The dynamics of mercury flow in a curved pipe

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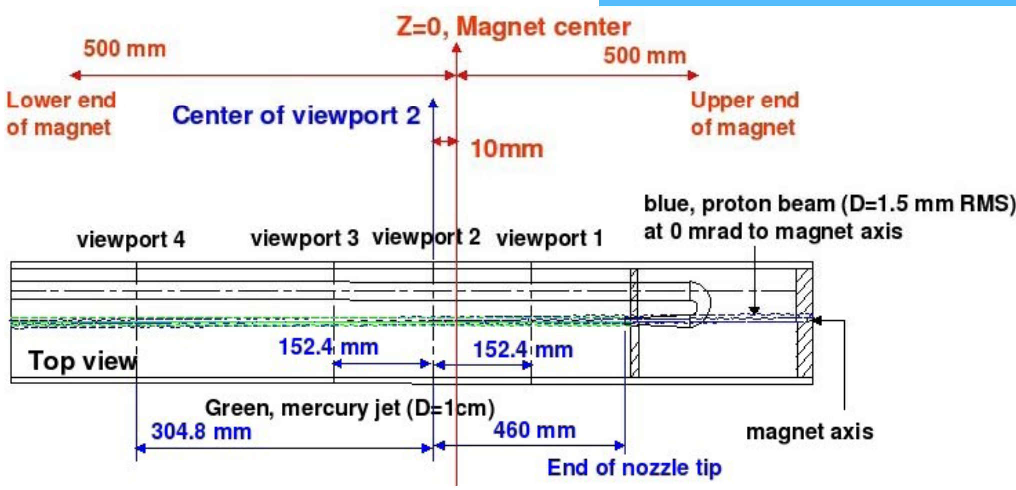
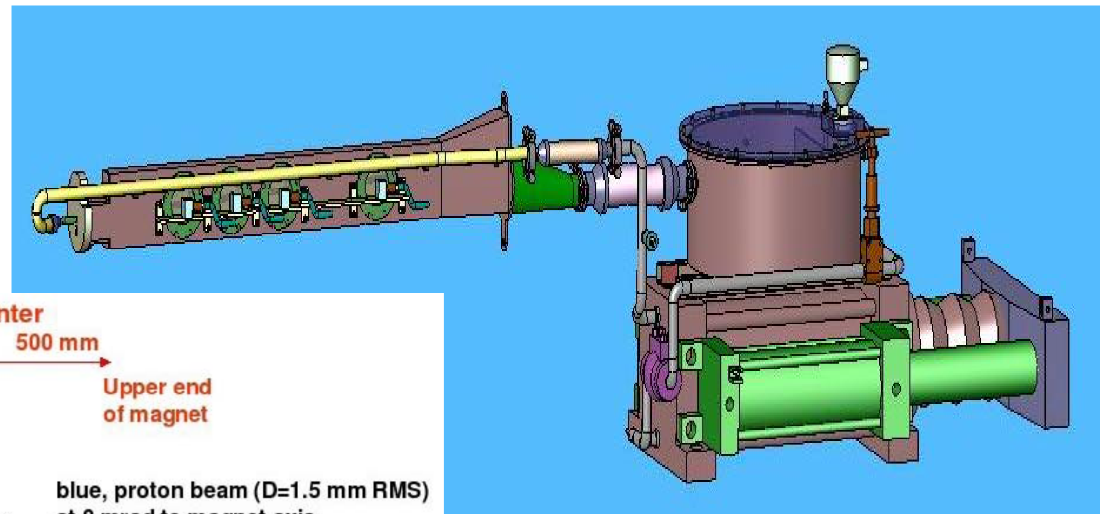
Introduction and Motivation

- Liquid mercury has been proposed as a potential high-Z target for the Moun Collider Accelerator Project
- An experimental setup at CERN(Geneva, Switzerland) is shown below
  - Delivery system requires a 90^0/ 90^0 elbow combination for the Hg supply and return
  - Jet exhausts into vacuum/air
  - High energy beam
  - Magnetic field





Hg delivery system at CERN

Overall Objectives

- To investigate the fluid dynamics of a liquid Hg target for the Moun Collider Accelerator Project
  - ➡ Dynamics of the Hg flow in a curve pipe
  - Influence of magnetic field on Hg pipe flow (MHD)
  - Hg exhaust jet flow
  - Effect of magnetic field on jet flow
  - Effect of high energy deposition on jet flow
  - Combined effects of magnetic field and high energy deposition on Hg jet flow
- Starting with the curved pipe subsystem
- Focus on laminar flow

