Peter Sievers, CERN
1211 – Geneva 23, Switzerland

Classical Solution of Wave equation

$$\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$; u : displacement

- One dimensional

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

$$u\,(x,t) = f\,(x-ct) + g\,(x+ct)$$

- Spherical Symmetry

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2u}{r^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

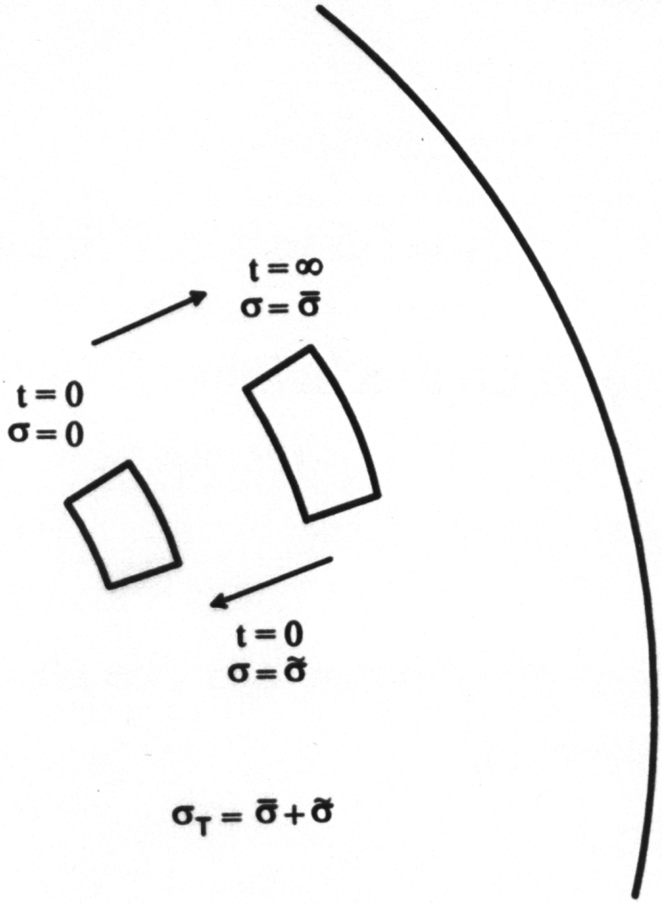
$$u(r,t) = f\,\frac{(r-ct)}{r} + g\,\frac{(r+ct)}{r}$$

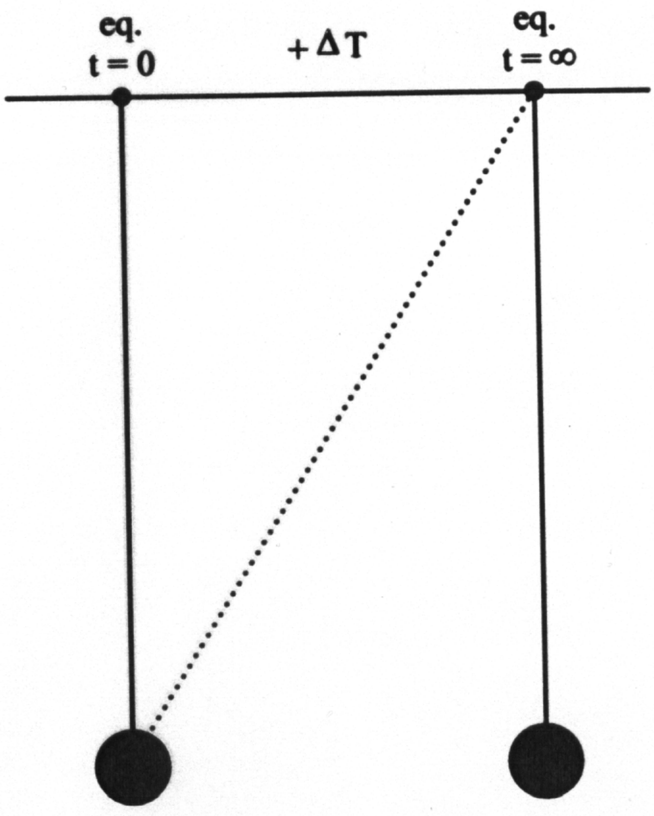
- Cylindrical Symmetry

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

$$u\,(r,t) = \sum_{n=1}^{\infty} c_n J_1(\varepsilon_n \xi)\cos(\varepsilon_n \theta)$$

$$\xi = r/R, \theta = ct/R$$


t = ∞
σ = σ̄
t = 0
σ = 0
t = 0
σ = σ̃
σ_T = σ̄ + σ̃


eq.
t = 0
+ ΔT
eq.
t = ∞

Exemple of Solutions : Free Solid, Parabolic Function

$v := \frac{1}{3}$ $\quad \xi := 0.01..1$ $\quad \theta := 0.1, 0.2..10$

r_n are the solutions of $J0(r_n) = (1 - 2v)\, J2(r_n)$

$$fp(\theta,\Theta,n) := \left| \begin{array}{l} \left(\frac{1}{r_n\cdot\Theta}\cdot\sin\left(r_n\cdot\theta\right)\right) \text{ if } (\theta\le\Theta) \\ \left[\frac{1}{r_n\cdot\Theta}\cdot\left[\sin\left(r_n\cdot\theta\right)-\left(\sin\left(r_n\cdot\theta\right)\cdot\cos\left(r_n\cdot\Theta\right)-\cos\left(r_n\cdot\theta\right)\cdot\sin\left(r_n\cdot\Theta\right)\right)\right]\right] \text{ if } (\Theta<\theta) \end{array} \right.$$

$$\sigma r(\xi,\theta,\Theta) := \frac{4}{(1-v)^2}\cdot\left[\sum_{n=0}^{200}\frac{1}{\left[\left(r_n\right)^2-\frac{(1-2\cdot v)}{(1-v)^2}\right]\cdot\left(r_n\right)^2\cdot J0\left(r_n\right)}\cdot\left[J0\left(r_n\cdot\xi\right)-\frac{(1-2\cdot v)}{(1-v)}\cdot\frac{J1\left(r_n\cdot\xi\right)}{r_n\cdot\xi}\right]\cdot fp(\theta,\Theta,n)\right]+\frac{1}{4\cdot(1-v)}\cdot\left(1-\xi^2\right)$$

$$\sigma\phi(\xi,\theta,\Theta) := \frac{4}{(1-v)^2}\cdot\left[\sum_{n=0}^{200}\frac{1}{\left[\left(r_n\right)^2-\frac{(1-2\cdot v)}{(1-v)^2}\right]\cdot J0\left(r_n\right)\cdot\left(r_n\right)^2}\cdot\left[\frac{v}{1-v}\cdot J0\left(r_n\cdot\xi\right)+\frac{(1-2\cdot v)}{(1-v)}\cdot\frac{J1\left(r_n\cdot\xi\right)}{r_n\cdot\xi}\right]\cdot fp(\theta,\Theta,n)\right]+\frac{1}{4\cdot(1-v)}\cdot\left(1-3\cdot\xi^2\right)$$

$$\sigma z(\xi,\theta,\Theta) := \frac{4\cdot v}{(1-v)^3}\cdot\left[\sum_{n=0}^{200}\frac{1}{\left[\left(r_n\right)^2-\frac{(1-2\cdot v)}{(1-v)^2}\right]\cdot J0\left(r_n\right)\cdot\left(r_n\right)^2}\cdot J0\left(r_n\cdot\xi\right)\cdot fp(\theta,\Theta,n)\right]+\frac{1}{4\cdot(1-v)}\cdot\left(4-2\cdot v-4\cdot\xi^2\right)$$

For solids believe in Hook's Law, negative stresses allowed.

For liquids: negative pressures lead to cavitation.
Wave equation fails as soon as

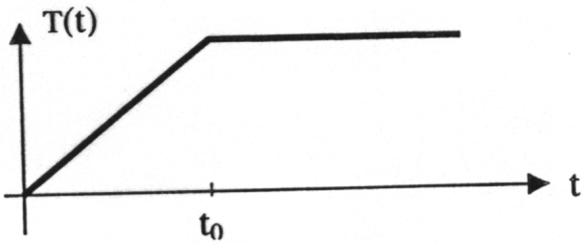
$$- P < P_{cavitation}$$

For instantaneous Heating at $t = 0$

All $\sigma_T\ (r,t = 0) = \ \overline{\sigma}\ (r) + \tilde{\sigma}(r,t = 0) = \dfrac{E\,\alpha\,\mathrm{T}(r)}{(1-2\nu)}$

independent of boundary condition.
However larger stresses may occur at later times !

For finite (non instantaneous) heating:



Convolution $\sigma(r,t,t_0) = \int_0^t \sigma(r,t-\tau)\dfrac{\partial T(\tau)}{\partial\tau}d\tau$

Sound Velocity: $c \sim \sqrt{\dfrac{E}{\rho}}$ for solids

$c \sim \sqrt{\dfrac{1}{\kappa\rho}}$ for liquids

Material Velocity: $\mathrm{v}/_{c} \approx \alpha_L \, \Delta T$ for solids

$\mathrm{v}/_{c} \approx 1/_{2} P\kappa = 1/_{2}\alpha_V \Delta T$ for liquids

Example Hg:

$\Delta T = 200\ K$

$\alpha_V = 18.1 \times 10^{-5}\,\mathrm{K}^{-1}$

$\kappa = 0.45 \times 10^{-10}\,\mathrm{m}^2/\mathrm{N}$

$P = 800\ \mathrm{MPa}$

$\mathrm{v}/_{c} = 1.8\%$ (not yet supersonic !)

