The Effects of Geometrical Configurations on Curved Pipe Flow for Muon Collider Project

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Abstract

Liquid **mercury** has been investigated as a potential high-Z target for the Muon Collider project. The objective of this part of the project is to develop a target delivery system that results in the least turbulent flow conditions at the exit of the nozzle. In the present work, several **curved pipe** configurations have been studied, in which we examined the dynamics of flow in those configurations using theoretical analysis and fluid flow modeling. Steady turbulent flows have been studied for 0°/0°, 30°/30°, 60°/60°, and 90°/90° elbow combinations, using several **turbulent models**, and comparing the results with experimental data for some of the cases. The generation of **vorticity** by pipe curvature is critically examined and is reported for the various pipe configurations.

Introduction and Motivation

- ❖ Target delivery system requires a $90^o/90^o$ elbow combination for the Hg supply and return
- ❖ Mercury jet exhausts into vacumm/air
- ❖ High energy beam interacts with Hg jet
- ❖ The whole system in the high magnetic field

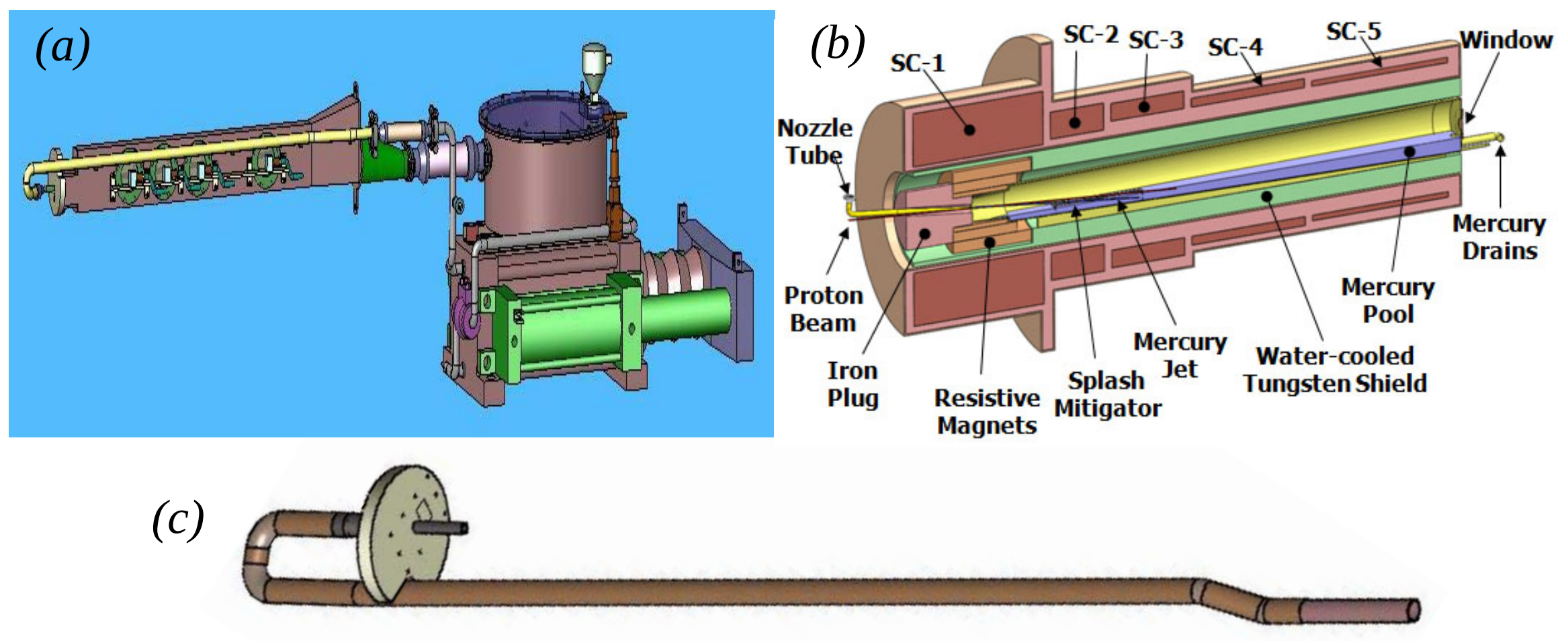

Fig. 1 (a) Mercury delivery system at CERN (b) Concept of a 4-MW target station based on a free mercury jet inside at 20-T solenoid [1] (c) Mercury supply piping (long curved pipe)