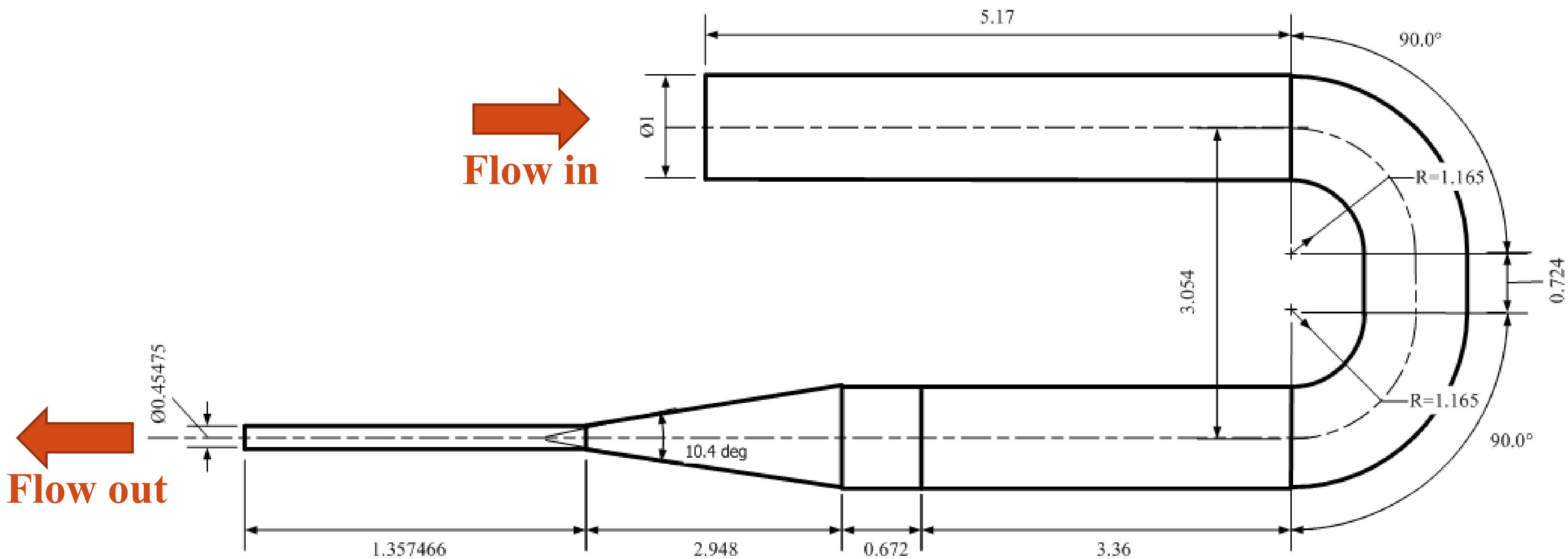
Background

$$\text{Dean Number} = \left(\frac{a}{R}\right)^{1/2} \mathrm{Re}_a$$

- Analytical work
  - Small Dean number (0<D<96)
    - asymptotic solution by Dean (1928)
  - Moderate Dean number (96<D<600)
    - Fourier series by McConalogue and Srivastava (1968)
- Numerical work
  - Moderate Dean number (96<D<600)
    - finite difference solution by Greenspan (1973)
    - entrance flow in curved pipe by Singh (1974) and Yao & Berger (1975)
  - Large Dean number (600<D<5000)
    - finite-difference method by Collins and Dennis(1975)
    - asymptotic boundary-layer theory by Barua(1963)
- Experimental work
  - Williams (1902), Grindley and Gybson (1908), Eustice (1910,1911), White (1929), Taylor(1929), Argawal, Talbot and Gong (1978)

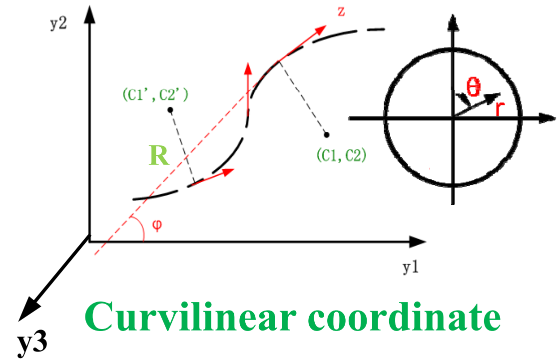
Problem Description

Dimension of the 90°-90° combination bend pipe



Governing Equations

- Curvilinear coordinate (r,θ,z)
- Governing equations $(\delta = \frac{a}{R}, \mathrm{Re} = \frac{a u_0 \rho}{\mu})$



Curvilinear coordinate

  - Continuity

$$\frac{\partial u^*}{\partial r^*} + \frac{u^*}{r^*}\frac{1+2\delta\sin\theta}{1+\delta\sin\theta} + \frac{1}{r^*}\frac{\partial v^*}{\partial\theta} + \frac{\cos\theta\delta}{1+r^*\delta\sin\theta}v^* + \frac{1}{1+r^*\delta\sin\theta}\frac{\partial w^*}{\partial z^*} = 0$$

  - r-Momentum

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial r^*} + \frac{w^*}{1+r^*\delta\sin\theta}\frac{\partial u^*}{\partial z^*} + \frac{v^*}{r^*}\frac{\partial u^*}{\partial\theta} - \frac{v^{*2}}{r^*} - \frac{\sin\theta\delta}{1+r^*\delta\sin\theta}w^{*2} = -\frac{1}{r^*}\frac{\partial P^*}{\partial r^*}$$
$$+\frac{1}{\mathrm{Re}}\Big[\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^{*2}}\frac{\partial^2 u^*}{\partial\theta^2} + \frac{1}{(1+r^*\delta\sin\theta)^2}\frac{\partial^2 u^*}{\partial z^{*2}} - \frac{2}{r^{*2}}\frac{\partial v^*}{\partial\theta} - \frac{2\sin\theta\delta}{(1+r^*\delta\sin\theta)^2}\frac{\partial w^*}{\partial z^*}$$
$$-\frac{\cos\theta\delta}{1+r^*\delta\sin\theta}\left(\frac{\partial v^*}{\partial r^*} + \frac{v^*}{r^*} - \frac{1}{r^*}\frac{\partial u^*}{\partial\theta}\right) + \delta\frac{w^* + r^*\frac{\partial u^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\sin\theta\frac{d\delta^*}{dz^*}\Big]$$

Governing Equations Cont'd

- θ-Momentum

$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}\frac{\partial v^*}{\partial \theta}+\frac{w^*}{1+r^*\delta\sin\theta}\frac{\partial v^*}{\partial z^*}+\frac{v^*u^*}{r^*}-\frac{\cos\theta\delta}{1+r^*\delta\sin\theta}w^{*2}=-\frac{1}{r^*}\frac{\partial P^*}{\partial\theta}$$
$$+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 v^*}{\partial r^{*2}}+\frac{1}{r^{*2}}\frac{\partial^2 v^*}{\partial\theta^2}+\frac{1}{(1+r^*\delta\sin\theta)^2}\frac{\partial^2 v^*}{\partial z^{*2}}+\frac{1}{r^{*2}}\frac{\partial u^*}{\partial\theta}-\frac{v^*}{r^{*2}}-\frac{\delta\cos\theta}{(1+r^*\delta\sin\theta)^2}\frac{\partial w^*}{\partial z^*}\right.$$
$$+\frac{\delta\cos\theta}{1+r^*\delta\sin\theta}\left(\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}-\frac{1}{r^*}\frac{\partial u^*}{\partial\theta}\right)+\delta\frac{\cos\theta w^*+r^*\sin\theta\frac{\partial v^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\frac{d\delta^*}{dz^*}$$

