Conceptual Design of a Magnet System to Generate 20 T in a 0.15 m Diameter Bore, Employing an Inductor Precooled by Liquid Nitrogen

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***Abstract*—The research program for an eventual neutrino factory or muon collider needs a magnet of ~0.15 m diameter bore to generate ~20 T over a length of ~0.3 m. Downstream for ~3 m the field should fall gradually to ~1.25 T, while the bore increases fourfold inversely as the square root of the field. A conventional magnet would require ~40 MW; a superconducting or hybrid magnet might cost tens of millions of dollars. An economically feasible system employs a pulse magnet precooled by liquid nitrogen, with two sets of coils energized sequentially. An outer set of coils of ~12 tons, energized in ~20 s by a 16 kA, 250 V supply available at Brookhaven National Laboratory, generates a peak field of ~9 T and stores ~20 MJ. A resistor of ~¼ Ω inserted across the terminals of the set introduces a voltage drop, initially ~4 kV, to energize an inner set of coils to ~10 kA in ~¼ s. This set adds ~13 T to the ~7 T remaining from the outer set, whose current has decayed to ~12 kA. Complicating the design is a superconducting coil, once part of the PEP-4 detector at the Stanford Linear Accelerator Center, that begins only ~2.2 m downstream and is sensitive to eddy-current heating by rapid flux changes. Therefore the proposed magnet system includes a conventional dc coil of ~0.7 MW to distance the pulse magnet from the PEP-4 coil. Also, a bucking coil in series with the outer set reduces by an order of magnitude the pulsed flux seen by the PEP-4 coil. The bucking coil serves also to reduce the axial force on the PEP-4 cryostat to below its limit of 200 kN.**

***Index terms*—pulse magnet; liquid nitrogen; inductor; conical bore; superconducting magnet; flux shielding; optimization**

I. Parameters and Function of Component Coils

To get pions, for decay into muons for a neutrino factory or muon collider, one bombards a target with protons of high energy. A typical target is cylindrical, ~10 mm in diameter and ~0.3 m long, tilted ~0.1 radians with respect to the field to reduce reabsorption of pions by the target. To capture the pions emitted by the target one can employ a horn, a lithium lens or a solenoidal magnet. A solenoid captures charged particles by bending their trajectories into helices. Capture to a transverse momentum of 225 MeV/c needs a field-bore product of 3 T-m: 20 T in a 0.15 m bore, for example. To retain the captured particles requires that the on-axis field profile decline not too rapidly with distance z from the downstream end of the target, while the bore increases inversely as the square root of the field. In the design presented here the field declines inversely as $1+5z$ until $z=3$ m, where the field levels off at ~1.25 T; meanwhile the bore diameter has increased fourfold. Fig. 1 sketches the cross section of the magnet windings proposed to achieve this field.

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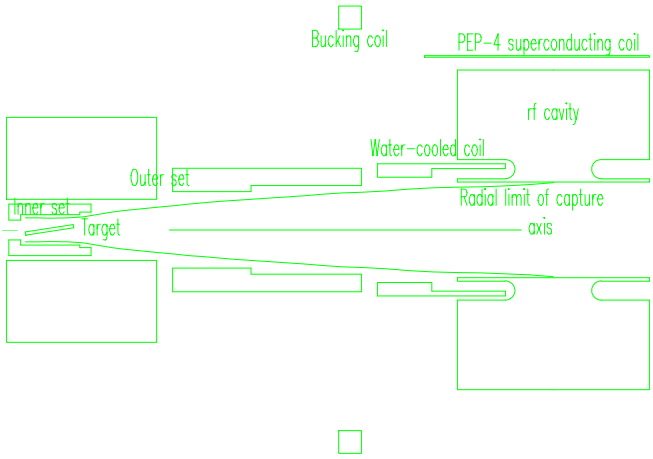

Fig. 1. Winding cross section of magnet system to generate ~20 T in a 0.15 m diameter bore. The pulse magnet weighs ~12 tons; its outer set of coils stores ~20 MJ from which to energize its inner set. The PEP-4 superconducting coil, 2.2 m in diameter and 3.3 m long (shown truncated) generates ~1.25 T in the rf cavity. The water-cooled and bucking coils reduce the pulsed flux that would quench the PEP-4 coil.