$c = 1.3\ \mathrm{km/s}$

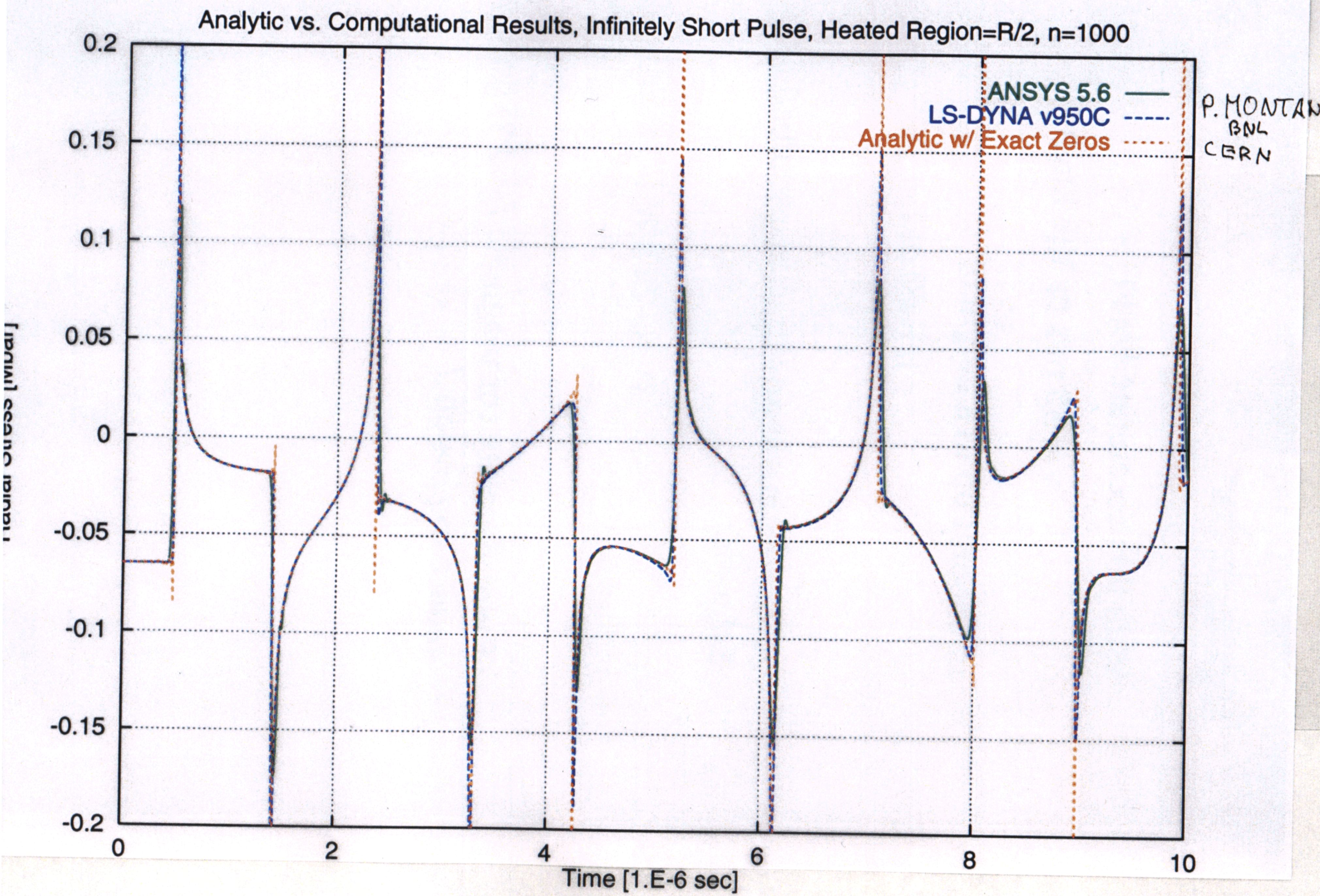
$\mathrm{v} = 23\ \mathrm{m/s}$

- Get some insight into the physics.

- Identify critical areas, times.

- Use it as a " Bench Mark " to check against FE-codes which are " Black Boxes ".

- Use it for scaling with R, c, t_0.


Analytic vs. Computational Results, Infinitely Short Pulse, Heated Region=R/2, n=1000
ANSYS 5.6
LS-DYNA v950C
Analytic w/ Exact Zeros
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
2
4
6
8
10
Time [1.E-6 sec]
P. MONTAN
BNL
CERN

Results

(All axially constrained)

- Dynamic: $\tilde{P} = \frac{\alpha_V T_0}{\kappa} f(\xi, \theta, \theta_0)$

$\xi = r/_R$, $\theta = ct/_R$, θ_0 : Burst Duration $ct_0/_R$

f : depends also on initial condition, $T(\xi, 0, 0)$

and boundary condition

- Static: $\bar{P} = \frac{\alpha_V T_0}{\kappa}\ g(\xi)$

g : depends on $T(\xi, 0, 0)$ and boundary condition

Total : $P_T = \tilde{P} + \bar{P} = \frac{\alpha_V T_0}{\kappa}\{f(\xi, \theta, \theta_0) + g(\xi)\}$

radially free: $g(\xi) = 0$

- Velocity: $\upsilon = \alpha_V T_0 c\ \ h(\xi, \theta, \theta_0)$

$$c \approx 1/\sqrt{\rho\kappa}$$

For solids:

$$\alpha_T = E\alpha_L T_0\{f(\xi, \theta, \theta_0) + g(\xi)\}$$

radially free: $g(\xi) \neq 0$

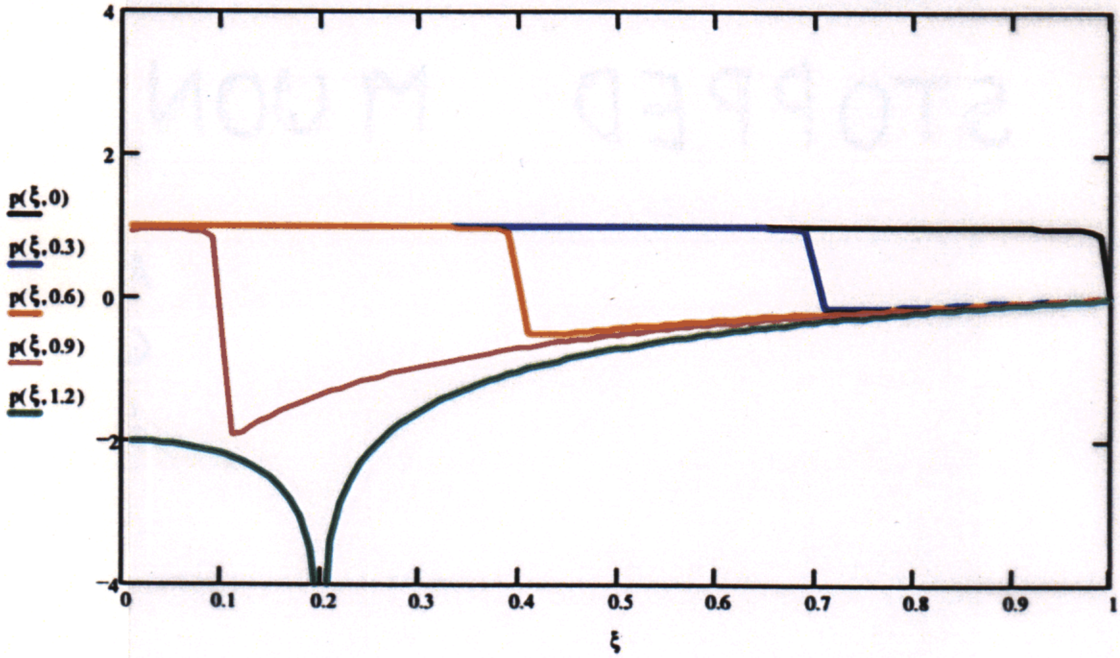
At $\theta = 0$ for $\theta_0 = 0$:

