BUNCH COALESCING IN A HELICAL CHANNEL*

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Abstract

A high-luminosity Muon Collider requires bunch recombination for optimal luminosity. In this paper, we take advantage of the large slip factor attainable in a helical transport channel (HTC) to coalesce bunches of muons into a single one over a shorter distance than can be achieved over a straight channel.

INTRODUCTION

A muon collider provides a possible path for the U.S. to return to the high energy leadership. One configuration of a muon collider [1] is shown in Figure 1. The start of the muon collider is at pion production from protons on a high-Z target. The pions are captured in a tapered solenoidal field (not shown at the target) and are allowed to simultaneously decay into muons and drift to develop an energy-time correlation that is used in a phase rotation to create a string of mono-energetic bunches. The bunch string is cooled transversely in the Linear Cooler, followed by further 6D cooling in the Helical Cooler. The cooled string of bunches are then merged into a single bunch, which is the subject of this paper, and re-cooled in 6D to produce a single bunch of muons that is ready for the final stage of cooling, followed by a series of accelerations to their final collision energy. The study reported here exploits the larger slip factor attainable in a helical channel to merge the muon bunches over a shorter distance than is possible over a straight channel.

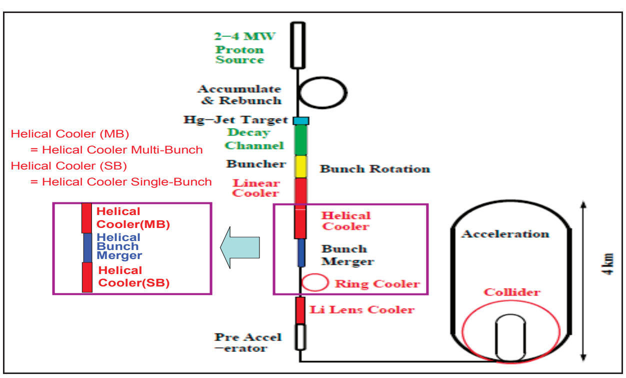

Figure 1: Muon Collider layout. To the right of the arrow is a configuration of a muon collider [1]. This study coalesces muon bunches in a helical channel, as shown to the left of the arrow.

ADVANTAGES OF BUNCH COALESCING IN A HELICAL CHANNEL

As stated above, muon bunches can be coalesced over a shorter distance in a helical channel than can be achieved in a straight channel, resulting in reduced decay losses and likely lower cost of infrastructure and operation. The shorter coalescing distance is afforded by the larger absolute value for the slip factor attainable in a helical channel rather than relying solely on kinematics of the particles in a straight one. Table 1 quantifies the difference for muons with a variety of kinetic energies where there is an initial energy-distance (E-D) relation of dE/d(ct) = 4.09 MeV/m. The distance needed to align bunches with a linear E-D relation in a straight channel is:

$$L_{z,straight} = \left(\beta^3 \gamma\, m_\mu\right) / \left(\eta / \left(dE/d(ct)\right)\big|_{bunches}\right) \tag{1}$$

where β and γ are the usual Lorentz kinematic parameters and η is the slip factor defined in a straight channel as:

$$\eta = -\frac{1}{\gamma^2} \tag{2}$$

The distance over which bunches can be coalesced in a helical channel is[2]:

$$L_{z,HC} = \frac{1}{\eta_{HC}} \frac{m_\mu}{\left(dE/d(ct)\right)\big|_{bunches}} \tag{3}$$

where the slip factor for a Helical Channel η_{HC} is derived in [3] and the value used is driven by that in a design of the upstream Helical Cooling Channel, HCC, [4,5]:

$$\eta_{HC} = \frac{\sqrt{1+\kappa^2}}{\gamma\beta^3}\left[\frac{\hat{D}\kappa^2}{1+\kappa^2} - \frac{1}{\gamma^2}\right] \cong 0.43 \tag{4}$$

$$\hat{D}^{-1} = \frac{\kappa^2 + (1-\kappa^2)}{1+\kappa^2}\left[\frac{B\sqrt{1+\kappa^2}}{pk} - 1\right] - \frac{\left(1+\kappa^2\right)^{3/2}}{pk^2}\frac{\partial b}{\partial a} \cong 0.59 \tag{5}$$

with values for other helical channel parameters[6]:

