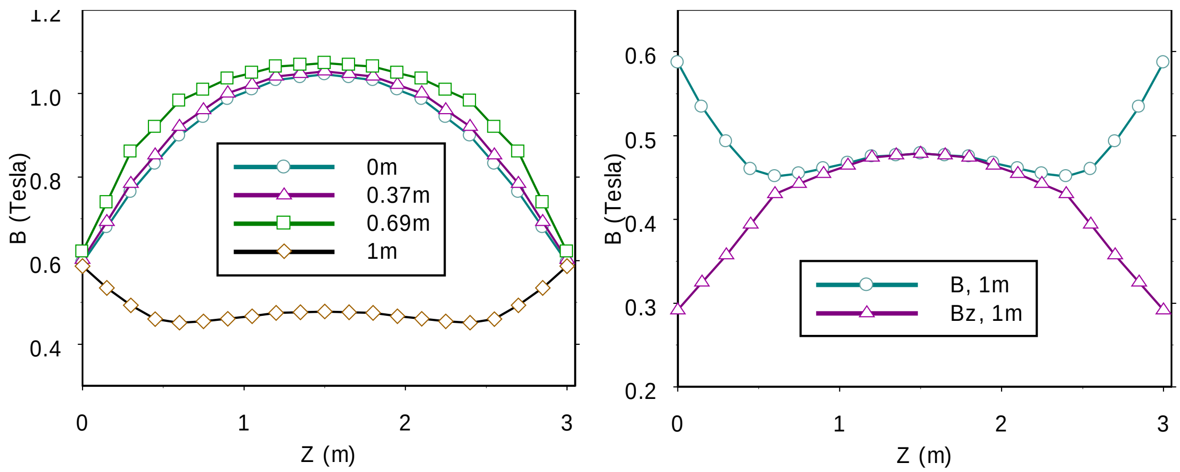

Figure 7. *B* and *Bz* distribution along the axis and several parallel lines to the axis

Magnetic force on the coil

The magnetic force vectors on the coil elements are shown in figure 9. The force will squeeze the coil on axial direction and expand it on radial direction.

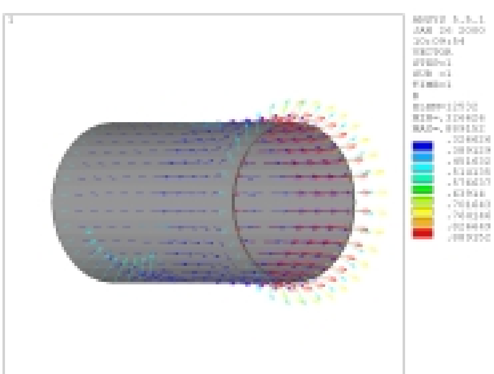

Figure 8. *B* vector distribution on the coil

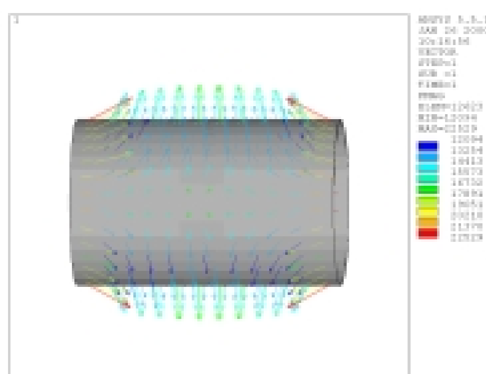

Figure 9. Magnetic force vector distribution on the coil

Stress

The first principal stress (hoop stress) distribution is shown in figure 10. In the middle section of the coil the maximum hoop stress is 32Mpa, and the minimum hoop stress is 23Mpa; in Table 1 the circumferential stress from Roark & Young's formulas is 28.3Mpa; they are consistent. The stress intensity of the coil is presented in figure 11; the maximum intensity is 45Mpa. Just for comparison the commercial aluminum alloy 3003 has tensile strength (ultimate strength) 110Mpa, and alloy 2014 is 185Mpa, which exceeds the maximum stress intensity by a big factor.

Deformation and Strain

The radial and axial deformation of the coil is provided in figure 12. The average radial increase is 0.3mm, and axial decrease is 0.53mm. According to previous calculation with Roark & Young's formulas they are 0.35mm and 0.46mm, respectively. The strain map of the coil is shown in figure 13. The maximum strain is 0.06%.