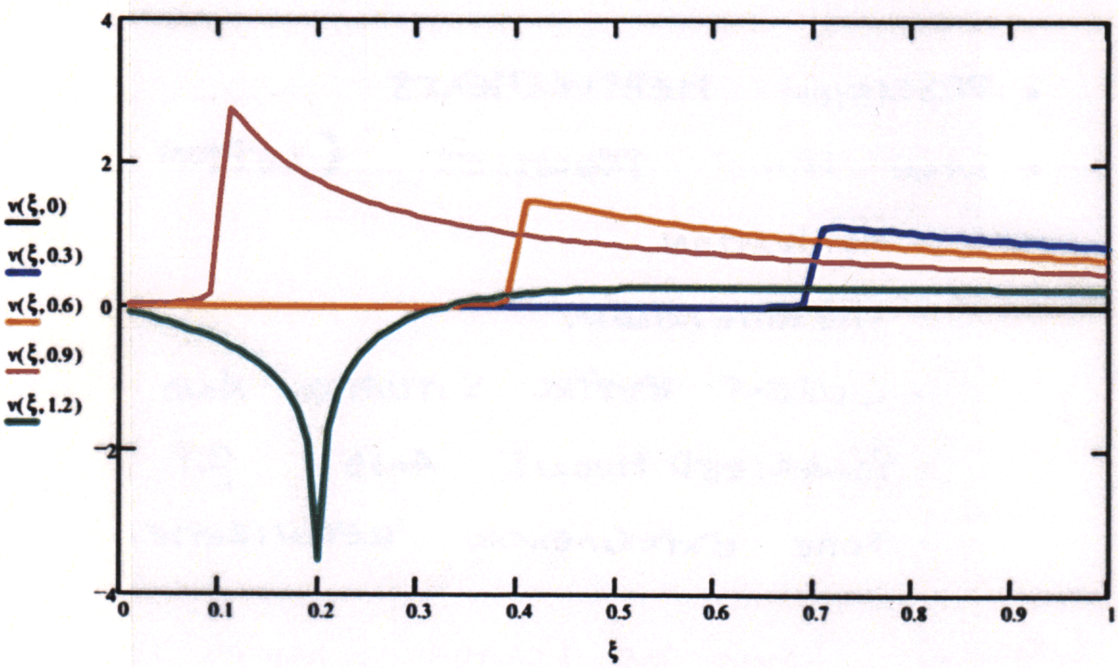
$$P_T(\xi,0,0) = \frac{\alpha_V T(\xi,0,0)}{\kappa}$$

$$\sigma_T(\xi,0,0) = \frac{E\alpha_L T(\xi,0,0)}{1-2\nu}$$

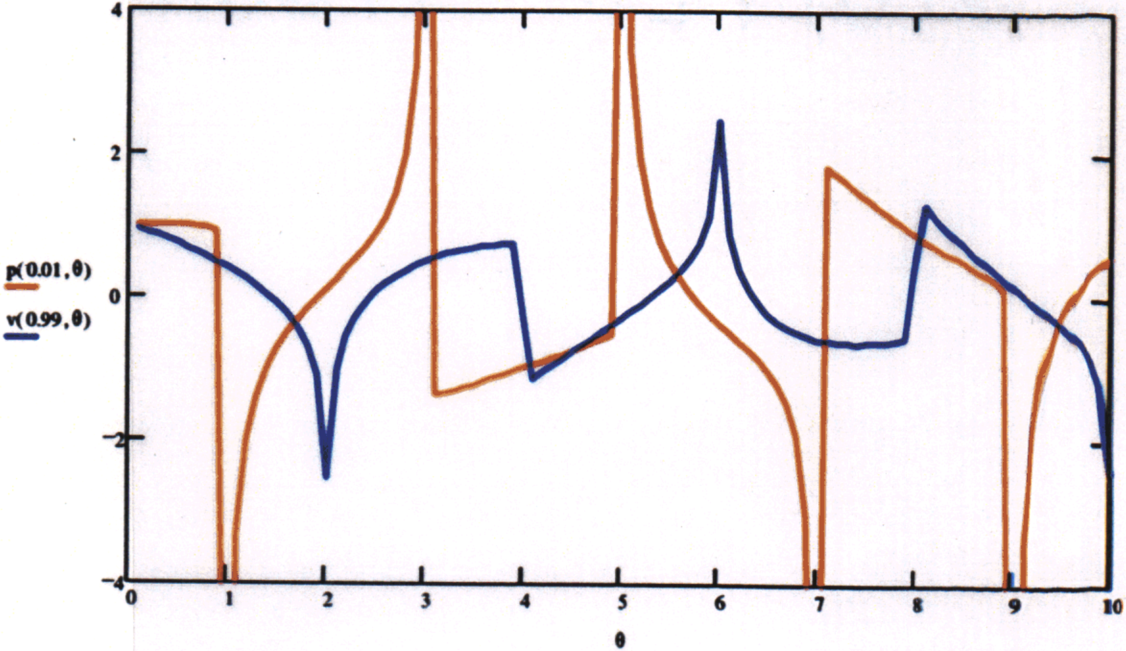
Does not depend on boundary conditions !

Liquid with free boundary conditions. Step Function

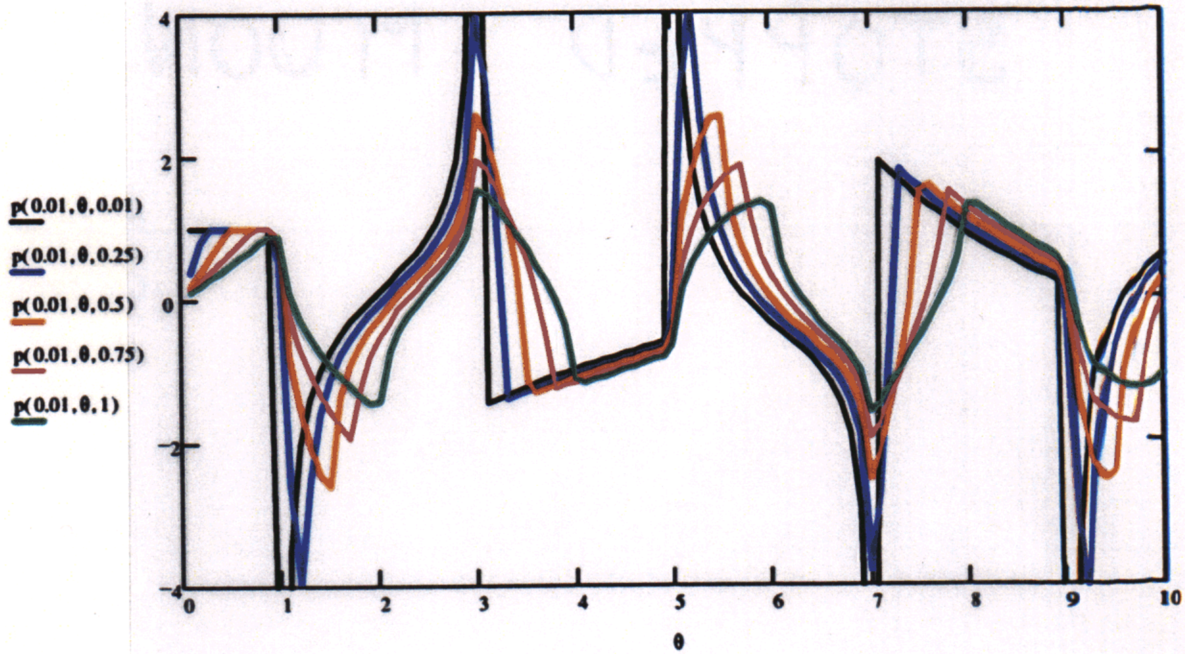


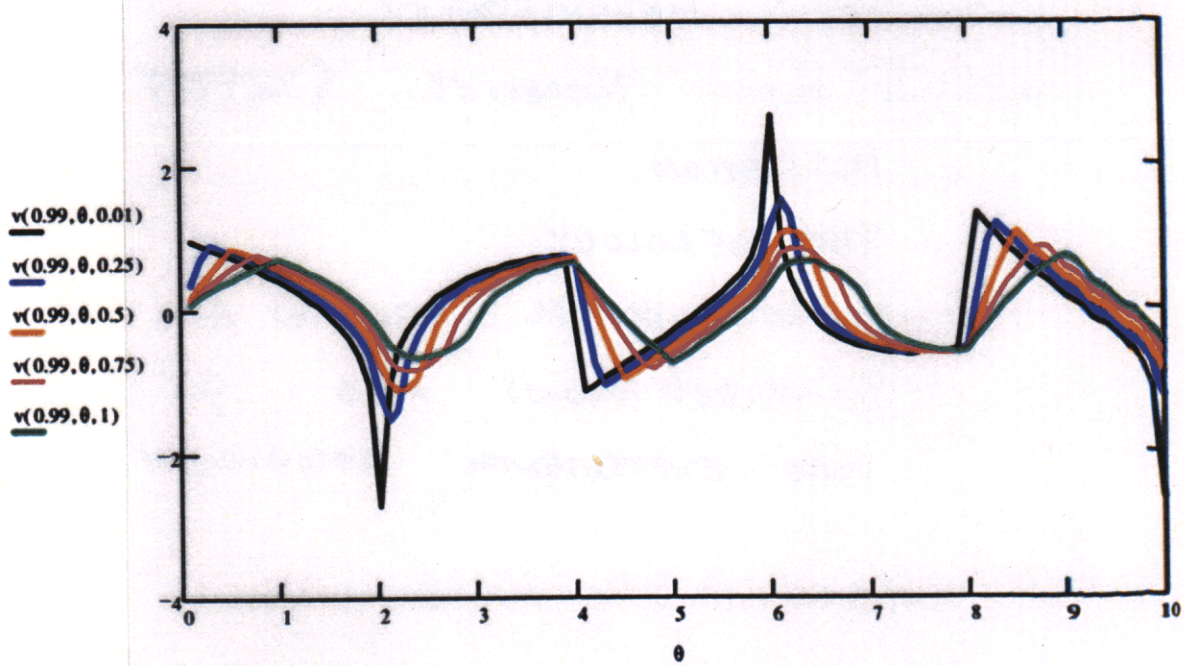


Liquid with free boundary conditions, Step Function



Free Liquid, Step Function, Heating Time Effects





Hg:

$\rho = 13.5 \times 10^3$ kg/m^3

$\kappa = 0.45 \times 10^{-10}$ m^2/N

$c = 1.28 \times 10^3$ m/s

$\alpha_V = 18.1 \times 10^{-5}$ K^{-1}

$c_V = 140$ J/kg K

$P_{cavitation} \approx -...$N/m^2 ??