Overall Objects and Procedures

Investigate the fluid dynamics of a liquid Hg target for the Muon Collider Accelerator Project:

- ✓ Dynamics of the Hg flow in a curved pipe
- ❖ Effect of magnetic field on Hg pipe flow (MHD)
- ❖ Hg exhaust jet flow
- ❖ Effect of magnetic field on jet flow
- ❖ Effect of high energy deposition on jet flow
- ❖ Combined effects of magnetic field and high energy deposition on Hg jet flow

Procedures to study Hg flow in a curved pipe:

- ❖ Analytical analysis for laminar curved pipe flow
- ❖ Numerical solution for turbulent flow in curved pipe without/with nozzle

Analytical Analysis for Laminar Curved Pipe Flow

Extra Terms for Laminar Curved Pipe Flow

❖Extra terms in the θ momentum

$$D_\theta^* = r^*\delta\sin\theta w^*\frac{\partial v^*}{\partial z^*}+\frac{\delta\cos\theta}{1+r^*\delta\sin\theta}w^{*2}+\frac{1}{\mathrm{Re}}\left[\frac{\delta\cos\theta}{1+r^*\delta\sin\theta}\left(\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}-\frac{1}{1+r^*\delta\sin\theta}\frac{\partial w^*}{\partial z^*}-\frac{1}{r^*}\frac{\partial u^*}{\partial\theta}\right)+\delta\frac{\cos\theta w^*+r^*\sin\theta\frac{\partial v^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\frac{d\delta^*}{dz^*}\right]$$

❖Extra terms in the z momentum

$$D_z^* = r^*\delta\sin\theta w^*\frac{\partial w^*}{\partial z^*}-\frac{\cos\theta\delta}{1+r^*\delta\sin\theta}v^*w^*-\frac{\sin\theta\delta}{1+r^*\delta\sin\theta}u^*w^*+\frac{1}{\mathrm{Re}}\{-2r^*\delta\sin\theta\frac{\partial^2 w^*}{\partial z^{*2}}+\frac{\delta}{1+r^*\delta\sin\theta}\left(\frac{\cos\theta}{r^*}\frac{\partial w^*}{\partial\theta}+\sin\theta\frac{\partial w^*}{\partial r^*}\right)+\frac{1}{r(1+r^*\delta\sin\theta)^2}$$

$$[(2r^*\delta\sin\theta+1)\frac{\partial u^*}{\partial z^*}-r^*\delta\cos\theta\frac{\partial v^*}{\partial z^*}+r^*\delta^2(\sin\theta\cos\theta+\cos\theta-\frac{1}{r^*\delta}\sin\theta-\sin^2\theta)w^*]+\frac{\delta\cos\theta v^*+\delta\sin\theta u^*-(1+\delta)r^*\sin\theta\frac{\partial w^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\frac{d\delta^*}{dz^*}\}$$

❖Extra terms in the r momentum

$$D_r^* = w^*r^*\delta\sin\theta\frac{\partial u^*}{\partial z^*}+\frac{w^{*2}\sin\theta\delta}{1+r^*\delta\sin\theta}+\frac{1}{\mathrm{Re}}\left[-2r^*\delta\sin\theta\frac{\partial^2 u^*}{\partial z^{*2}}+\frac{u^*}{r^{*2}}-\frac{2\sin\theta\delta}{(1+r^*\delta\sin\theta)^2}\frac{\partial w^*}{\partial z^*}-\frac{\cos\theta\delta}{1+r^*\delta\sin\theta}\left(\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}-\frac{1}{r^*}\frac{\partial u^*}{\partial\theta}\right)+\delta\frac{w^*+r^*\frac{\partial u^*}{\partial z^*}}{(1+r^*\delta\sin\theta)^3}\sin\theta\frac{d\delta^*}{dz^*}\right]$$