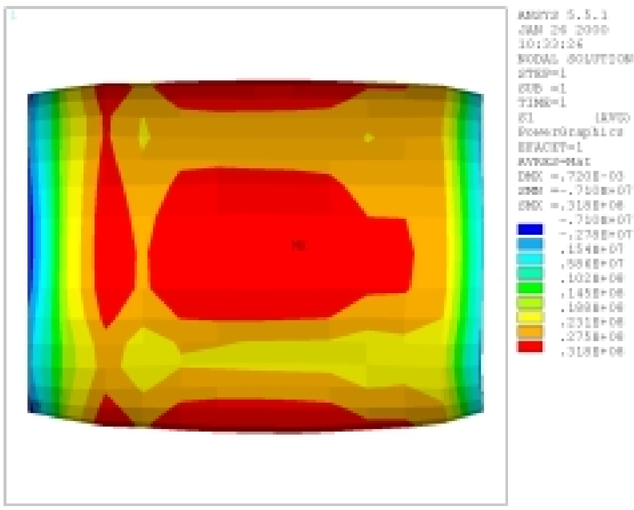

Figure 10. Hoop stress distribution

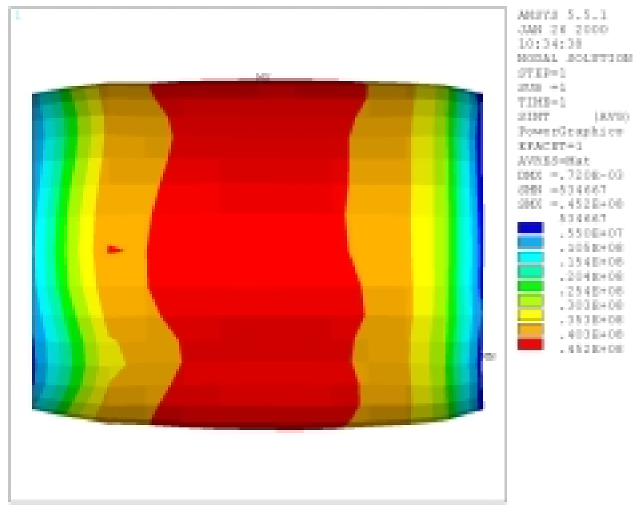

Figure 11. Stress intensity of the coil

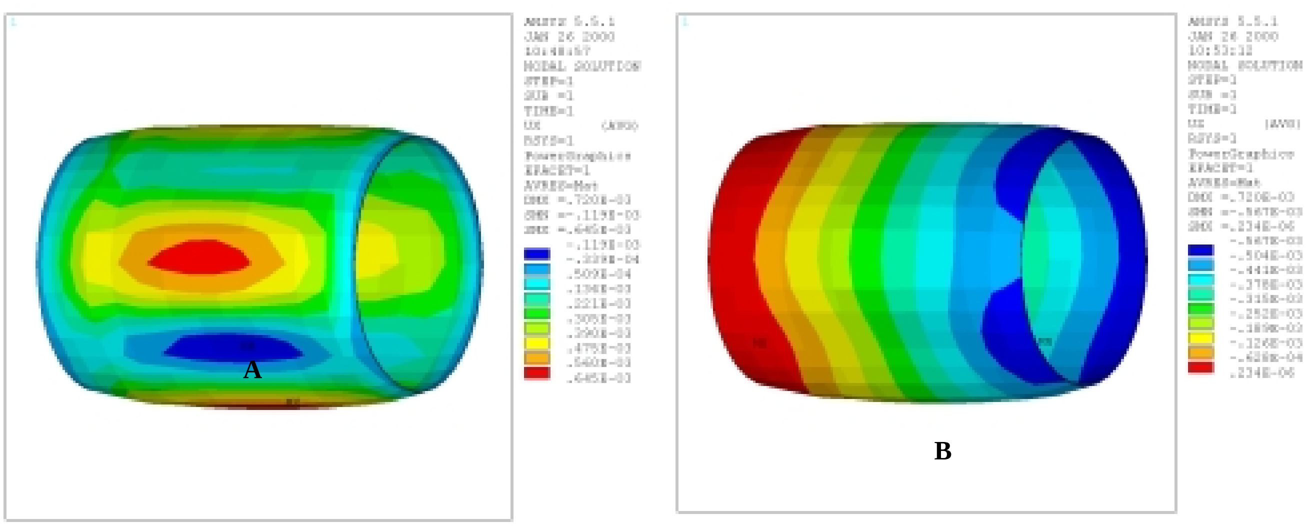

Figure 12. Deformation of the coil, (A) radial, (B) axial, the circles outlines the undeformed shape.

Lifting force

The steel ceiling and wall will exert the magnetic force on the coil when the solenoid is energized. We set the relative permeability of the ceiling, right and left walls equal to 10^6. The floor, the front and back walls have the same permeability as air. If the height of the hall is 4m, the center of the solenoid coil is at 2m height, the overall lifting force on the coil is about 3.6Ton. If we place 5mm thick steal plate on the floor, the lifting force will be reduced to 0.7Ton.