As shown in Fig. 1 the magnet system proposed by the Muon Collider Collaboration for its Targetry Experiment is to include three types of magnets. One is pulsed, with about twelve metric tons of copper precooled by liquid nitrogen. Another is dc, of water-cooled hollow conductor, consuming 0.7 MW. The third type is superconducting, salvaged from the Stanford Linear Accelerator PEP-4 detector. It is of 2 m bore, 3.3 m long and capable of a central field of 1.5 T [1].

The pulse magnet has two sets of coils, energized sequentially. The outer set, with about 98% of the mass, is of hollow conductor, for ease of containment of liquid nitrogen. Energized at 250 V to 16 kA in about 20 s, the set generates a peak field of about 9 T and stores about 20 MJ. Discharged through a resistor of about ¼ Ω it provides the voltage, initially about 4 kV, to energize the inner set of coils to about 10 kA in about ¼ second. While doing so, its current falls to about 12 kA, because of dissipation and inductive coupling.

The inner set employs rectangular conductor, cold worked and reinforced with stainless steel. The set contributes about 13 T to the field in the target, which is about 0.3 m long. Discharge of the inner set through a resistor of about 0.4 Ω (to give the same 4 kV peak voltage) limits its temperature rise to about 40 K. The energy dissipated in the magnet is about 30 MJ per pulse; re-cool may take 200 liters of nitrogen.

The dc coil downstream of the pulse magnet extends the tail of its field profile while avoiding the generation of eddy currents whose heating would quench the superconducting coil further downstream.

The superconducting coil is to generate most of the 1.25 T in which to test a radio-frequency cavity centered about three meters beyond the downstream end of the target. The rf cavity is about 2 m in diameter and 1.2 m long.

II. On-axis Field of Coil with Conical Mouth

The gradually increasing bore of the system calls for a formula for the field from a coil with a conical mouth, i.e., with a winding cross section that is trapezoidal rather than rectangular. The field is the sum of four terms, one for each of the four corners of the winding cross section. If the trapezoid's sloping side depends on radius r as $b = b_0 + sr$, where s is the slope of the line and b_0 its axial intercept, then the indefinite integral for the field at axial position z is:

$$\frac{B}{\mu_0 j} = u^3 b_0 \sinh^{-1}\left(\frac{r + s\Delta b}{|b_0|}\right) + u^2 s\sqrt{r^2 + \Delta z^2}\ . \qquad (1)$$

j is the current density, $u \equiv 1/\sqrt{1+s^2}$, $\Delta b \equiv b - z$ and $\Delta z \equiv b_0 - z$. For zero slope the formula simplifies to that for a coil with a rectangular winding cross section.

III. Estimation of Self and Mutual Inductances

The pulse magnet interacts inductively with all coils of the magnet system. To avoid recomputing inductances exactly at every iteration when optimizing the system, one can estimate each inductance from the dimensions of the coil and the on-axis field of the flux that links it. Consider a coil of rectangular winding cross section, inner radius a_1, outer radius a_2 and length $\Delta b = b_2 - b_1$, where b_1 and b_2 are the upstream and downstream ends, respectively. Integrating over the coil cross section the flux linkages from another coil, and dividing by the two coil currents i and i^*, gives the mutual inductance M between the coils. For a field invariant with radius:

$$M = \pi j B \Delta b \left(a_2^3 - a_1^3\right)/3ii^*\ . \qquad (2)$$

B is the field averaged over the length of the coil. For self inductance L a more appropriate radial dependence of field is uniform from the axis to the inner radius, thereafter falling linearly to zero at the outer radius, for which:

$$L = \pi j B \Delta b \left(a_2 - a_1\right)\left(3a_1^2 + 2a_1 a_2 + a_2^2\right)/6i^2\ . \qquad (3)$$

For nested coils, where the field falls linearly from its peak at a radius $a_0 \le a_1$ to zero field at $a_3 \ge a_2$, the inductance is:

$$L = \pi j B \Delta b \frac{2a_3(a_2^3 - a_1^3) - (a_2^4 - a_1^4) - 2a_0^3(a_2 - a_1)}{6(a_3 - a_0)i^2}\ . \qquad (4)$$

These inductance estimates can be accurate indeed if calibrated with a system of similar geometry. For the mutual inductances in this magnet system the calibration factors range from 84% to 101%. Self inductance factors range from 96% to 111%, except for the bucking coil, which is so short as to have twice the inductance estimated from its on-axis field.