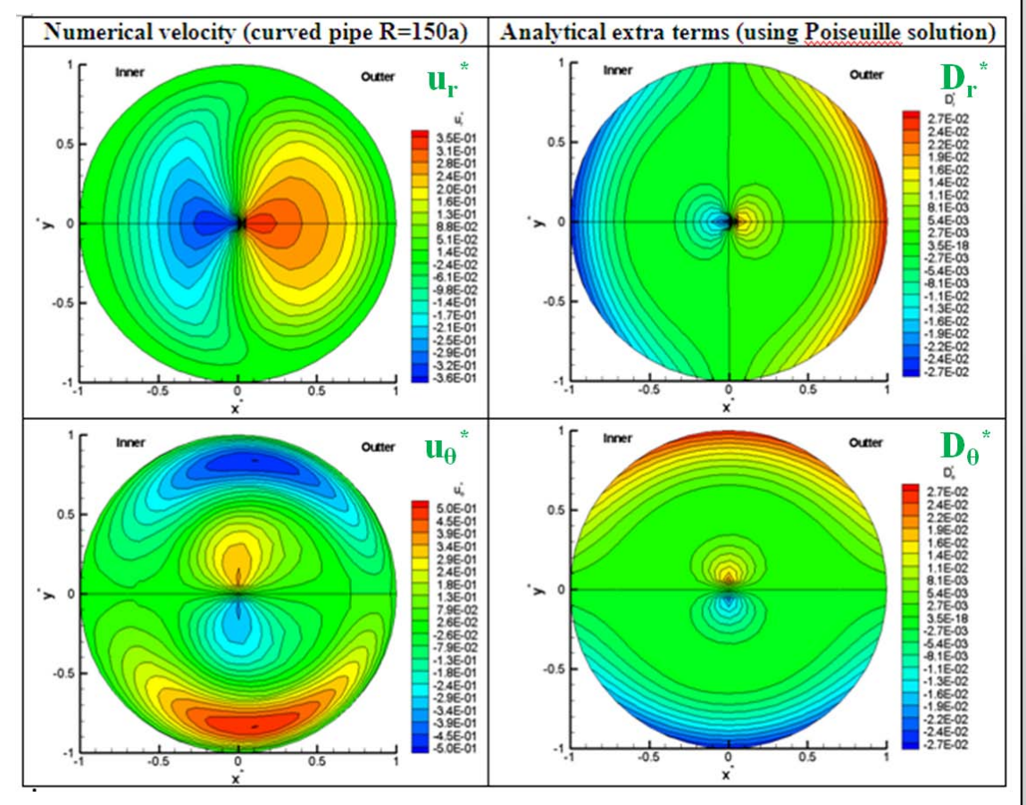

Fig. 2 Assessment of extra terms[2][3]

Numerical Analysis for Turbulent Curved Pipe Flow (Hg)

Turbulence Models Comparison[4]

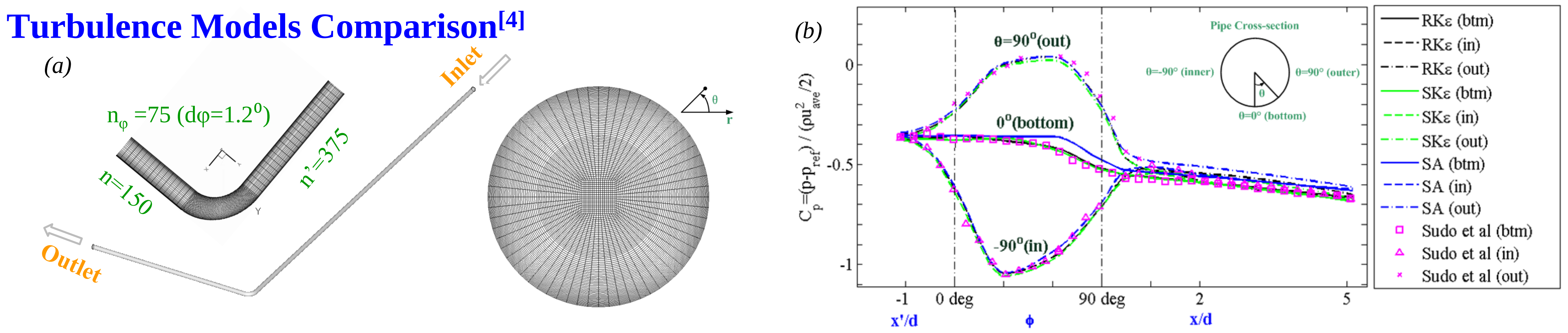

Fig. 3 (a) Mesh for the 90° test pipe (cross-sectional mesh: $n_r \times n_\theta$ =152×64) (b) Longitudinal distribution of static pressure at the inner, outer and bottom of the pipe