$\sigma_{el.} = 10^6 [\Omega m]^{-1}$

$T_0 = 200$ K

$\frac{\alpha_V T_0}{\kappa} = 804$ MPa (8 000 Bar)

$\alpha_V T_0 = 3.6\,\%$

$\alpha_V T_0$ c = 46 m/s

Ta:

$\rho = 16.8 \times 10^3$ kg/m^3

$E = 16 \times 10^{10}$ N/m^2

$c = 3.1 \times 10^3$ m/s

$\alpha_L = 6.5 \times 10^{-6}$ K^{-1}

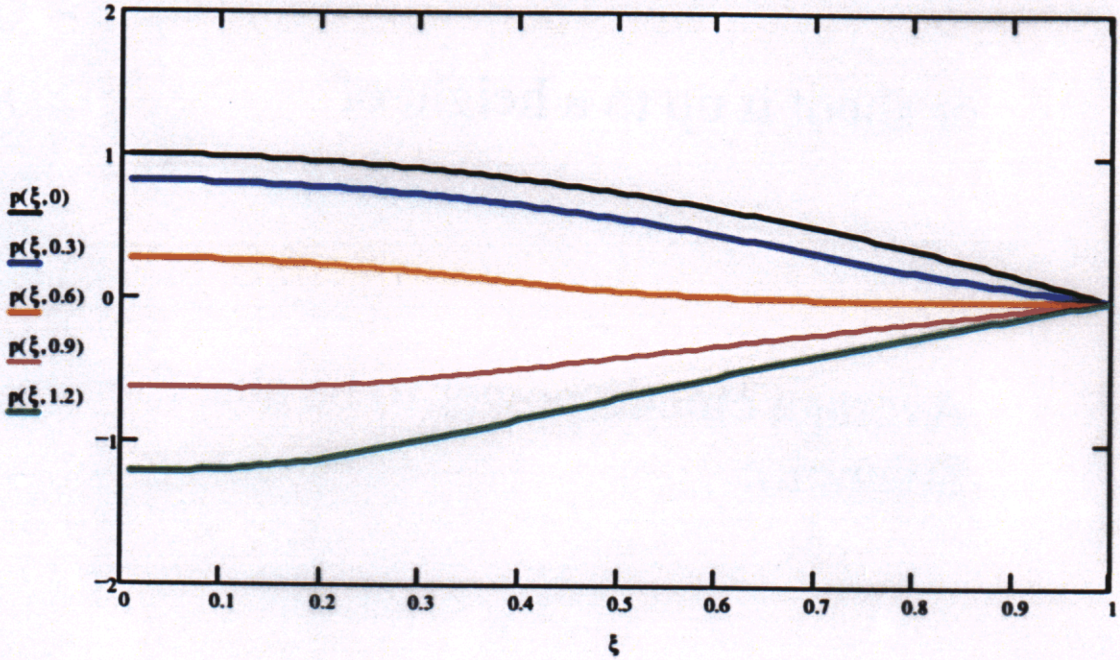
$c_V = 151$ J/kg K (at room temperature)