IV. Current, Temperature and Resistance vs. Time

The prediction of magnet performance is complicated by the increase in resistance of each coil as it heats up during the pulse. This complexity precludes an analytical solution. Instead, one partitions the pulse into dozens of time intervals, in each of which one approximates the current and resistance of each coil as a power series at least cubic in current and at least quadratic in resistance.

Three equations describe the charging of the outer set of coils:

$$V = Ri + Li'\ ;\ \ R = r\rho\ ;\ \text{and}\ \ T' = j^2\rho^*\ . \qquad (5a,b,c)$$

The primes indicate differentiation with respect to time. V is the voltage from the power supply. R, L and T are the resistance, inductance and temperature, respectively, of the set. i is its current, j its current density, r its resistance with unit resistivity, ρ its electrical resistivity and ρ^* its ratio of resistivity to heat capacity. For copper both of these material properties are linear above ~70 K: $\rho = \rho_0 + \rho_T T$ and $\rho^* = \rho_0^* + \rho_T^* T$. Linearity permits integration of the third equation to predict temperatures from current densities:

$$T = \left[\left(\rho_o^* + \rho_T^* T_0\right)\exp\left(\rho_T^* \int j^2 dt\right) - \rho_0^*\right]/\rho_T^*\ . \qquad (6)$$

To predict the current density and resistance as functions of time, one solves alternately for successive coefficients in each of the two power series. Equation (5a) yields i' as $\left(V - Ri\right)/L$. Differentiation of (5b) yields R' as $r\rho'$, which is $r\rho_T T'$ and hence, from (5c), $r\rho_T j^2\rho^*$. Differentiation of (5a) gives the quadratic coefficient of current as $-\left(Ri' + R'i\right)/2L$.

Charging of the inner set of coils, via discharge of the outer set through an external resistor R_e, introduces additional complexity: two currents instead of one, and inductive cross-talk between coils, but the above technique still works. The coupled equations (subscript 1: outer set, 2: inner set) are:

$$\left(R_1 + R_e\right)i_1 + L_1 i_1' + M i_2' = R_e i_2\ , \qquad (7a)$$

$$\left(R_2 + R_e\right)i_2 + L_2 i_2' + M i_1' = R_e i_1\ . \qquad (7b)$$

The leading terms are:

$$i_1' = -\left[\left(L_2 + M\right)V_e + L_2 V_1 - M V_2\right]/L^2\ , \qquad (8a)$$

$$i_2' = \left[\left(L_1 + M\right)V_e - L_1 V_2 + M V_1\right]/L^2\ , \qquad (8b)$$

$$R_1' = \rho_T r_1 T_1'\ ,\ \text{and}\ \ R_2' = \rho_T r_2 T_2'\ . \qquad (8c,d)$$

$V_1 \equiv R_1 i_1$, $V_2 \equiv R_2 i_2$, $V_e \equiv R_e\left(i_1 - i_2\right)$, $L^2 \equiv L_1 L_2 - M^2$ and (5c) gives T_1' and T_2'. Fig. 2 shows that charging the outer coil set to 16 kA takes about 20 s. Charging the inner coil to ~10 kA takes only ~¼ s if $R_e = ¼\ \Omega$; the outer current meanwhile has decayed to ~12 kA, due to inductive coupling and resistive losses.