Solutions for Mercury Flow in Curved Pipe without Nozzle

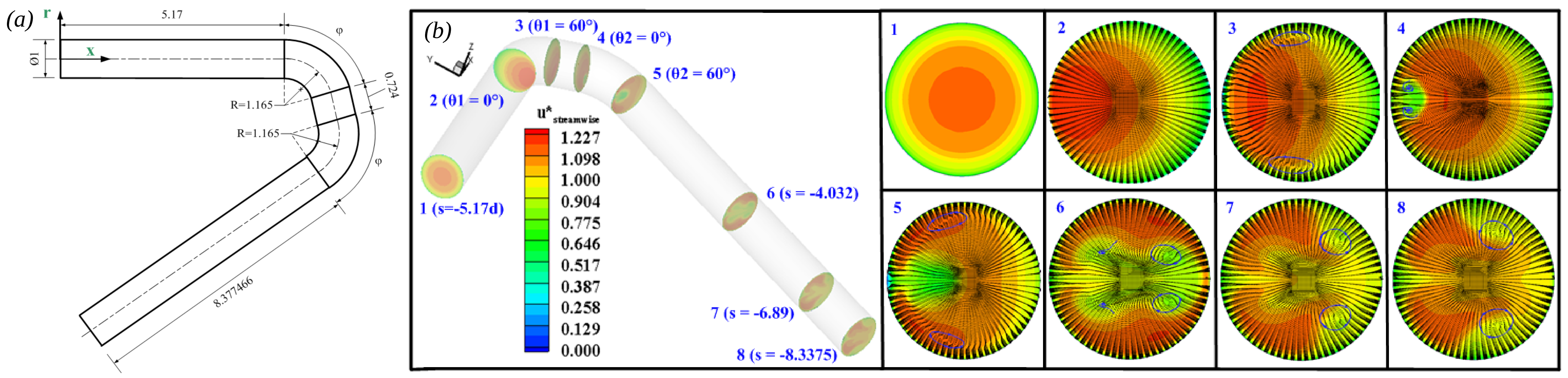

Fig. 4 (a) Geometry of pipes without nozzle in varying angles (φ: 0^0, 30^0, 60^0, 90^0) (b) Stream-wise velocity and velocity vector plots (φ=60°, inner: left, outer: right)