- z-Momentum

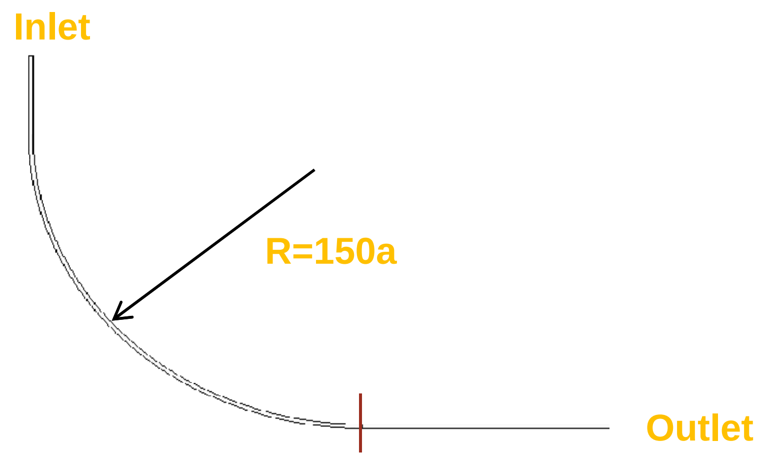
$$\frac{\partial w^*}{\partial t^*}+u^*\frac{\partial w^*}{\partial r^*}+\frac{v^*}{r^*}\frac{\partial w^*}{\partial\theta}+\frac{w}{1+r^*\delta\sin\theta}\frac{\partial w^*}{\partial z^*}+\frac{\cos\theta\delta}{1+r^*\delta\sin\theta}v^*w^*+\frac{\sin\theta\delta}{1+r^*\delta\sin\theta}u^*w^*=-\frac{1}{1+r^*\delta\sin\theta}\frac{\partial P^*}{\partial z^*}$$
$$+\frac{1}{\mathrm{Re}}\left\{\frac{\partial^2 w^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial w^*}{\partial r^*}+\frac{1}{r^{*2}}\frac{\partial^2 w^*}{\partial\theta^2}+\frac{1}{(1+r^*\delta\sin\theta)^2}\frac{\partial^2 w^*}{\partial z^{*2}}+\frac{\delta}{1+r^*\delta\sin\theta}\left(\frac{\cos\theta}{r^*}\frac{\partial w^*}{\partial\theta}+\sin\theta\frac{\partial w^*}{\partial r^*}\right)\right.$$
$$+\frac{1}{r(1+r^*\delta\sin\theta)^2}\left[(2r^*\delta\sin\theta+1)\frac{\partial u^*}{\partial z^*}+r^*\delta\cos\theta\frac{\partial v^*}{\partial z^*}-r\delta^{*2}(\sin\theta\cos\theta+\cos\theta-\frac{1}{r^*\delta}\sin\theta-\sin^2\theta)w\right]$$
$$\left.+\delta\frac{\sin\theta u^*+\cos\theta v^*-r^*\sin\theta(1+\frac{1}{\delta})\frac{\partial w^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\frac{d\delta^*}{dz^*}\right\}$$

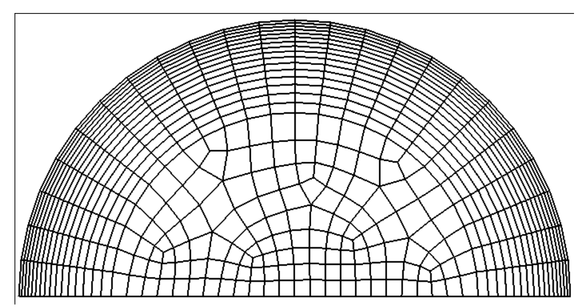
Procedures

- Analytical solution for a constant curvature pipe (Dean, 1927)
- Numerical solution for a constant curvature pipe
- Assessment of the extra terms for a curved pipe
- Numerical solution for the target delivery pipe

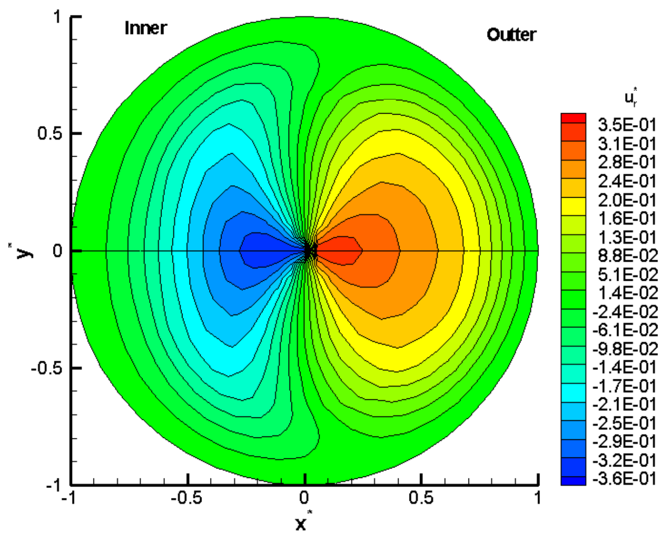
Constant Curvature Case

Reynolds number	**1000**
Pipe diameter	1.127 mm
Pipe Length	20 diameter before bend and 100 diameter after bend
Inlet condition	Fully developed velocity profile and static pressure of 18.5bar
Mesh ($\delta 2$=0.00667)	Axial direction 1600 Radial direction 56 (Δ=0.005) Circumferential direction 24 Total 2150400