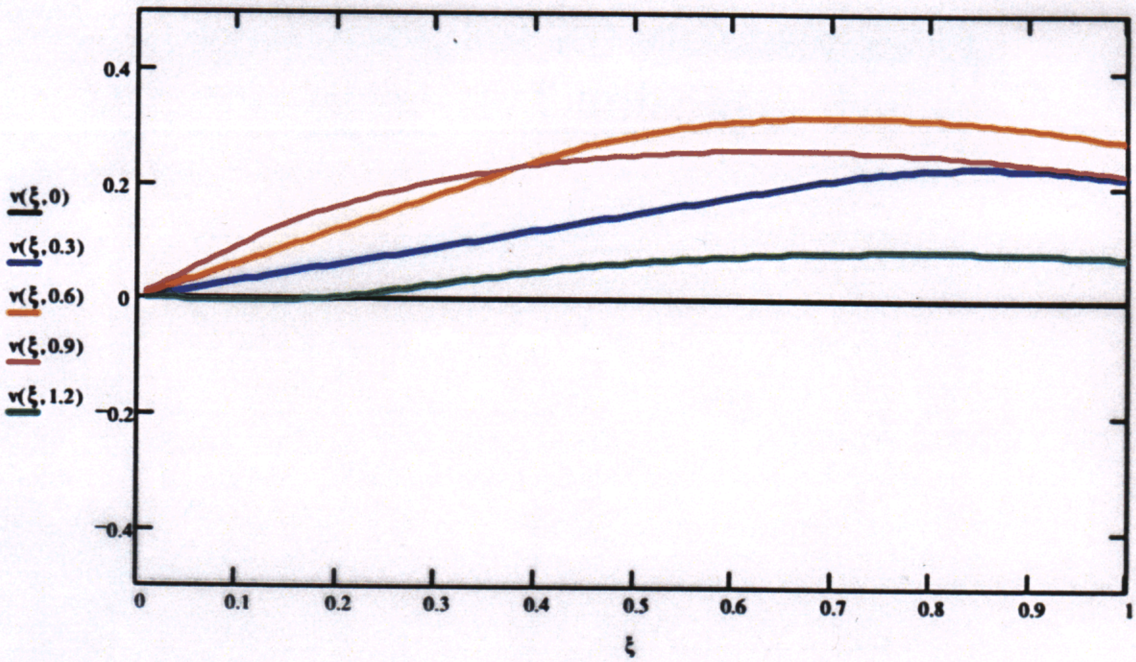
$\sigma_0 = 500...\ 1000$ MPa

$T_0 = 200$ K

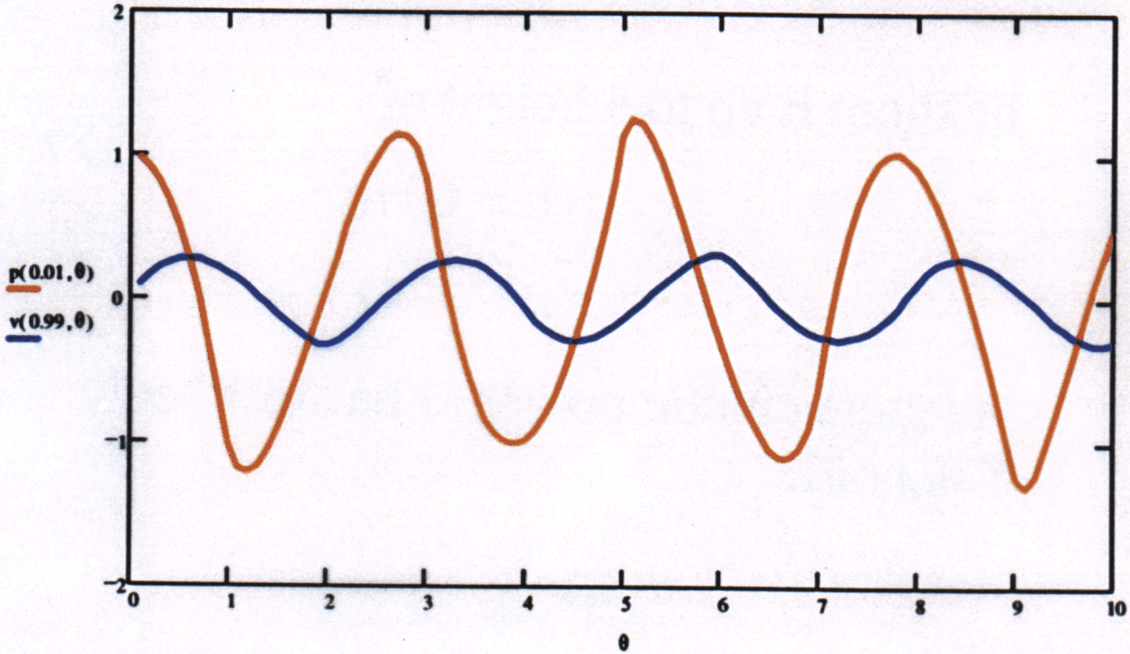
$E\alpha_L T_0 = 208$ MPa

Liquid with free boundary conditions. Parabolic Function

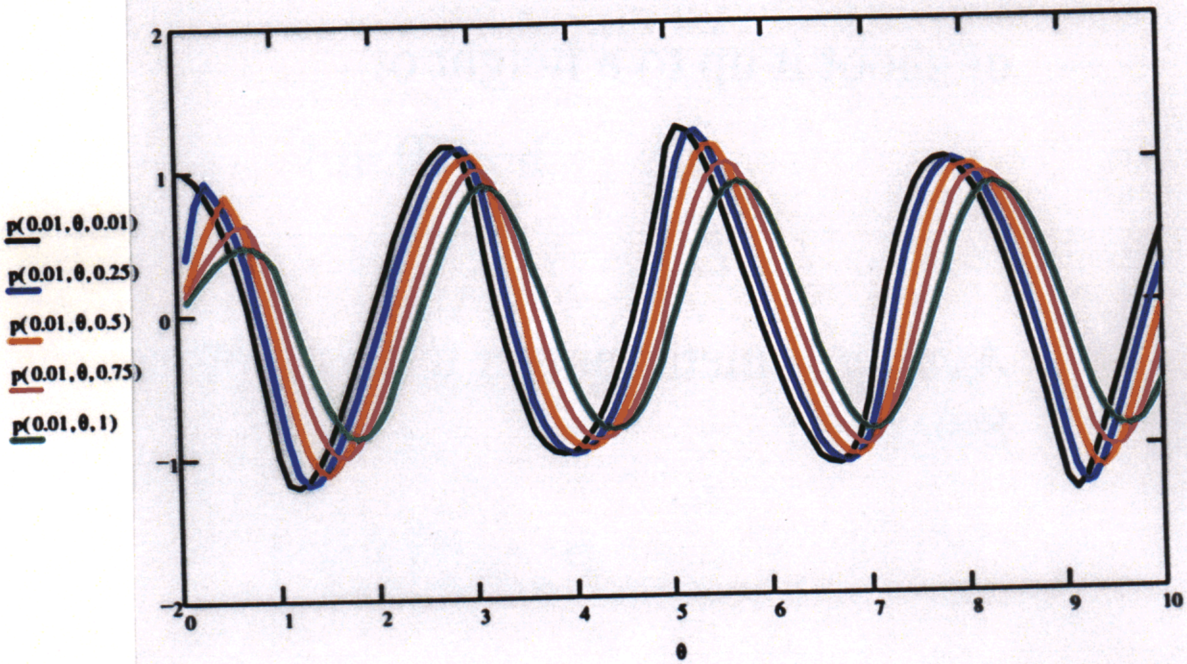


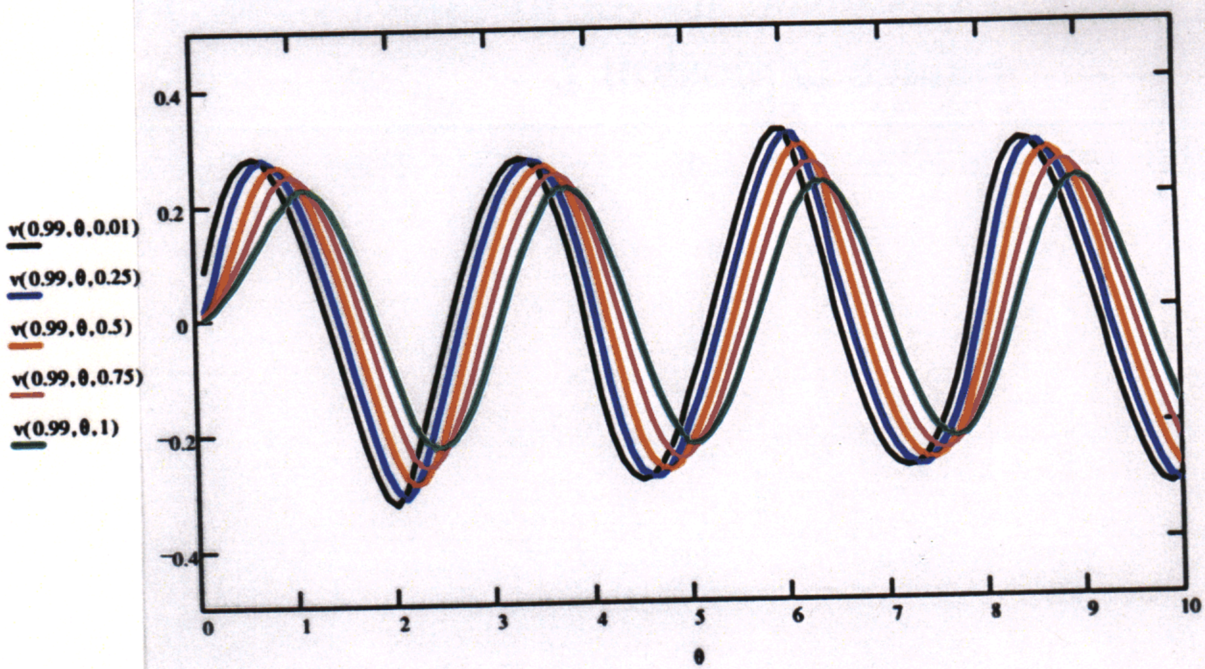


Liquid with free boundary conditions. Parabolic Function



<u>**Free Liquid, Parabolic Function, Heating Time Effects**</u>





- Free Liquid: $T(\xi,0,0) = T_0(1-\xi^2)$, *parabolic*

  all much smoother, dP/dξ lower

  Similar to harmonic oscillator

  $P \approx +1....-1$: ± 800 MPa all the time

  $\upsilon \approx \pm 1/_3$: ± 15 m/s (1.2%)

- Effect of $\theta_0 > 0$:

  P at $\theta_0 = 1$, initially $\dfrac{P(\theta_0 = 1)}{P(\theta_0 = 0)} \sim 1/_2$ (400 MPa)

  later ± 1 (± 800 MPa)

  υ at $\theta_0 = 1$ ± 0.2 (9 m/s)

- Free Solid:

  in center $\sigma_r = \sigma_\varphi \approx +3...-2$: $+624$ MPa...-416 MPa

  $\sigma_z = 3...-0.5$: $+624$ MPa...-104 MPa

  at rim $\sigma_\varphi \approx 0...-1.5$: 0...$-312$ MPa

  $\sigma_z \approx 0...-0.5$: 0...$-104$ MPa

  Von Mieses $\sigma^2_{eq.} = \sigma_r^2 + \sigma_\varphi^2 + \sigma_z^2 - \sigma_r\sigma_\varphi - \sigma_r\sigma_z - \sigma_\varphi\sigma_z$

  Von Mieses at center: at $\theta = 0$ $\sigma_{eq.} = 0$ (hydrostatic equilibrium)

  at $\theta \approx 1.5$ $\sigma_{eq.} \approx 1.5$

  oscillates between 0....312 MPa

Von Mieses at rim: at $\theta = 0$ $\sigma_{eq.} = 0$

at $\theta \approx 1.5$ $\sigma_{eq.} \approx 1.3$

oscillates between 0….275 MPa

Looks → ok for one shot !

But for 50 H_Z (4Mio / Day) ???

Conclusions: For pulse duration

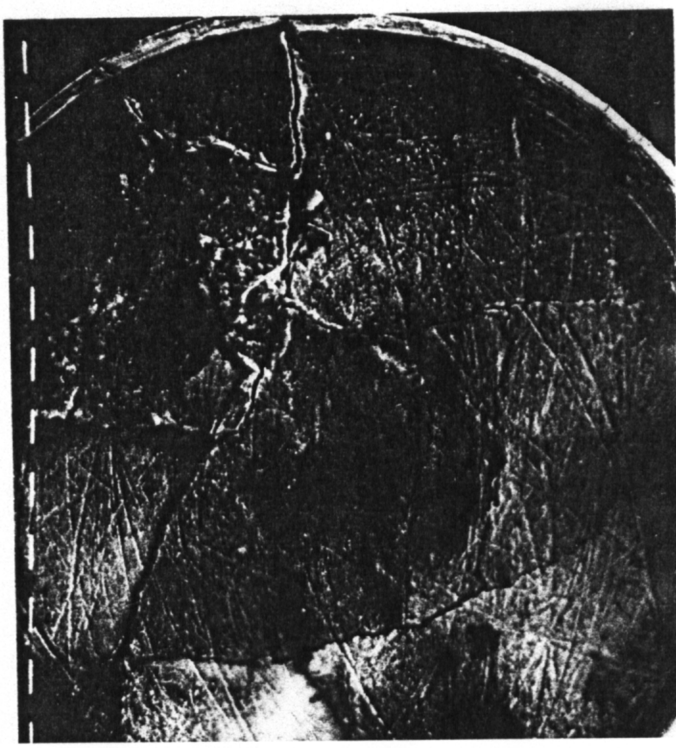
$$\theta_0 \leq 1,\ t_0 = \frac{R}{C} = \frac{5\,mm}{c_{Hg}} \simeq 4\,\mu s$$

Pressures, stresses of the same order as for $\theta_0 = 0$

For small R correspondingly smaller.

Problem of cavitation in liquids (negative pressures) to be solved.

Problems of fatigue in solids in high temperature to be assessed.