Solutions for Mercury Flow in Curved Pipe with Nozzle

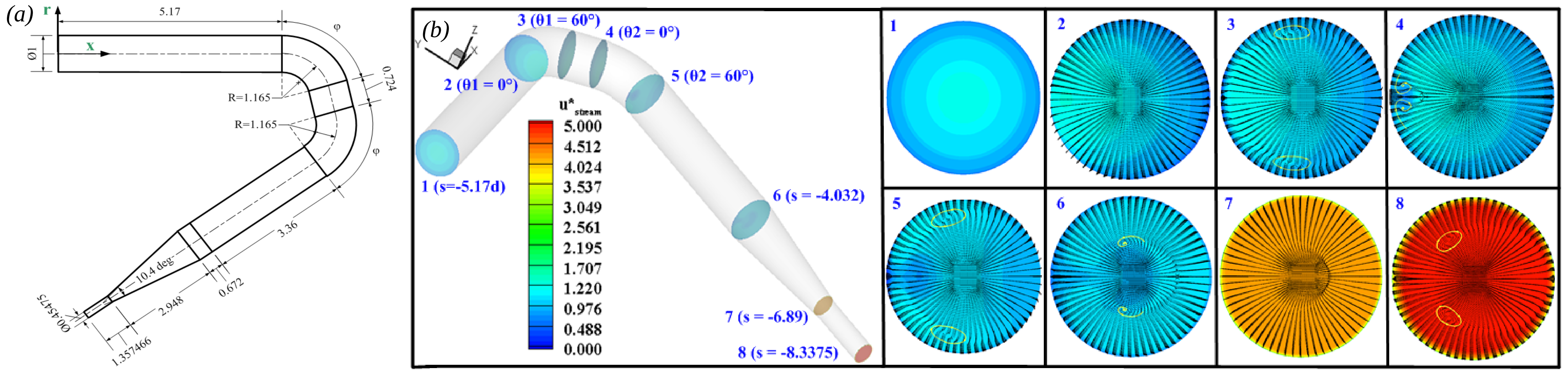

Fig.5 (a) Geometry of pipes with nozzle in varying angles (φ: 0^0, 30^0, 60^0, 90^0) (b) Stream-wise velocity and velocity vector plots (φ=60°, inner: left, outer: right)

Assessment of Static Pressure Through the Bernoulli's Law

$$P_{loss}=P_{main}+\rho g(2h_{elbow}+h_{contr})\approx\rho g(2\xi u_{in}^2/2g+Ku_{out}^2/2g)=198196944\ \mathrm{Pa}$$

$$P_{in}+\rho u_{in}^2/2+\rho gh_1=P_{out}+\rho u_{out}^2/2+\rho gh_1+P_{loss}\Rightarrow P_{out}=208465.3534\ \mathrm{Pa}$$

$$Error=\left|P_{num}-P_{ana}\right|/\left|P_{ana}\right|\times 100\%=\left|189516.3-208465.4\right|/208465.4\times 100\%=9.09\%$$

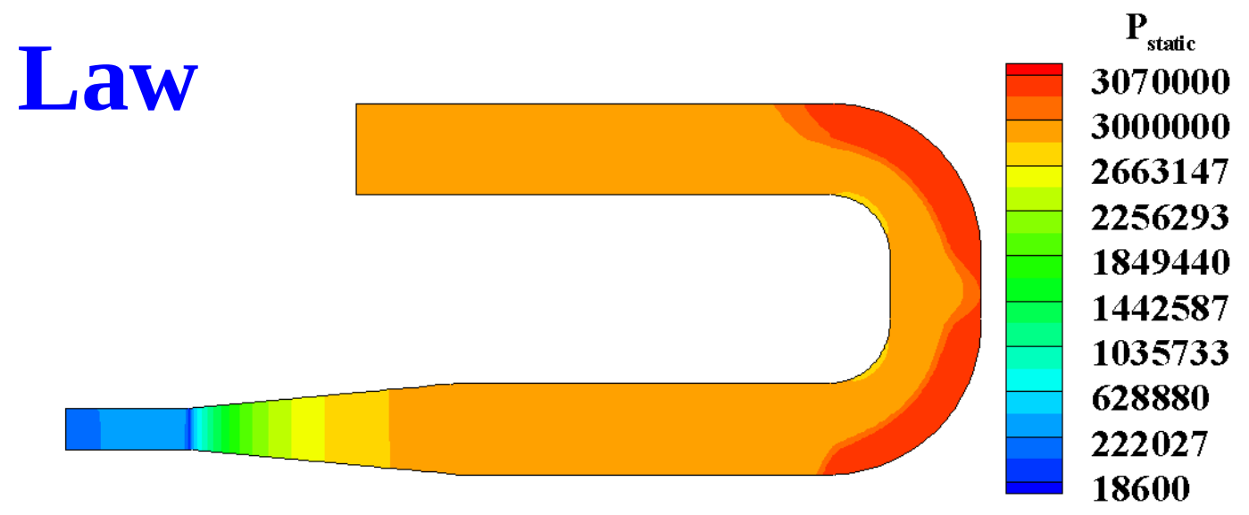

Fig. 6 Static pressure Contour for the $90^0/90^0$ pipe (assume smooth pipe, then the friction loss =zero)