- λ = longitudinal spatial period = 1 m
- r_{ref} = reference radius = 16 cm
- $\kappa = p_T/p_z = 2\pi r_{ref}/\lambda = 1$ = tangent of the pitch angle of the reference trajectory
- $B_z|_{z\text{-axis}} = 5.7$ T, $B_z|_{\text{on reference}} = 5.0$ T
- $b_\varphi(r)|_{\text{on reference}} = 0.72$ T
- $\delta b_\varphi/\delta\rho(\text{ref}) = -1.2$ T/m.

Table 1: Coalescing distances and muon decay losses in straight and helical channels for muon bunches with initial energy-distance relation of dE/d(ct) = 4.09 MeV/m. Reference particle kinetic energy is 200 MeV.

KE (MeV)	P (MeV/c)	$L_{z,straight}$ (m)	$L_{z,HC}$ (m)	ΔN/N straight (%)	ΔN/N HC (%)
120	199	174	60	13	6.6
200	287	517	60	25	4.6
250	340	858	60	33	3.9

HELICAL CHANNEL BUNCH COALESCING SUBSYSTEM

The Helical Channel Bunch Coalescing (HCBC) subsystem consists of five subcomponents, each performing a specific operation:

1. Acceleration to desired initial energy

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- Accelerates bunches out of HCC to higher energies where bunch coalescing works better.

2. Creation of linear energy-time correlation
   - Creates an energy-time correlation amongst the bunches, so they can be aligned at end of the drift.
3. Drift to align bunches in time
   - The bunches drift and are aligned and the end.
4. Capture into a single RF bucket
   - RF is turned on where the bunches are aligned.
5. Deceleration and longitudinal cooling
   - Decelerate bunch to match the downstream HCC.
   - Longitudinal cooling may also be needed.

Acceleration to Desired Initial Energy

Bunch coalescing works best at higher energies due to the more linear relationship between the relativistic parameter β and energy, but must also consider the acceptance of the channel. These points are best illustrated via simulations, as shown in Figure 2, where this is illuminated in two ways. In Figure 2(a), muons of differing energies start their drift in the Helical Channel and 60 m downstream they have the energy-time (E-T) relation shown in Figure 2(b), where a linear E-T relation has developed at the higher energies. Conversely, if a linear E-T relation exists at the start as in Figure 2(c), free drifts over 60 m result in the E-T relation in Figure 2(d), where the higher energy muons with kinetic energies between ~170 to ~270 MeV have now been aligned in time and longitudinal location. Note that muons exiting the upstream HCC will be in a consecutive series of RF buckets having kinetic energy of 120 MeV (p = 200 MeV/c). So, the first subcomponent will adiabatically accelerate those cooled bunches from 120 to 200 MeV, while maintaining the cooled normalized emittances of $\varepsilon_{T,N}$ = 0.34 mm-rad and $\varepsilon_{L,N}$ = 1.1 mm [4].

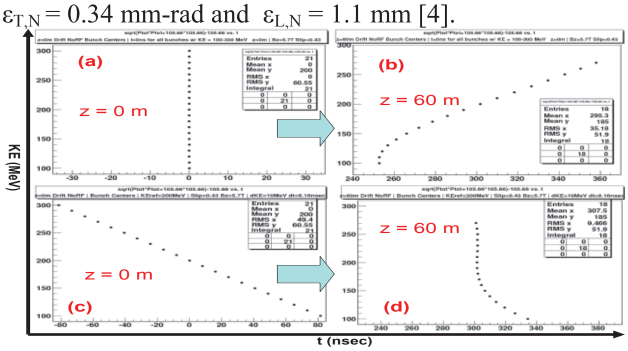

Figure 2: Simulations illustrating kinematics that motivate use of higher energies to coalesce bunches. See text for explanation.