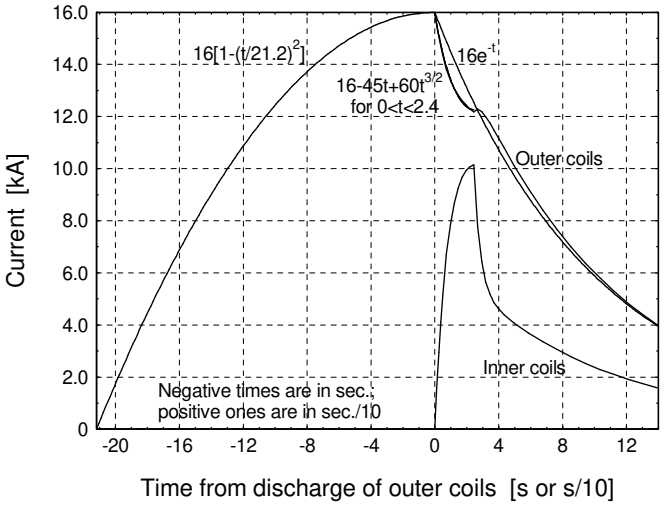

Fig. 2. Current vs. time for outer and inner sets of coils of pulse magnet. The graph expands the time scale by a factor of ten when t>0. The curve fits are for predicting the eddy-current heating caused by flux from the pulse magnet.

V. Generation of Desired On-Axis Field Profile

The desired field profile is ~20 T throughout the target, $-0.3 \le z \le 0.0$ m, and falls inversely with $1+5z$ over the next 3 m, thereafter holding at 1.25 T. Adjusting the dimensions and current densities of the many coils of the system matches this field profile within a few percent (see Fig. 3).

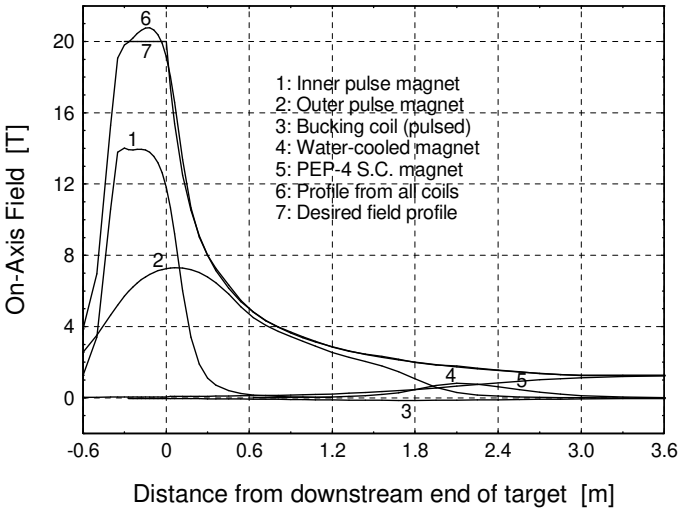

Fig. 3. On-axis field profile of 20 T magnet and its subsystems. The inner set of the pulse magnet generates most of the field within the target, from –0.3 m to 0 m, the outer set the most from 0.2 m to 1.8 m, and the PEP-4 coil the most within the rf cavity, from 2.4 m to 3.6 m.

VI. Optimization; Choice of Discharge Voltage

The computer program adjusts the dimensions and current densities of coils to satisfy numerous goals and constraints. Minimized is a weighted combination of conductor mass, energy dissipation (liquid nitrogen consumption), maximum temperature rise (for ease of re-cool) and deviation from the desired field profile. Constraints include limits on hoop stresses and clearances between each coil and the flaring envelope of pion capture.

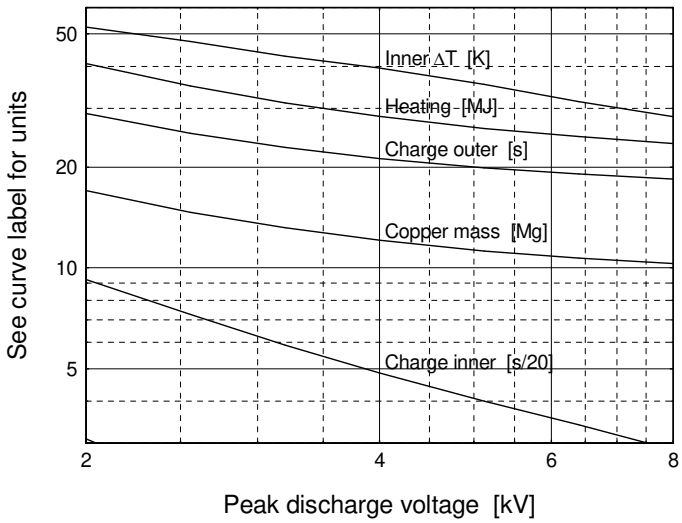

Fig. 4. Parameters of 20 T systems vs. peak discharge voltage of pulse magnet. All parameters decrease ever more slowly with increasing discharge voltage. The copper mass at 4 kV is within 15% of its value at 8 kV.