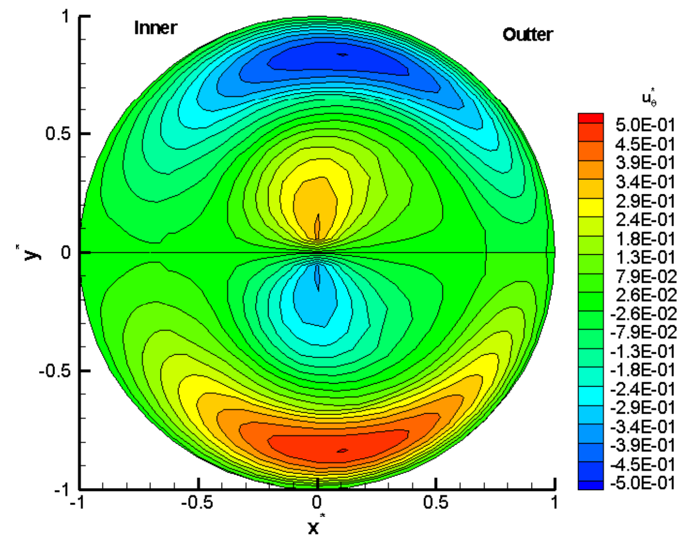




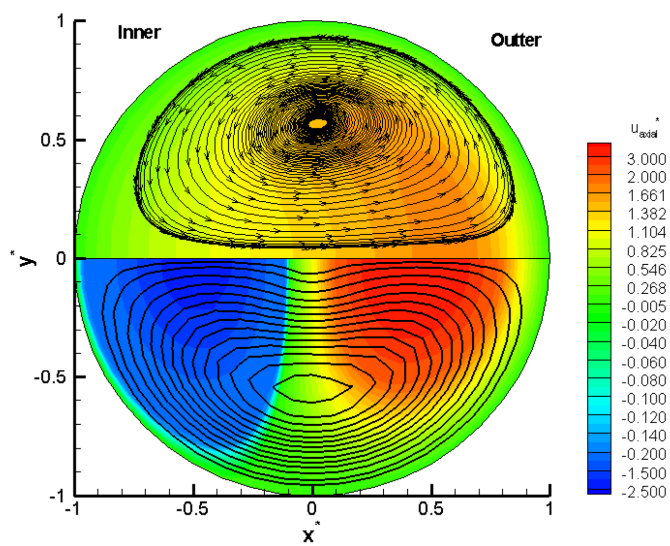
Constant Curvature Case



u^*_r (curved)



u^*_θ (curved)



Streamline (curved)
(Numerical/Analytical)

Assessment of the Extra Terms

Extra terms relative to the straight pipe

Assume $$\frac{1}{1+r^*\delta\sin\theta}\approx 1-r^*\delta\sin\theta+o(\delta^2)$$

- Extra terms in r-momentum

$$D_r^* = w^* r^*\delta\sin\theta\frac{\partial u^*}{\partial z^*}+\frac{w^{*2}\sin\theta\delta}{1+r^*\delta\sin\theta}+\frac{1}{\text{Re}}[-2r^*\delta\sin\theta\frac{\partial^2 u^*}{\partial z^{*2}}+\frac{u^*}{r^{*2}}-\frac{2\sin\theta\delta}{(1+r^*\delta\sin\theta)^2}\frac{\partial w^*}{\partial z^*}$$
$$-\frac{\cos\theta\delta}{1+r^*\delta\sin\theta}(\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}-\frac{1}{r^*}\frac{\partial u^*}{\partial\theta})+\delta\frac{w^*+r^*\frac{\partial u^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\sin\theta\frac{d\delta^*}{dz^*}]$$

- Extra terms in θ-momentum

$$D_\theta^* = r^*\delta\sin\theta w^*\frac{\partial v^*}{\partial z^*}+\frac{\delta\cos\theta}{1+r^*\delta\sin\theta}w^{*2}+\frac{1}{\text{Re}}[\frac{\delta\cos\theta}{1+r^*\delta\sin\theta}(\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}-\frac{1}{1+r^*\delta\sin\theta}\frac{\partial w^*}{\partial z^*}-\frac{1}{r^*}\frac{\partial u^*}{\partial\theta})$$
$$+\delta\frac{\cos\theta w^*+r^*\sin\theta\frac{\partial v^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\frac{d\delta^*}{dz^*}]$$

Assessment of the Extra Terms Cont'd

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- Extra terms in z-momentum

$$D_z^* = r^*\delta\sin\theta w^*\frac{\partial w^*}{\partial z^*} - \frac{\cos\theta\delta}{1+r^*\delta\sin\theta}v^*w^* - \frac{\sin\theta\delta}{1+r^*\delta\sin\theta}u^*w^* + \frac{1}{\mathrm{Re}}\{-2r^*\delta\sin\theta\frac{\partial^2 w^*}{\partial z^{*2}}$$

$$+\frac{\delta}{1+r^*\delta\sin\theta}\left(\frac{\cos\theta}{r^*}\frac{\partial w^*}{\partial\theta} + \sin\theta\frac{\partial w^*}{\partial r^*}\right) + \frac{1}{r(1+r^*\delta\sin\theta)^2}[(2r^*\delta\sin\theta+1)\frac{\partial u^*}{\partial z^*}$$