Discussions

Momentum Thickness at the Pipe Exit

$$\theta_t=\int_0^R\frac{u}{U}(1-\frac{u}{U})dr$$

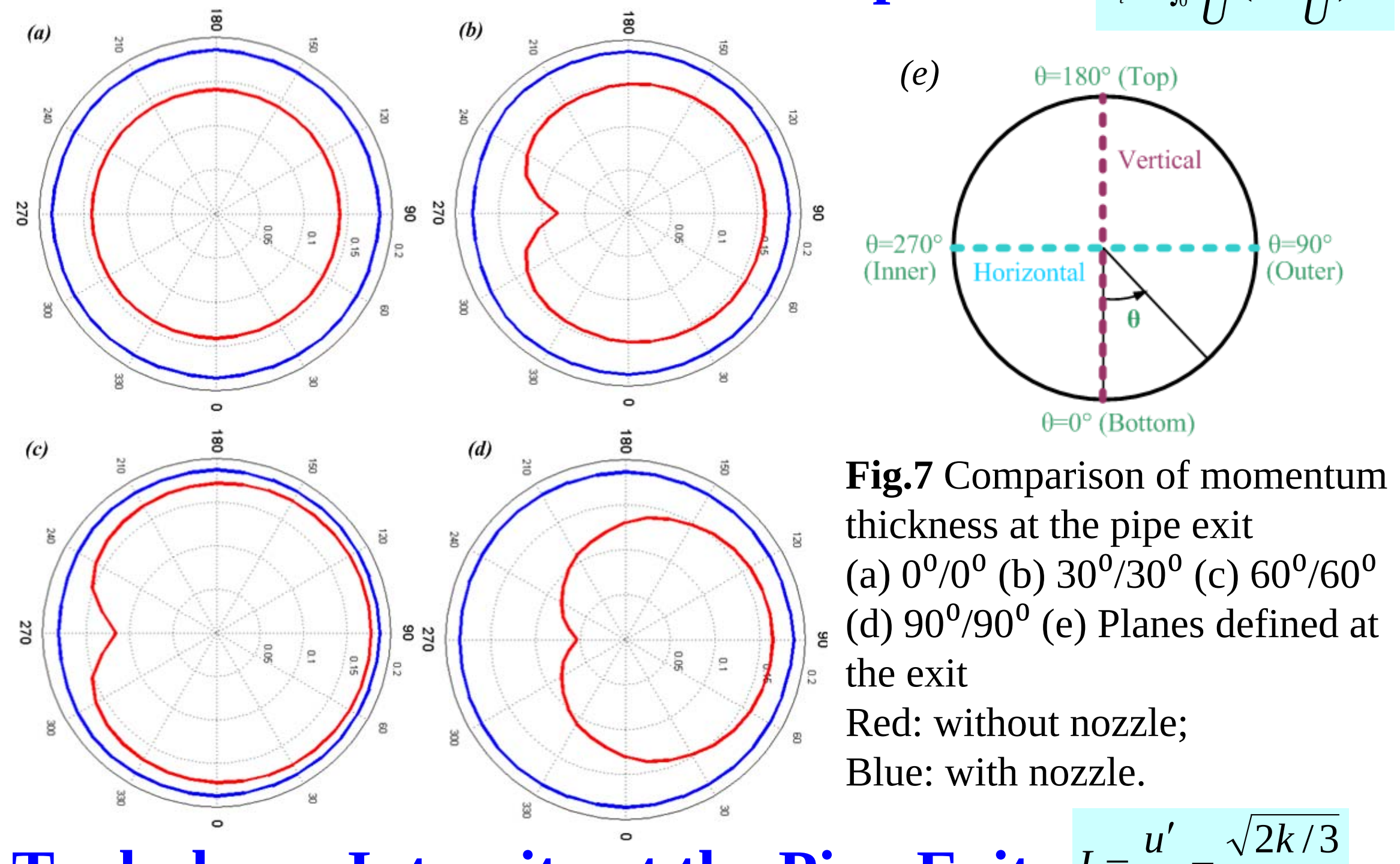

Fig.7 Comparison of momentum thickness at the pipe exit
(a) $0^0/0^0$ (b) $30^0/30^0$ (c) $60^0/60^0$ (d) $90^0/90^0$ (e) Planes defined at the exit
Red: without nozzle;
Blue: with nozzle.