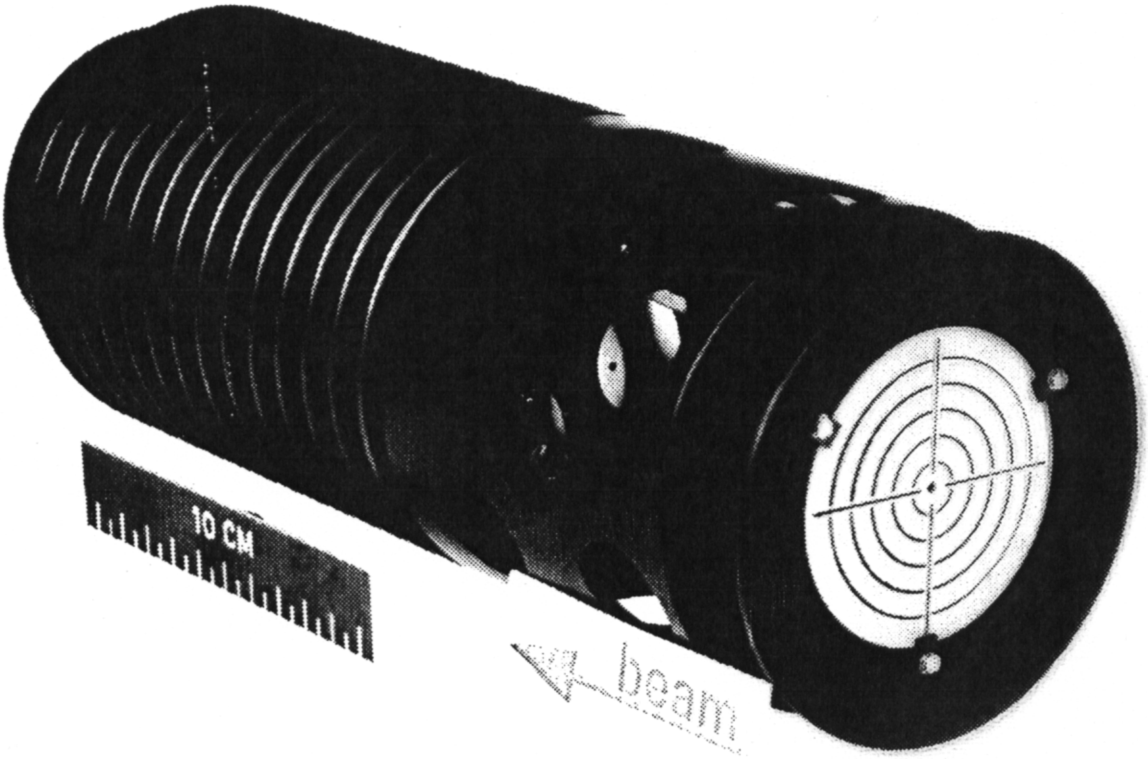


Fig. 5 - Overall view of the target assembly. The aluminium container is anodised black to aid cooling via radiation. The hole in the upstream luminescent screen avoids premature aging and radiation damage of the latter.

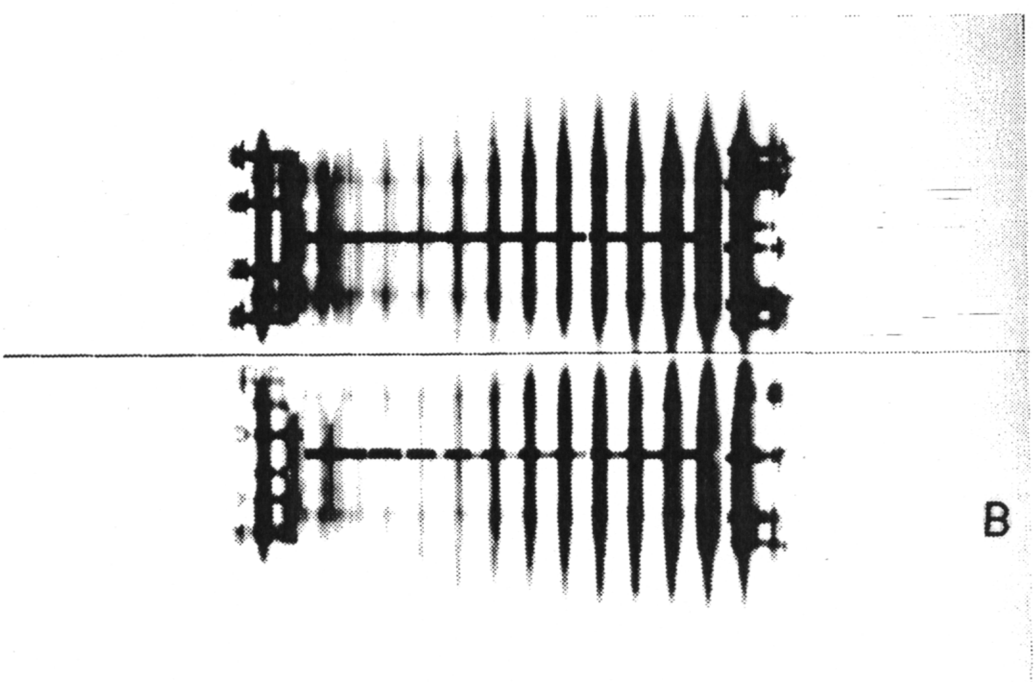


Fig. 6 - X-ray photo of the target ensemble after irradiation.

- To reduce ΔE/Target

Increase	f
Need	$\Delta s = 3$ cm
Increase	$\varnothing \sim$ f ! !
Const.	ω

- Thermal stress (Static + Dynamic at t = 0)

$$\sigma \sim \frac{E\,\alpha\,\Delta T}{1-2\nu}$$

$$\nu \approx 1/3$$

Tungsten $E \sim 400$ GPa
$\alpha \sim 4.5 \times 10^{-6}\ K^{-1}$
$\Delta T \sim 200$ K
$\sigma \sim 1$ GPa $< \sigma_{max} \sim 1.5$ Gpa

$\Delta T = 625$ K, Target will crack

- Target must be contained !

  Container Material with

  Absorption length $\lambda_C >> \lambda_{Target}$

  λ W ~ 10 cm

  λ Carbon ~ 30 cm

  C A R B O N !

  Can stand < 3000 °C in Vacuum

  Good heat conductor

  Excellent experience at CERN-$\overline{P}$-Source

- Need 200 Targets
  Target Wheel Ø 2 m
  Target Spacing $\Delta s = 3$ cm

r.p.s.	0.25 t/s
r.p.m.	15 t /min.
Linear velocity	1.5 m/s (5.4 km/h)
Centr. Force	0.25 g

- 1 MW in 200 Targets : 5 kW / Target
  1 MW evacuated through surface
  of ~ 1 m^2 ($\pi \times \varnothing = 6.3 \times l_{Target} = 0.15$ m)

$$100 \text{ W / cm}^2$$

into Water Cooling System

$l_{Target} = 0.15$ cm:
longer and lighter target helpful for cooling,
but $\lambda_C \approx \lambda_{Target}$

– Average steady state temperature

ΔT Graphite	~ 60 K
ΔT Cu, 1 cm	~ 30 K
ΔT Water	~ 60 K
$\overline{\Delta T}$	~ 150 K < ΔT per shot

- Water Cooling System

Water close to target :	~1 cm
Flow :	5 l/s
ΔT_W :	50 K
Ø pipe :	5 cm
v_{water} :	2.5 m/s
$b_{(Coriolis)}$:	0.8 g
b_{centr} :	0.25 g

- Need well-designed heat exchanger from Cu → Water

- Precision of Rotation

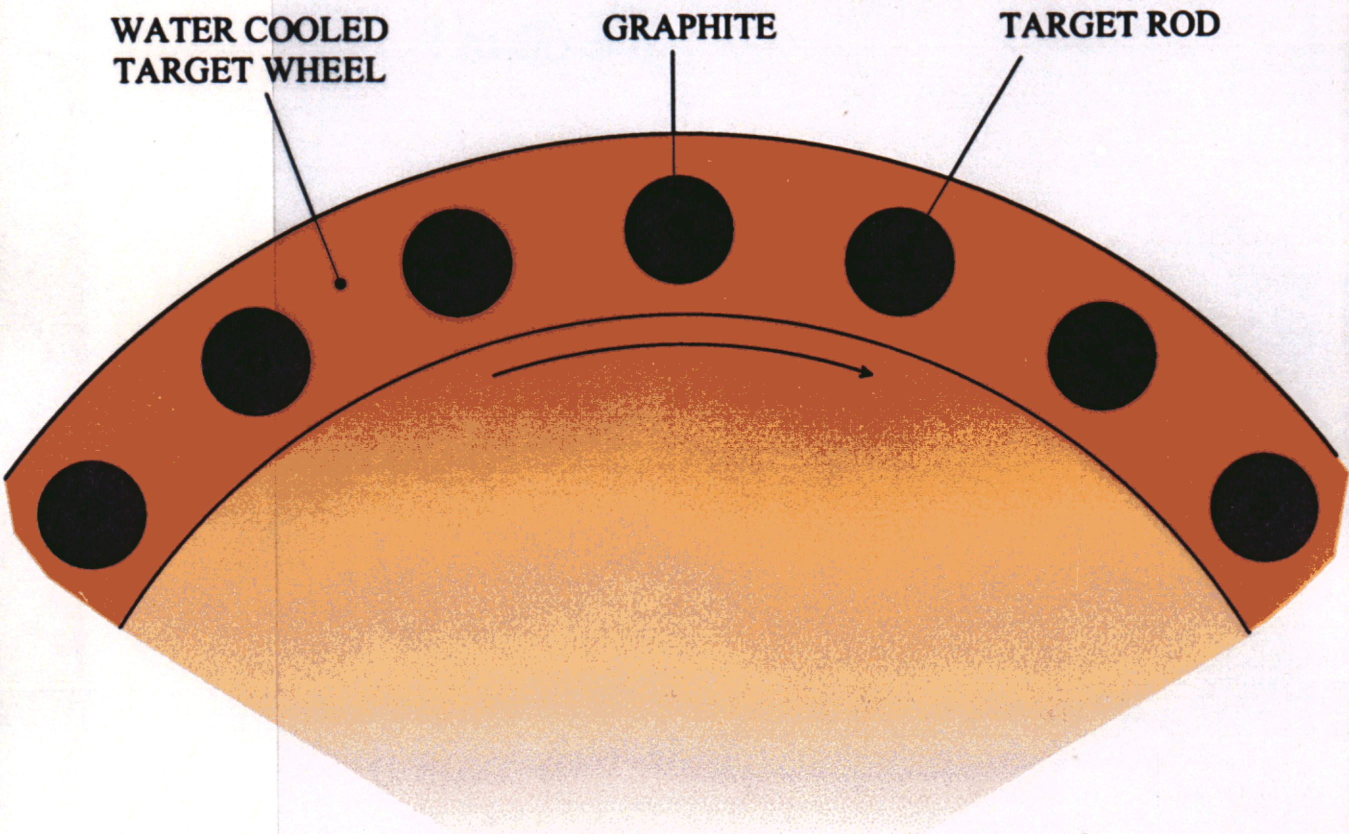
Assume a <u>change</u> of friction by 1% ≈ 65 W
(expected friction of rotation < 10 %, ≈ 650 W)
Targets out of beam line by 1 mm
after 300 ms