VII. Eddy-Current Heating by Pulse Magnet

Even with the water-cooled magnet the PEP-4 coil sees flux changes great enough to trigger quenching. To reduce these flux changes one can employ an iron shield to divert pulse magnet flux that otherwise would link the PEP-4 coil. However, to limit the average flux density in the iron to 1.6 T, so as to maintain high permeability, one needs a shield 0.4 m thick, which implies a mass of a hundred tons to make flux detour around the PEP-4 coil. An appealing alternative is a bucking coil of only a ton. Such a coil can reduce the flux changes felt by the PEP-4 coil by more than an order or magnitude to safely below the levels that would trigger a quench.

The mechanism for quenching is overheating of the superconducting wire from eddy currents induced in the wire itself or in the bore tube. According to Mike Green, the designer of the PEP-4 system, its operating current is approximately half its capacity at 4.2 K. This implies a temperature margin of about 2.4 K, which will accommodate only about 14 kJ/m^3.

One source of heat that can quench a superconducting wire is from shielding currents. The 40 mm twist pitch of the PEP-4 wire, however, is amply tight for stability. The energy density q_e dissipated in a wire of characteristic time τ_c in an exponential field ramp of magnitude ΔB and time constant τ [2] is:

$$q_e = \tau_c \, \Delta B^2 \big/ \mu_0 \left(\tau + \tau_c \right); \quad \Delta B = \sqrt{\mu_0 q_e \left(\tau + \tau_c \right) / \tau_c} \; . \quad \text{(9a,b)}$$

For PEP-4 wire, τ_c is only ~0.1 s. Fig. 2 shows that the pulse magnet discharges with a time constant of ~1.0 s. An energy density of 14 kJ/m^3 allows a ΔB of about 0.4 T; this is several times the maximum pulse field that the wire will see.

A more serious threat to the stability of the PEP-4 superconducting coil is heating from eddy currents in its bore tube, which is of aluminum of quite high conductivity. Heating occurs during both the charging and the discharging of the pulse magnet; the two heat inputs add.

Charging of the pulse magnet is sufficiently gradual that its flux fills the PEP-4 bore, diminished only modestly by eddy currents induced in the bore tube. The eddy current density j is $-\Phi'/2\pi r\rho$, where r is the mean radius of the bore tube, ρ is its electrical resistivity, and the prime symbol indicates differentiation with respect to time. During charging of the magnet the flux Φ threading the bore tube is approximately quadratic with time (see Fig. 2), $\Phi_m\left[1-\left(t/\tau\right)^2\right]$, where Φ_m is the maximum flux linkage and τ the charging time. The predicted current density is $\Phi_m t/\pi r\rho\tau^2$. The heating rate per unit volume $j^2\rho$ integrated from $-\tau$ to zero, to give the total heating density q, is $\left(\Phi_m/\pi r\right)^2/3\rho\tau$. Inverting gives the permissible flux as $\Phi_m=\pi r\sqrt{3\rho\tau q}$. r is 1.07 m, ρ is ~1.1x10^{-9} Ω m, and τ is 21 s. Fig. 5 shows that without a bucking coil Φ_m is about 0.6 Wb when the outer set of coils is at its peak current of 16 kA. To limit the heat input to 14 kJ/m^3 requires limiting the flux change to about 0.11 Wb. Thus the bucking coil can leave uncompensated a fifth of the peak flux generated by the pulse coil and linked by the superconductor. This estimate is conservative to the extent that neglecting flux exclusion overestimates the induced eddy current densities.