Turbulence Intensity at the Pipe Exit

$$I\equiv\frac{u'}{u_m}=\frac{\sqrt{2k/3}}{u_m}$$

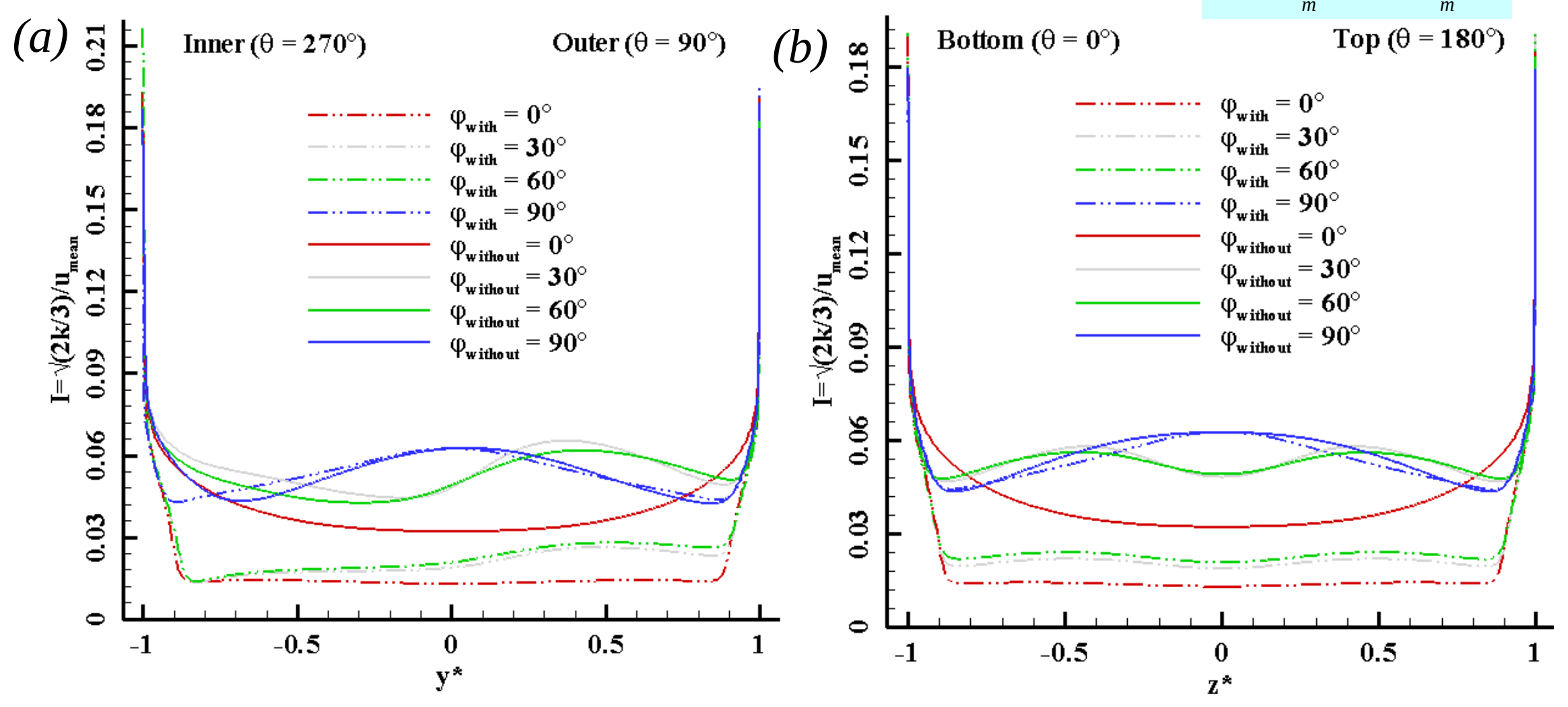

Fig.8 Turbulence intensity distribution comparison at the pipe exit
(a) Along the horizontal plane (b) Along the vertical plane

Merits and Impacts

Turbulent flow conditions are analyzed at the exit of the mercury target supply pipe. The results show :

- ❖**Realizable k-e** turbulence model is able to simulate turbulent mercury flow in curved pipe.
- ❖Bend effects: **Bigger θ_t near the inner side** of the curved pipe, which is even obvious in the $90^0/90^0$ pipe; the $90^0/90^0$ pipe has advantages of symmetry I.
- ❖Nozzle effects: Nozzle **decreases θ_t**, **uniforms θ_t** and **reduces I**.

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