Creation of Linear Energy-Time Correlation

After the cooled muon bunch string from the HCC has been accelerated to 200 MeV, the bunch string must now be manipulated to have a linear E-T relation, as in Figure 2(c). Our method is designed to coalesce 9 bunches and uses a modest RF gradient is used (V'_{max} = 1 MV/m) with frequencies varying along the channel, where the frequency at any particular location is determined by the time difference between the center reference bunch that is not accelerated and the fourth bunch ahead of it that is accelerated with a phase of 30º; hence, there are $4^1/_{12}$ number of wavelengths between these four bunches. The resultant range of frequencies is 204.17 to 271.84 MHz and application of this RF system to 13 bunches (recall designed for 9) is shown in Figure 3, where the end of this section is chosen at 40m, since ΔE within each bunch is minimized. The resulting slope between the bunches is dE/dt = -1.229 MeV/nsec (or dE/d(ct) = 4.09 MeV/m).

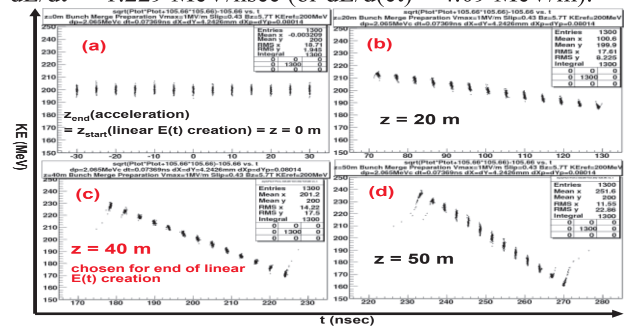

Figure 3: Creation of linear energy-time relation. The bunch string at end of the initial acceleration to 200 MeV is shown in (a) and linear energy-time relation at 20m, 40m, and 50m are shown in (b), (c), and (d), respectively.

Drift to Align Bunches in Time

The distance over which the bunches are aligned is determined in the linear approximation by the dE/dt slope of the bunch string and the slip factor of the channel. In particular,

$$L_z = \frac{1}{\eta_{HC}} m_\mu c^2 \frac{c}{\left|\frac{dE}{dt}\right|} = 60m \qquad (6)$$

where

- L_z is the length over which bunches are aligned
- η_{HC} = 0.43 is the slip factor in equation (4)
- dE/dt=-1.229 MeV/nsec from Figure 3(c)

Simulations shown in Figure 4 confirm the calculated bunch coalescing distance of 60 m.

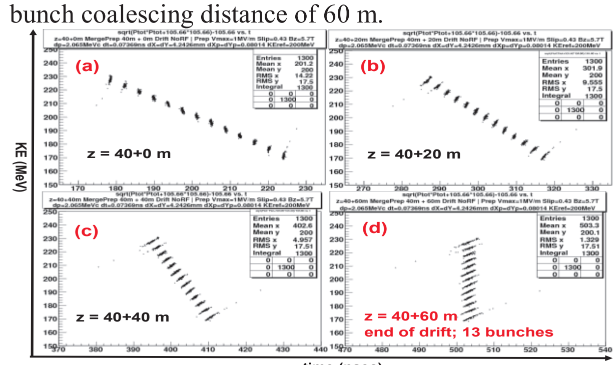

Figure 4: Drift of bunches to time align. Bunches enter the drift region in (a) and drift 20m, 40m, and 60m in (b), (c), and (d), respectively, aligning at 60 m as calculated.

Capture into a Single RF Bucket

As the multiple bunches are time aligned, RF is applied to hold them together in a single bunch. We used 200 MHz with a voltage gradient of 10 MV/m in vacuum; the longitudinal dynamics 5 m into the RF capture are shown in Figure 5. The rate of capture of the original 9, 11, and 13 bunches are 99.7%, 98.4%, and 94.2%. Muon decays are ignored, which would be ~8% for the 105m (horizontally) long channel, which ignores the section that provides the initial equal acceleration to all bunches.

