The Bethe-Heitler Process as a Source of Muons

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(April 15, 2015; updated May 20, 2015)

A possible source of muons of both signs for a Muon Collider [1] or Neutrino Factory [2] is the Bethe-Heitler lepton-pair-production process [3, 4, 5, 6],

$$\gamma + A \to A' + \mu^+\mu^-, \tag{1}$$

in high-energy photon interactions with a nucleus A, as recently suggested in [7] (slide 23). Of course, muon-pair production by this process is suppressed relative to electron-positron pair production by the ratio $(m_e/m_\mu)^2 \approx 1/40,000$, so there is an issue of the efficiency of muon production via reaction (1).

The Bethe-Heitler process (1) is further suppressed near threshold, so it would be appropriate to consider photons of energy ≈ 400 MeV, which would yield muons with average kinetic energy ≈ 100 MeV, high enough that the threshold-suppression factor is only $\approx 3/4$, and also the kinetic energy desired for the muon source at Muon Collider/Neutrino Factory. Approximately 10^{14} muons/s of each sign are desired, so the beam power of these muons is roughly 3 kW. But, in addition to the desired muons there are 40,000 times as many electrons/positrons, whose beam power is therefore ≈ 120 MW. Thus, the Bethe-Heitler process (1) for muon production would require a 120-MW beam of 400-MeV photons, essentially all of which power must then be dissipated as "waste" in the muon beam transport.

The 400-MeV photon beam would be generated by backscattering of a laser off an electron (or proton or heavy ion) beam, which latter beam would have to provide the 120 MW of power.

In sum, the Bethe-Heitler production process for muons is too inefficient to be useful for a Muon Collier/Neutrino Factory.[1]

A Appendix: Transverse Emittance

The muon beams of a Muon Collider should have very low transverse (and longitudinal) emittance. In particular, the rms transverse emittance needs to be about 0.001 of that of the initial muon beam as derived from the decay of pions produced in p-N collisions [9]. This reduction of emittance is to be accomplished by ionization cooling, at significant cost. The source emittance of muons produced in the Bethe-Heitler process can be quite small, which is a potential advantage that might compensate for its low efficiency.

However, the source emittance is less relevant than the rms transverse emittance of the muons captured in some kind of beam transport. Such rms emittance is a conserved quantity only in the approximation that the transport is "linear," meaning that all particles trajectories are sinusoidal oscillations about a central ray, which is not the case for, say, propagation of particles in straight lines or within a solenoidal magnetic field. That is, while the local density of (noninteracting) particles in phase space is an invariant (Liouville's) theorem, the

[1] This conclusion was reached in 1994 by Barletta and Sessler [8].

distribution of particles in this phase space becomes “filamented” during “nonlinear” beam transport such that rms measures of emittance increase rapidly along a beam transport system.

Analytic expressions for rms emittance growth during field-free beam transport (drift) are given in [10], and numerical examples are given on slide 8 of [11].

The beam-transport-system concept for a Muon Collider [1] or Neutrino Factory [2] is a solenoid magnet that begins already around the pion/muon production target. The rms transverse emittance grows with distance along a solenoid transport system, until it stabilizes at a value related to the average transverse momentum $\sigma_{P_\perp}$ of the beam and the field strength B of the solenoid magnet,

$$\epsilon_\perp = \frac{\sigma_\perp \sigma_{P_\perp}}{m} \approx \frac{2c\sigma_{P_\perp}^2}{eBm} = \frac{2\left\langle r_c \right\rangle \sigma_{P_\perp}}{m}\,, \tag{2}$$

where $\langle r_c \rangle = c\,\sigma_{P_\perp}/eB$ is the average cyclotron radius for particles in the beam.[2] Note that for particles produced in a target on the axis of the solenoid, $2\langle r_c \rangle$ is the average of the maximum distance of their trajectory from that axis as they move on helical trajectories. The argument for eq. (2) is that when computing the rms transverse emittance, the relevant value for $\sigma_\perp$ is not the small radius of the target, but the much larger diameter of the helical trajectories of the beam particles.