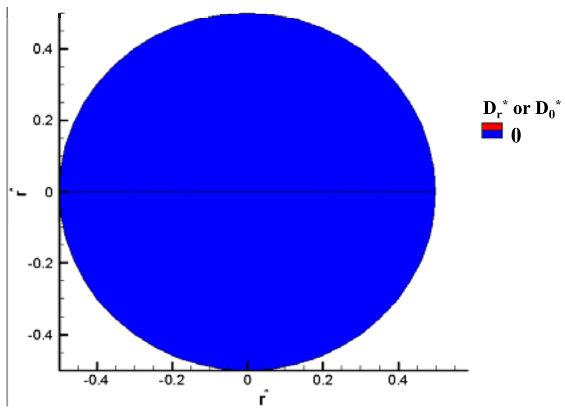
$$-r^*\delta\cos\theta\frac{\partial v^*}{\partial z^*} + r\delta^{*2}(\sin\theta\cos\theta + \cos\theta - \frac{1}{r^*\delta}\sin\theta - \sin^2\theta)w]$$

$$+\frac{\delta\cos\theta v^* + \delta\sin\theta u^* - (1+\delta)r^*\sin\theta\frac{\partial w^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\frac{d\delta^*}{dz^*}]$$

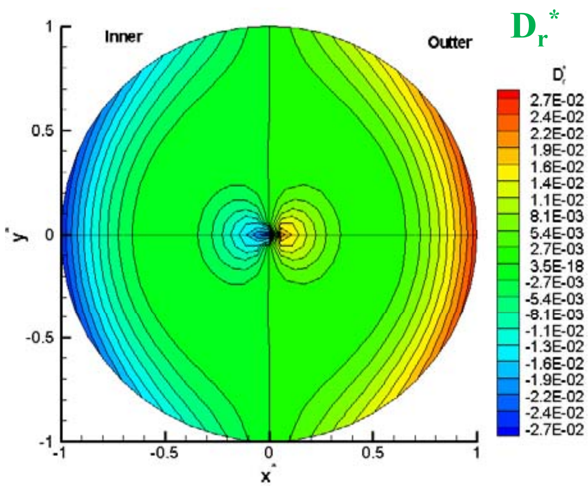
Assessment of the Extra Terms Cont'd

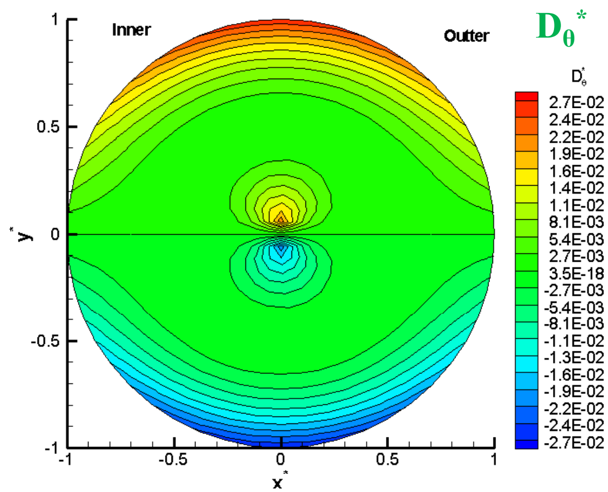
- Case1 Straight pipe
  - Poiseuille flow

D_r^* and D_θ^* for the poiseuille flow



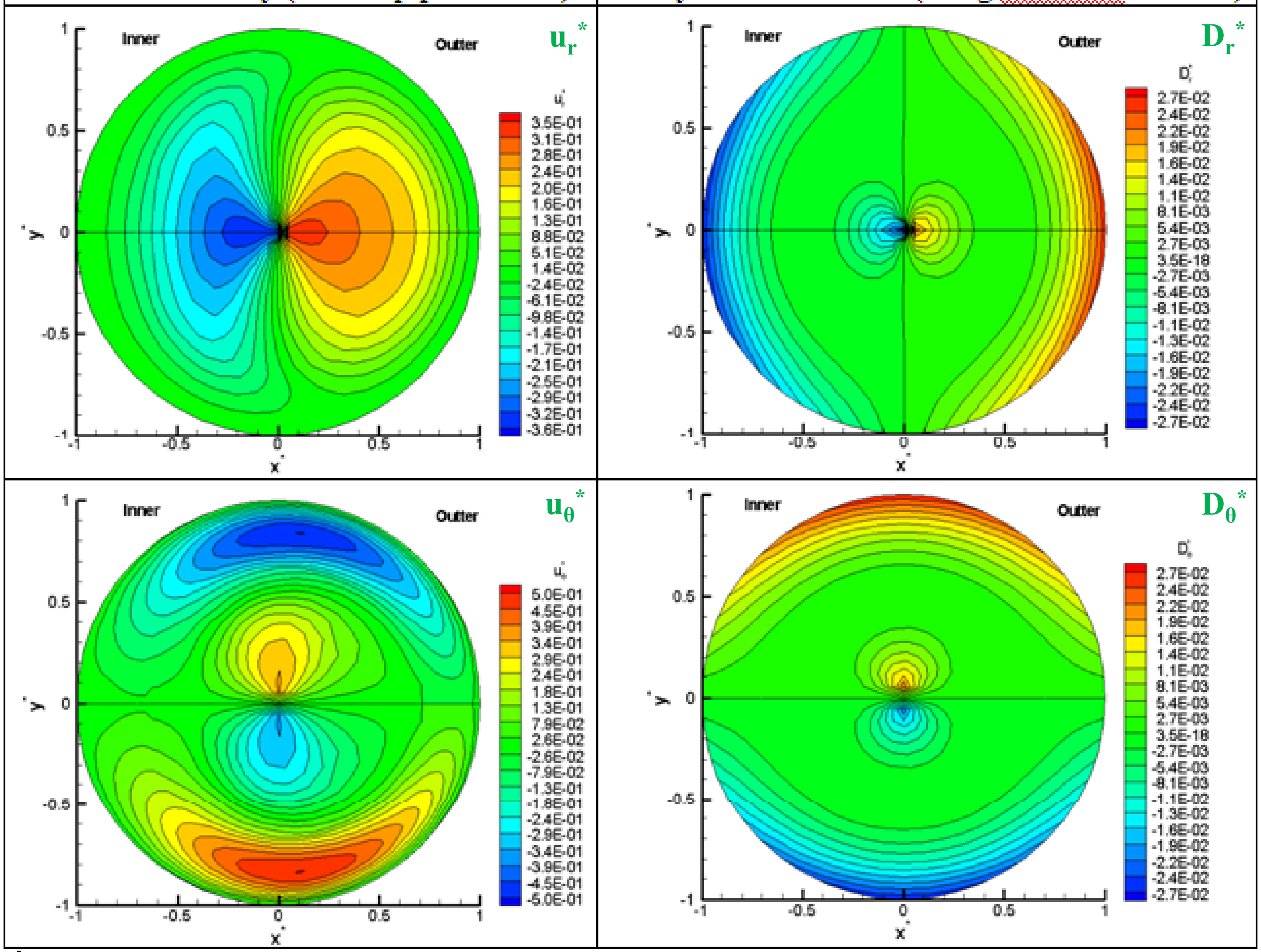
- Case2 Curved pipe
  - Poiseuille flow + extra terms
  - Evaluate of the extra terms using profile for Poiseuille flow