Need control loop locked to accelerator
via angle encoder with 6000 lines / turn

+

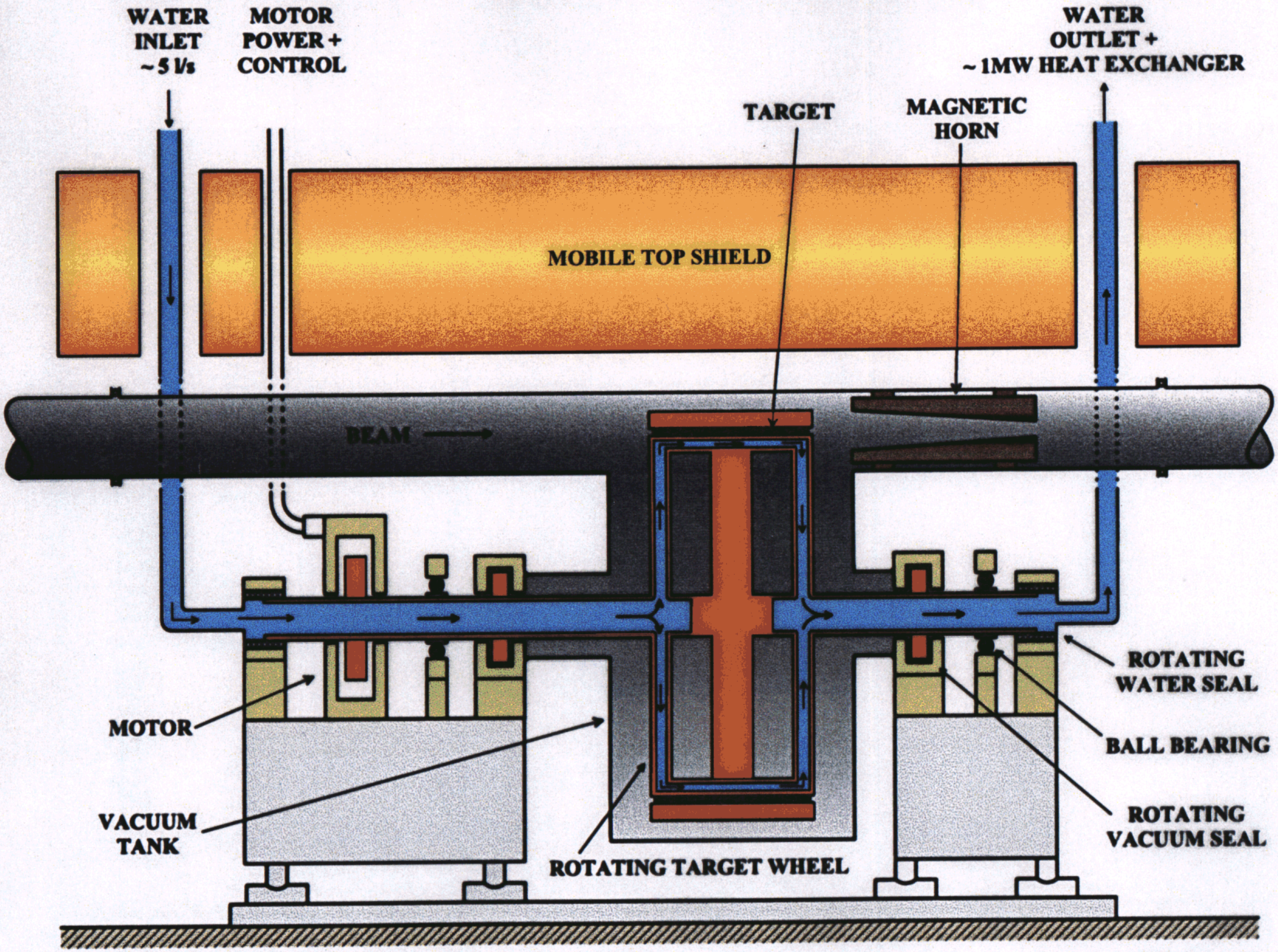
Interlock to stop the beam

ROTATING TARGET WHEEL



P. SIEVERS 23/05/2000

WATER COOLED ROTATING TARGET WHEEL + MAGNETIC HORN



P. SIEVERS 23/05/2000

- Eddy Currents

To move 1 cm^3 of Cu with $v = 5$ m/s into the solenoid takes a peak force of

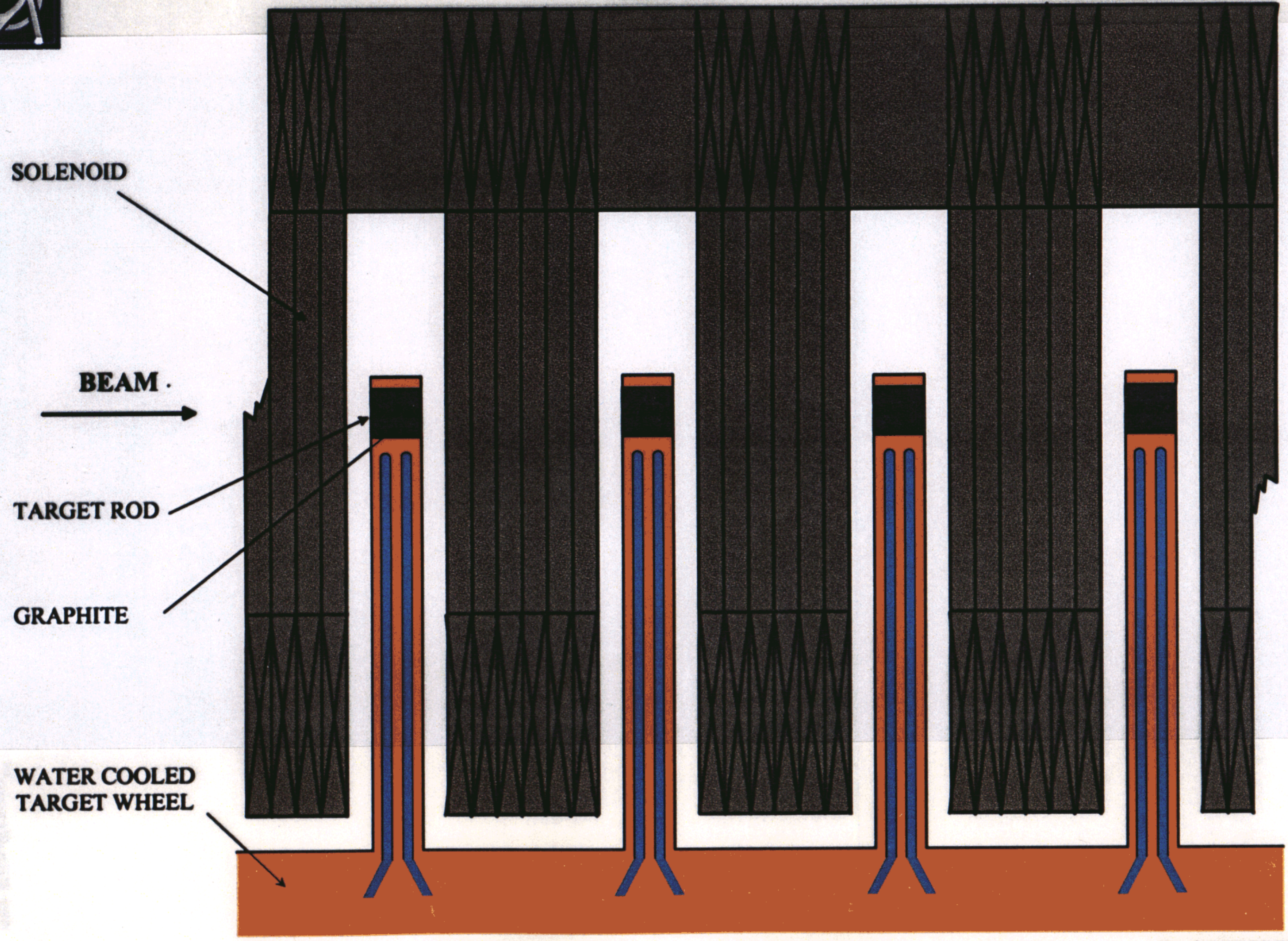
~ 600 N !