However, one must also add the eddy-current heating during discharge of the pulse coil, which may be over an order of magnitude faster. The discharge is so fast, in fact, that the eddy currents buck almost completely the imposed flux. To generate the field $B=\Phi/\pi r^2$ needed for flux exclusion, the currents must be of strength $\Phi/\mu_0\pi r^2$ A/m. To generate a flux of 0.6 Wb in a bore of 2.14 m diameter requires about 130 kA/m. Such a current uniformly distributed throughout a bore tube wall of 9.35 mm thickness implies a current density of 14 MA/m^2. The corresponding power density $j^2\rho$ is approximately 210 kW/m^3. Persisting throughout an exponential ramp of one-second time constant implies a heating density of about 40 kJ/m^3. To limit the heating to 14 kJ/m^3 one need reduce the pulsed flux by only a factor of two. (Caveat: this prediction is overly optimistic, to the extent that the eddy currents are not uniform, but to some extent concentrate toward the outer radius of the bore tube.)

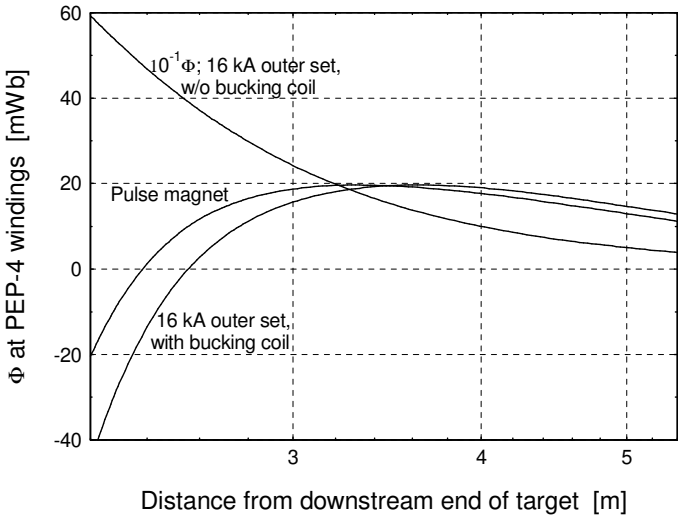

Fig. 5. Flux at mouth of PEP-4 magnet, from pulse magnet with and without bucking coil. The bucking coil reduces the maximum residual flux by a factor of more than ten.

To predict this non-uniform distribution of current density, one can draw upon published solutions to the diffusion equation. For an imposed field that ramps steadily as $t^{n/2}$, the solution as a function of depth x and time t is $t^{n/2}i^n erfc\left(x/2\sqrt{Dt}\right)$ [3]. $i^2 erfc$ is the nth integral of the complementary error function, and D is the magnetic diffusivity, ρ/μ_0. However, our simplified approximations suggest that, with a bucking coil that reduces the imposed flux by an order of magnitude, there should be little danger of quenching from eddy current heating.

VIII. Axial Forces Between Component Coils

Another concern is intercoil forces. Table I gives the axial forces between coils of the magnet system at three stages of energization. The forces on the inner and outer coil sets of the pulse coil are very large indeed. The force on the PEP-4 coil, however, remains within the 200 kN design limit of its cryostat. For this reason, along with the outstanding performance and economic feasibility of the system, we feel comfortable in proceeding further with the design.

TABLE I

AXIAL FORCES ON MEMBERS OF 20 T MAGNET SYSTEM

	W-C & S.C. coils only	Inner set off I_{out}=16 kA	All coils on I_{in}=12.3 kA
Inner set of pulse coil	0	0	1802 kN
Outer set of pulse coi	0	233 kN	-1612 kN
Water-cooled coil	196 kN	-108 kN	-39 kN
PEP-4 superconducting coil	-196 kN	-125 kN	-151 kN

References

[1] A. Dubois/M. Green, LBL Engineering Note LBID-665, Code P41000, Serial M6018, 1983.

[2] Martin N. Wilson, *Superconducting Magnets*, Oxford University Press, Oxford, 1983.

[3] J. Crank, *The Mathematics of Diffusion*, Clarendon Press, Oxford, 1973, p. 34.