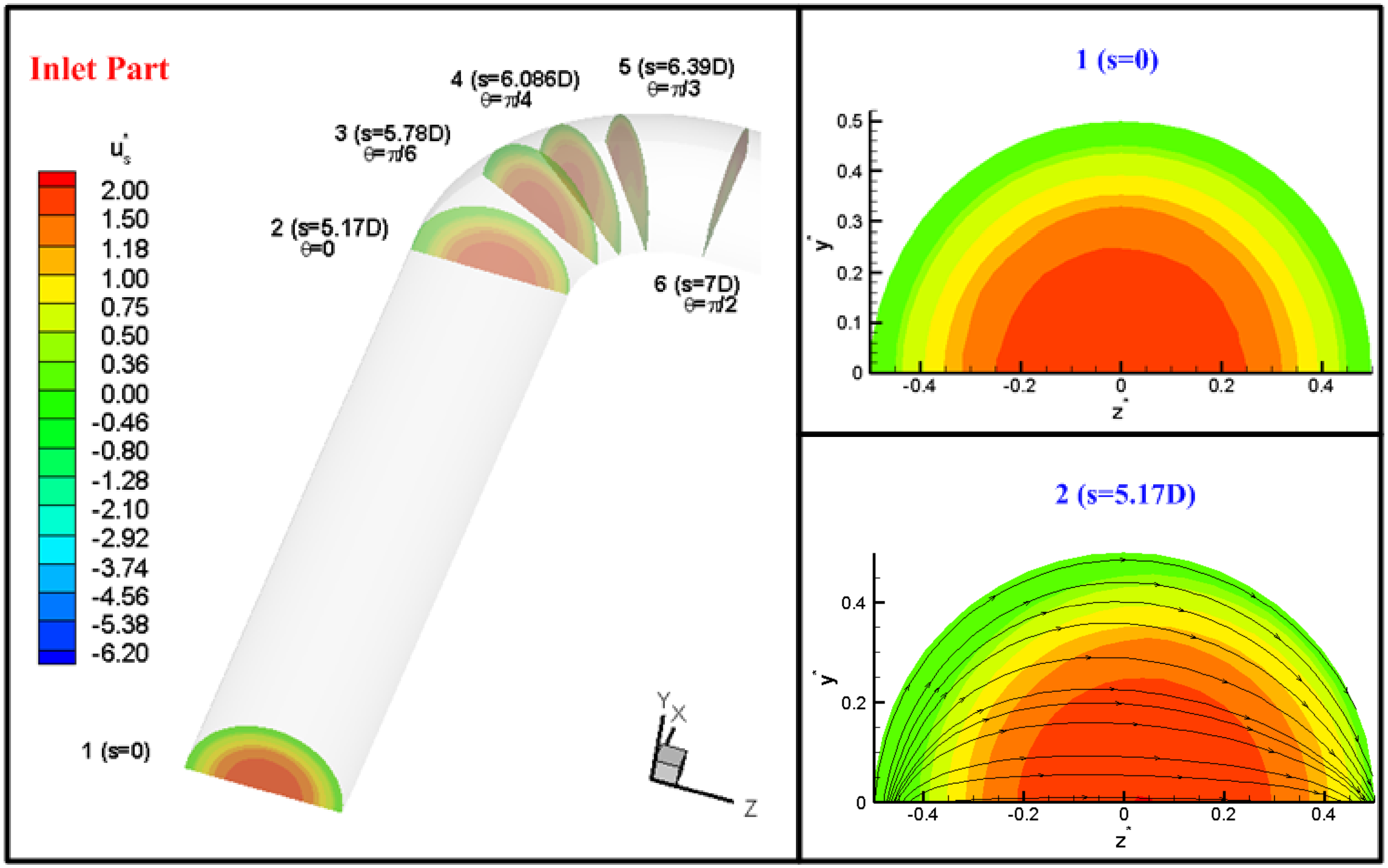


Assessment of the extra terms Cont'd

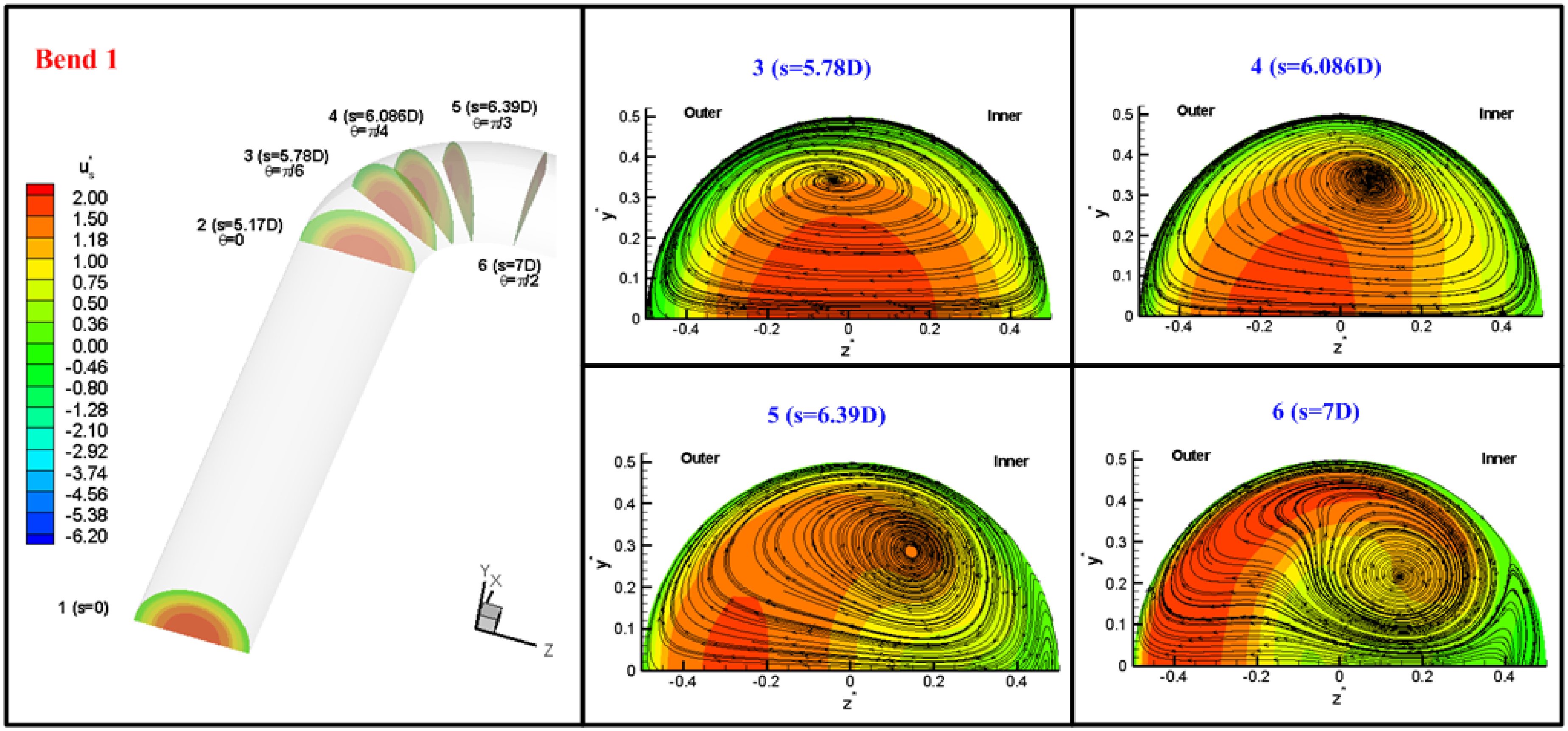
Numerical velocity (curved pipe R=150a)	Analytical extra terms (using Poiseuille solution)