Comparison with the calculations of original PEP-4 magnet coil engineering note

The following formulae are adopted from the original engineering note of PEP-4 magnet coil[4]:

$$
\begin{aligned}
&\sigma_1 A_1 + \sigma_2 A_2 = F_t \\
&\sigma_1^* A_1^* + \sigma_2^* A_2^* = F_t^* \\
&\delta_1 = \delta_2 \\
&\delta_1^* = \delta_2^*,
\end{aligned}
$$

where σ and δ represent the stress and strain, respectively. A is the cross-sectional area perpendicular to the direction indicated by its superscript, since we already know the thickness of the aluminum and Nb-Ti cylinders: 14.1mm and 1.74mm, respectively, all of the areas are easy to get: A_1=A_1^*=0.00174m^2, A_2=0.0141m^2. But A_2^* = 0.00953m^2 is somewhat different from A_2, because on axial direction the cooling aluminum pipe, which is wrapped around the aluminum bore tube, is not glued together, the effective aluminum thickness of the cooling pipe (4.6mm) is not added to the thickness of the aluminum bore tube (9.53mm). The subscript represents the material: 1 for aluminum and 2 for Nb-Ti supercondutor. The variables and parameters with superscript * represent longitudinal direction, ones without the superscript are on the hoop direction. F_t and F_t^* are the magnetic forces on the hoop and longitudinal directions, respectively. Besides those four independent equations, there are another four equations due to Hook's law to relate the stress and strain

$$
\begin{aligned}
\delta_1 &= \frac{\sigma_1}{E_1} - \nu_1^* \frac{\sigma_1^*}{E_1^*} \\
\delta_2 &= \frac{\sigma_2}{E_2} - \nu_2^* \frac{\sigma_2^*}{E_2^*} \\
\delta_1^* &= \frac{\sigma_1^*}{E_1^*} - \nu_1 \frac{\sigma_1}{E_1} \\
\delta_2^* &= \frac{\sigma_2^*}{E_2^*} - \nu_2 \frac{\sigma_2}{E_2}
\end{aligned}
$$

where ν is the Poisson coefficient. Now we only consider the hoop stress, neglect the axial force. The hoop force of unit length along with axial direction is

[4] Time Projection Chamber, Magnet coil (M5349), M. A. Green, Lawrence Berkley Laboratory, June 1979.

$$F_t = \frac{N \cdot i \cdot B \cdot R}{2L}$$

where N is the total turns of the coil, i is the current in each turn, B is the magnetic flux density in the central region of the magnet, R and L are the radius and length of the coil. In our case N = 1772, i = 1463A, B = 1Tesla, therefore F_t = 4.692×10^5 N. Solve the set of equations we get the following results: $\delta_1 = 4.025\times10^{-4}$, $\delta_1^* = -1.288\times10^{-4}$, which correspond to $\Delta R = \delta_1 R = 0.4$mm, $\Delta H = \delta_1^* L = -0.386$mm. These numbers are also similar to the results from ANSYS.

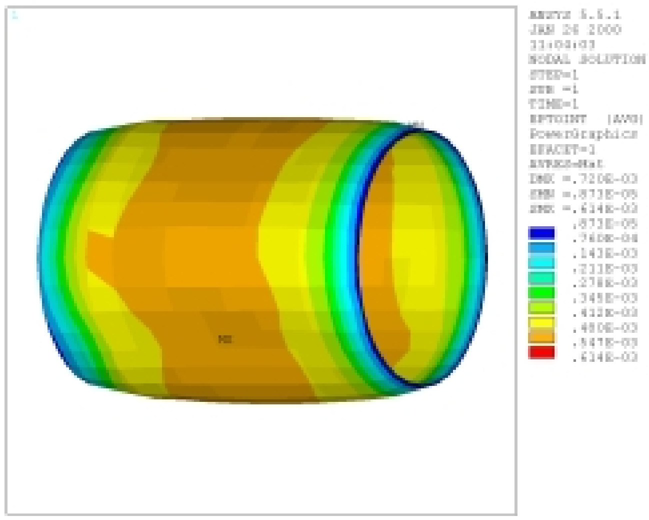

Figure 13. Strain distribution of the coil

Conclusion

The stress and strain analysis by Roak & Young's formulas and ANSYS FEA calculation on the bare superconductor coil provides consistent results. The maximum stress from FEA calculation is 45Mpa, which is much less than the tensile strength for most of the commercial aluminum alloys. With FEA model we can simulate various real situations, such as tilting the coil, placing a steal plate on the floor to balance the lifting force, etc.