To maintain good collection efficiency of muon produce in p-N collisions, which is the baseline production process considered for a Muon Collider, $\sigma_{P_\perp} \approx 250$ MeV/c, which is the average transverse momentum of the pions whose decay produces the muons. For Bethe-Heitler production of muons with reasonable rate, $\sigma_{P_\perp} \approx m_\mu c \approx 100$ MeV/c. Hence, the ratio of the rms transverse emittance in a solenoidal beam transport system for muons from the Bethe-Heitler process to that for muon from p-N collisions is about 1/6, so the initial transverse emittance for Bethe-Heitler muons is still a factor of 60 larger than that required for a Muon Collider. Substantial ionization cooling is required for muons from both p-N collisions and the Bethe-Heitler process,

The nominally low emittance of a Bethe-Heitler muon source does not lead to a significant advantage over a source based on p-N collisions, and does not compensate for the low efficiency of the Bethe-Heitler process for muon production.

References

[1] M.M. Alsharo'a *et al.*, *Recent progress in neutrino factory and muon collider research within the Muon Collaboration*, Phys. Rev. ST Accel. Beams **6**, 081001 (2003), http://physics.princeton.edu/~mcdonald/examples/accel/alsharoa_prstab_6_081001_03.pdf

[2] R.J. Abrams *et al.*, *International Design Study for a Neutrino Factory: Interim Design Report* (Dec. 2011), http://arxiv.org/abs/1112.2853

[3] H. Bethe and W. Heitler, *On the Stopping of Fast Particles and on the Creation of Positive Electrons*, Proc. Roy. Soc. London A **146**, 83 (1934), http://physics.princeton.edu/~mcdonald/examples/QED/bethe_prsla_146_83_34.pdf

[2]This stabilization of transverse emittance by a solenoid field is illustrated on slide 10 of [11].

[4] L.C. Maximon, *Simple Analytic Expressions for the Total Born Approximation Cross Section for Pair Production in a Coulomb Field*, J. Res. Nat. Bur. Stand. **72B**, 79 (1968), http://physics.princeton.edu/~mcdonald/examples/QED/maximon_jresnbs_72b_79_68.pdf

[5] J.W. Motz, H.A. Olsen and H.W. Koch, *Pair production by photons*, Rev. Mod. Phys. **41**, 581 (1969), http://physics.princeton.edu/~mcdonald/examples/QED/motz_rmp_41_581_69.pdf

[6] A. Mastichiadis, R.J. Protheroe, and J.G. Kirk, *Spectral and temporal signatures of ultrarelativistic protons in compact sources I. Effects of Bethe-Heitler pair production*, Astron. Astrophys. **433**, 765 (2005), http://physics.princeton.edu/~mcdonald/examples/QED/mastichiadis_aa_433_765_05.pdf

[7] M.W. Krasny, *The HIGS proposal and its highlights* (Jan. 2015), http://physics.princeton.edu/~mcdonald/examples/accel/krasny_0115.pdf

[8] W.A. Barletta and A.M. Sessler, *Characteristics of a high energy $\mu^+\mu^-$ collider based on electro-production of muons*, Nucl. Instrum. Meth. A **350**, 36 (1994), http://puhep1.princeton.edu/~mcdonald/examples/accel/barletta_nim_a350_36_94.pdf

[9] B.B. Palmer *et al.*, *A Complete Scheme of Ionization Cooling for a Muon Collider*, Proc. PAC07, p. 3193, http://physics.princeton.edu/~mcdonald/examples/accel/palmer_pac07_3193.pdf

[10] K.T. McDonald, *Emittance Growth from Weak Relativistic Effects* (Mar. 11, 2011), http://physics.princeton.edu/~mcdonald/examples/growth.pdf

[11] K.T. McDonald, *Comments on Emittance Calculations* (Apr. 8, 2011), http://www.physics.princeton.edu/~mcdonald/mumu/target/emittrans1.pdf