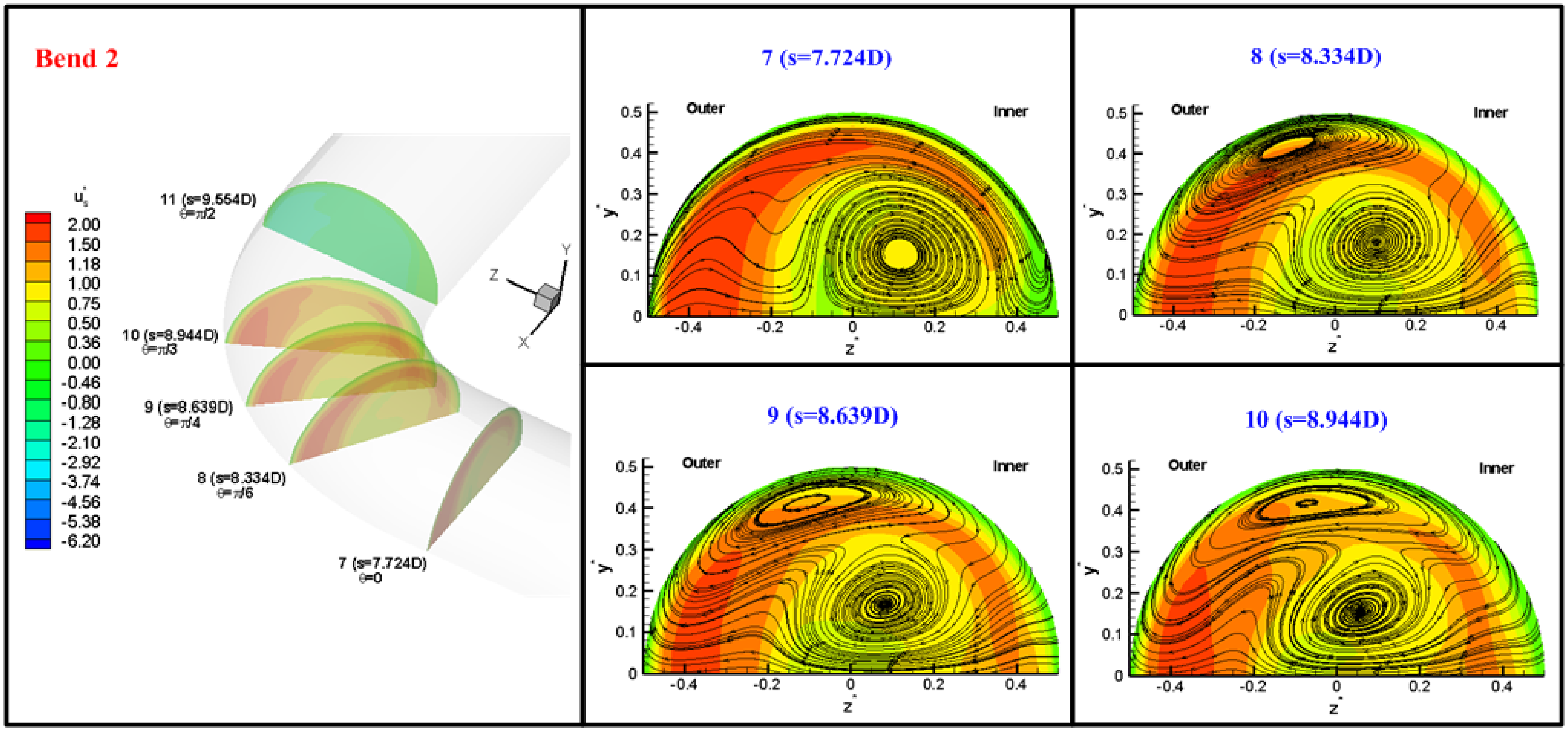
Target Delivery: Inlet + First Bend



Inlet + First Bend

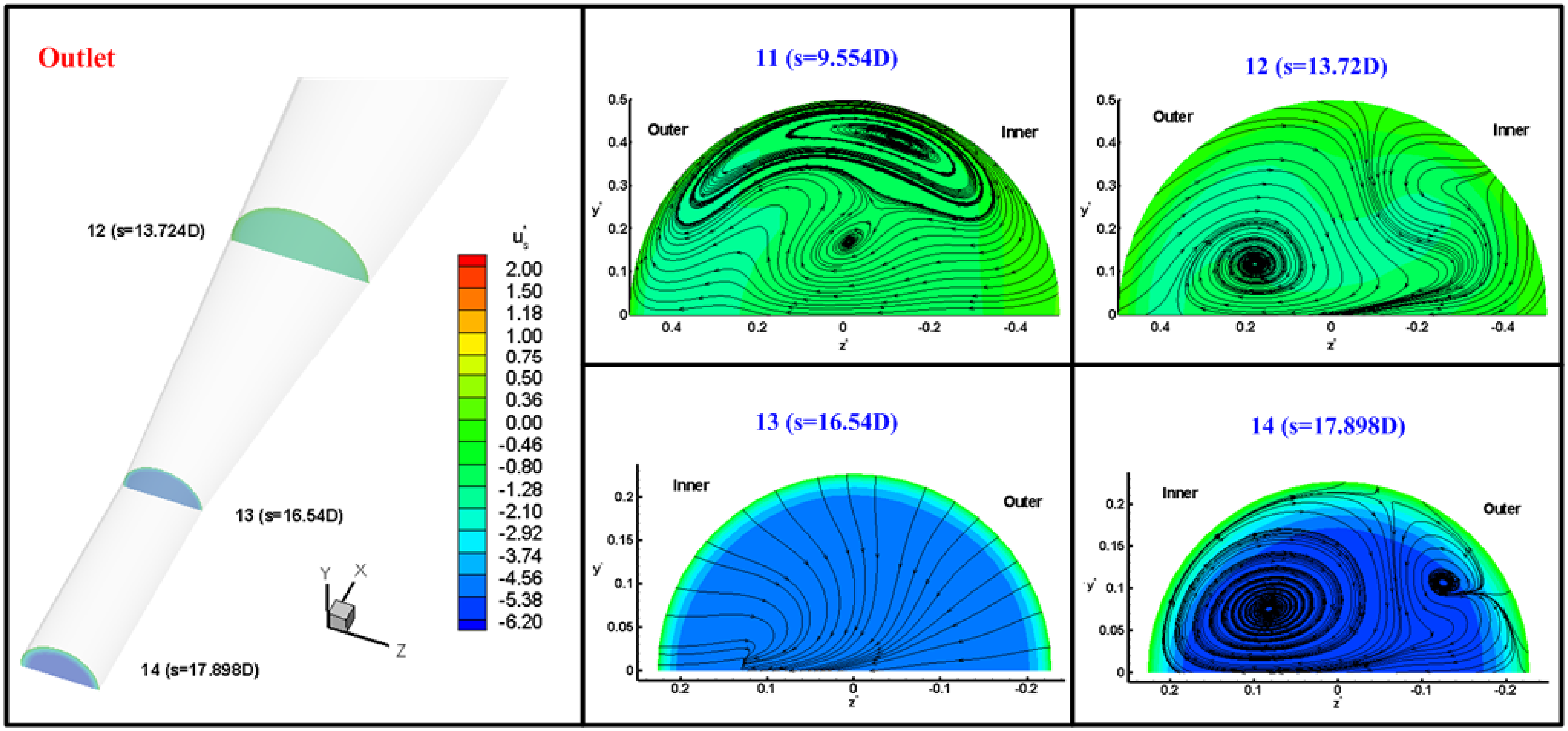


Second Bend



Bend 2
u*s
2.00
1.50
1.18
1.00
0.75
0.50
0.36
0.00
-0.46
-0.80
-1.28
-2.10
-2.92
-3.74
-4.56
-5.38
-6.20
11 (s=9.554D) θ=π/2
10 (s=8.944D) θ=π/3
9 (s=8.639D) θ=π/4
8 (s=8.334D) θ=π/6
7 (s=7.724D) θ=0
7 (s=7.724D)
8 (s=8.334D)
9 (s=8.639D)
10 (s=8.944D)
Outer
Inner
y*
z*

Outlet



Outlet
12 (s=13.724D)
13 (s=16.54D)
14 (s=17.898D)
Y
X
Z
u_s^*
2.00
1.50
1.18
1.00
0.75
0.50
0.36
0.00
-0.46
-0.80
-1.28
-2.10
-2.92
-3.74
-4.56
-5.38
-6.20
11 (s=9.554D)
Outer
Inner
12 (s=13.72D)
Outer
Inner
13 (s=16.54D)
Inner
Outer
14 (s=17.898D)
Inner
Outer
y^*
z^*

Summary

- Preliminary investigation of the flow in an Hg target delivery system for the Muon Collider Accelerator project
- Investigated the source of the secondary flow relative to a straight pipe
- Further work will consider the effects of MHD addition for control and energy deposition for the production of pions, mouns and neutrinos