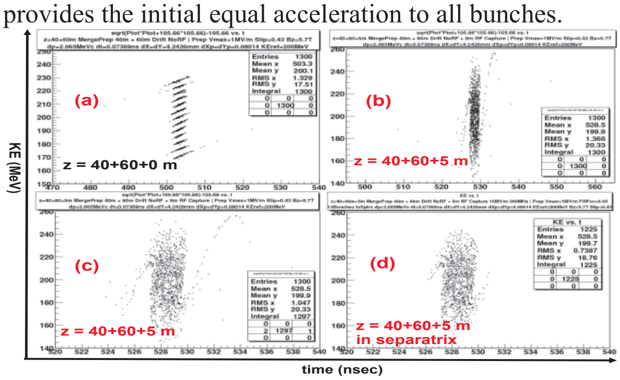

Figure 5: RF capture into a single bunch. Bunches at end of drift section in (a) and captured into a single bunch in (b). Zoomed in views are shown in (c) and (d), where only muons in the separatrix are shown in (d).

Deceleration and Longitudinal Cooling

A single bunch HCC has not yet been designed, but if one is to use the multi-bunch HCC's acceptance for guidance, longitudinal cooling may be necessary at expense of growing transverse emittance [6]. A design that addresses this is to implement radial wedges and phase the RF to zero crossing at the single bunch center, which realizes the following benefits:

- Deceleration is accomplished by material, not RF, so the bucket size remains large.
- Emittance exchange shrinks ε_L at expense of growing ε_T, where growth of ε_T is tolerable.

The amount of deceleration and longitudinal cooling needed will be determined from the future design of the second HCC.

SIMPLIFICATION OF THE HELICAL BUNCH COALESCING SUBSYSTEM

The most complicated section in the bunch coalescing subsystem is the portion that creates the linear energy-time correlation via a large number of RF cavities, each having its own frequency and short length of 10 cm. We investigated the effect of engineering simplifications that use longer RF cavities (25 cm long), less fill factors (~50% and ~25%), and larger gradients (2.2 MV/m and 4.4 MV/m) as described in Table 2. Simulations are carried to the end at the RF capture, with efficiency being calculated by muons inside the separatrix, as shown in Figure 5(d) with results in Figure 6. The configurations with the reduced fill factor (~50% and ~25%) performed equally well and were consistently ~4% less efficient compared to the original configuration that consists of 400 cavities, thus adding confidence in the robustness of the technique to coalesce bunches in a HTC.

Table 2: RF fill factor (FF) characteristics of original and simplified versions

	Configuration		
	Original	~50% FF	~25% FF
RF Field FF (%)	95%	49%	24.5%
Cavity Length (cm)	10	25	25
Number of Cavities	400	80	40
V'max (MV/m)	1.0	2.2	4.4

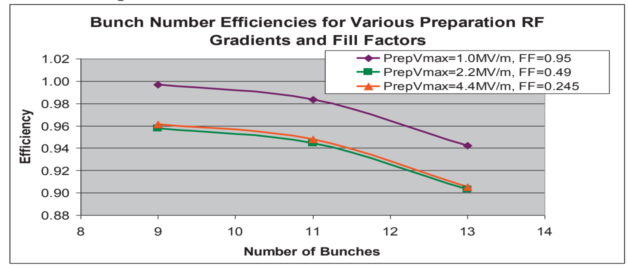

Figure 6: Efficiency of bunch coalescing for different configurations to create the linear energy-time relation for different number of bunches, as calculated by muons in the bunch separatrix.

SUMMARY AND FUTURE

A bunch coalescing subsystem in a helical channel has been designed that makes minimal modifications to the magnetic fields of an existing HCC design that cool muons. The ~105m long coalescing subsystem that is designed to merge 9 bunches is able to achieve efficiencies of 99.7%, 98.4%, and 94.2% for 9, 11, and 13 bunches, respectively. Simplified designs incorporating fill factors for RF cavities of ~25% and ~50% obtained efficiencies of 96%, 94-95%, and 90-91% for 9, 11, and 13 bunches, respectively. The efficiencies do not include decay losses, which would be ~8% for muons with kinetic energy of 200 MeV. Radial wedges in the helical channel may be needed to reduce the longitudinal emittance and reduce the operating energy to that of the second HCC, which is yet to be designed.

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