* Thin laminations for metallic parts with high electrical resistivity (Ti-alloys)

* Non-conducting materials (ceramics), again « Edge Technology »

TARGET WHEEL ROTATING THROUGH A SOLENOID

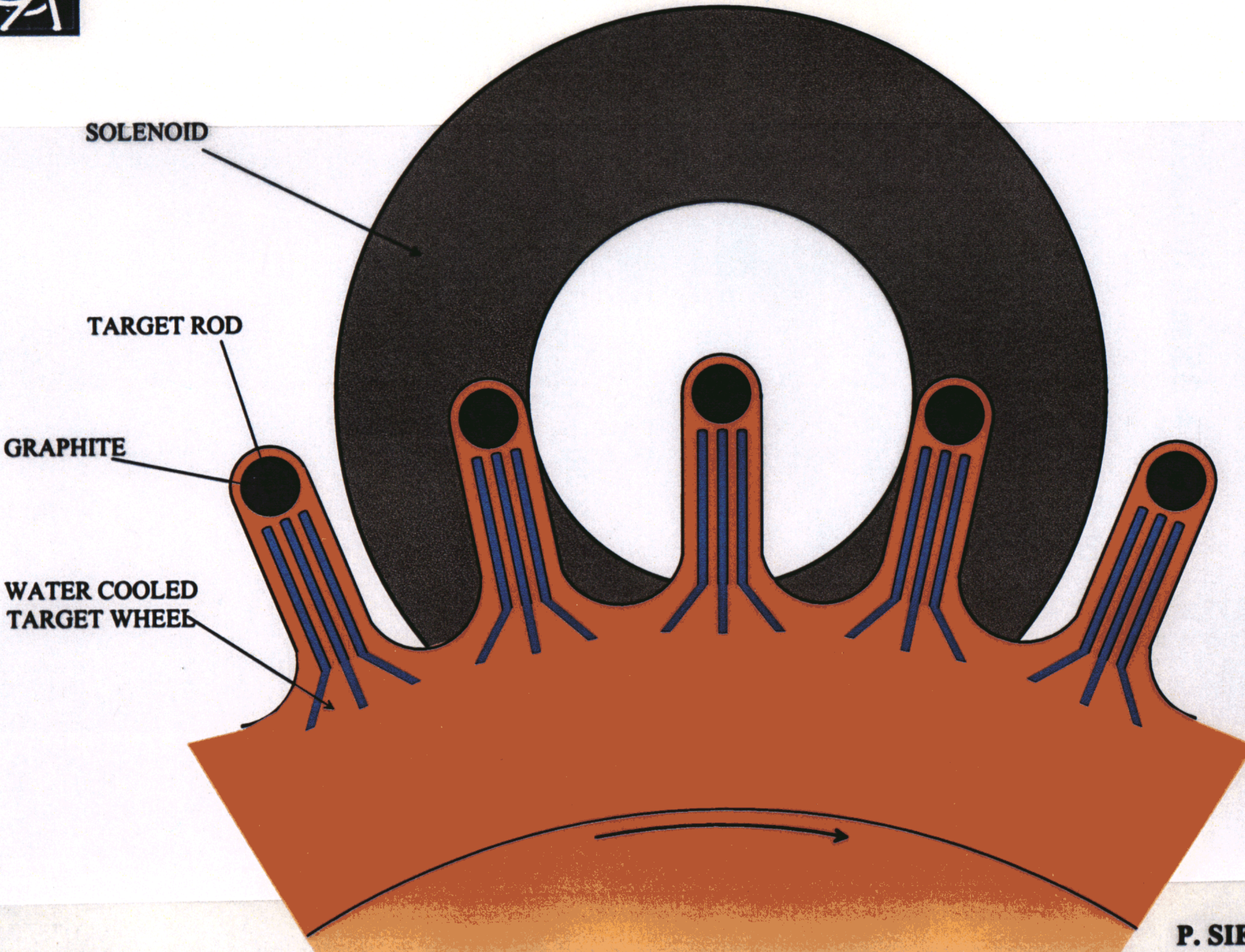
SIDE VIEW



P. SIEVERS 2

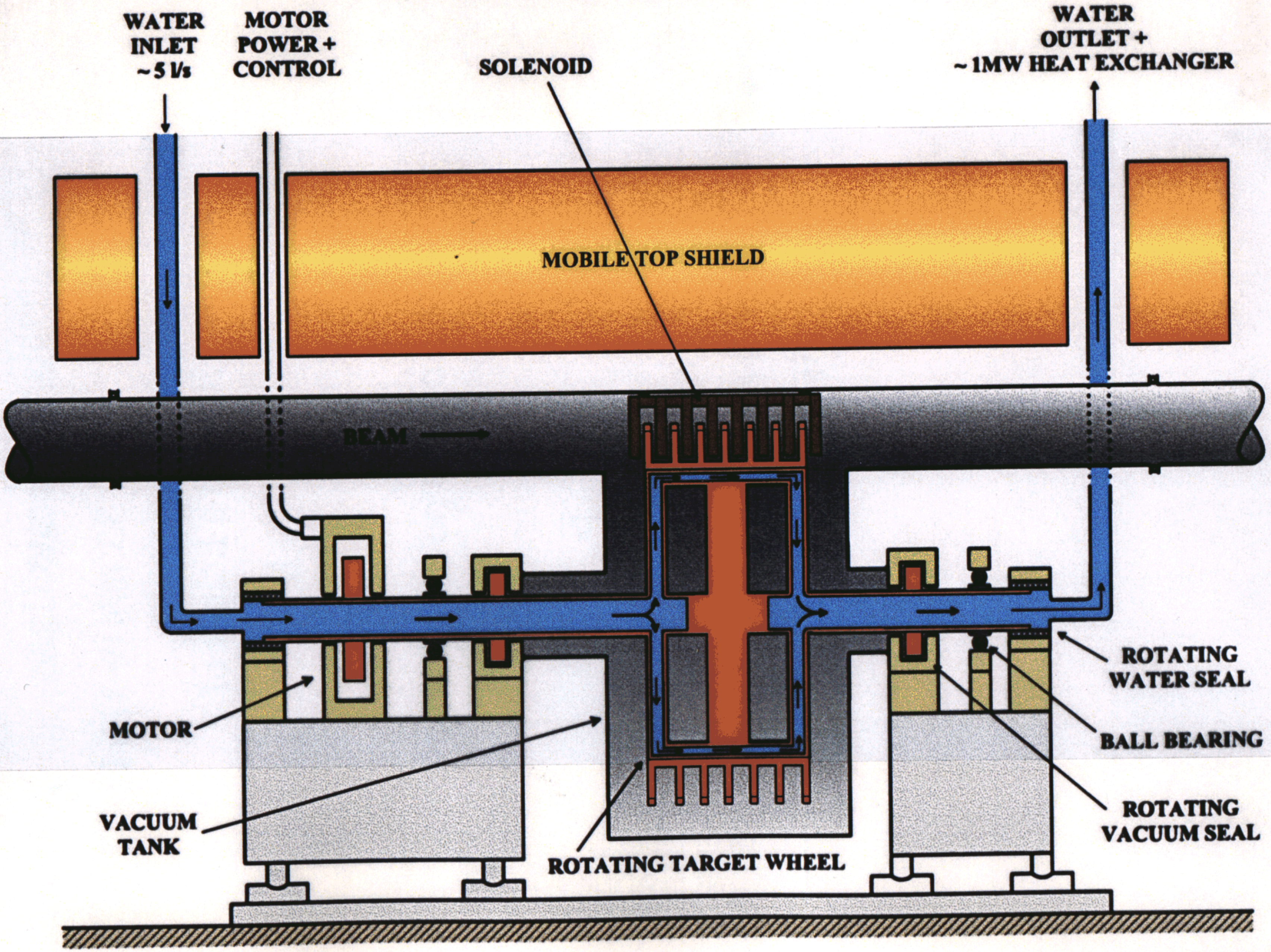
TARGET WHEEL ROTATING THROUGH A SOLENOID

FRONT VIEW



P. SIEVERS 23/05/2000

WATER COOLED ROTATING TARGET WHEEL + SOLENOID



Peter Sievers, CERN
1211 – Geneva 23, Switzerland

Liquid Metal Target Inside Solenoid

- Injection of liquid metal target into Solenoid hampered by
  forces, friction, pressure:

$\sim \frac{dB}{ds}$, $B_{(s)}$, $v_{(s)}$, target diameter, electrical conductivity of liquid.

- Let drop « Target-lets » from above into the center of the Solenoid

- Supply « shower head » with pipes of large cross-section to keep v low

In supply pipe: $v_{shower} \approx \frac{v_{axial\ injection}}{\text{no. of shower holes}}$

At shower exit $v \downarrow \approx \varnothing_{Target} \times f \approx 0.2 - 0.5$ m/s

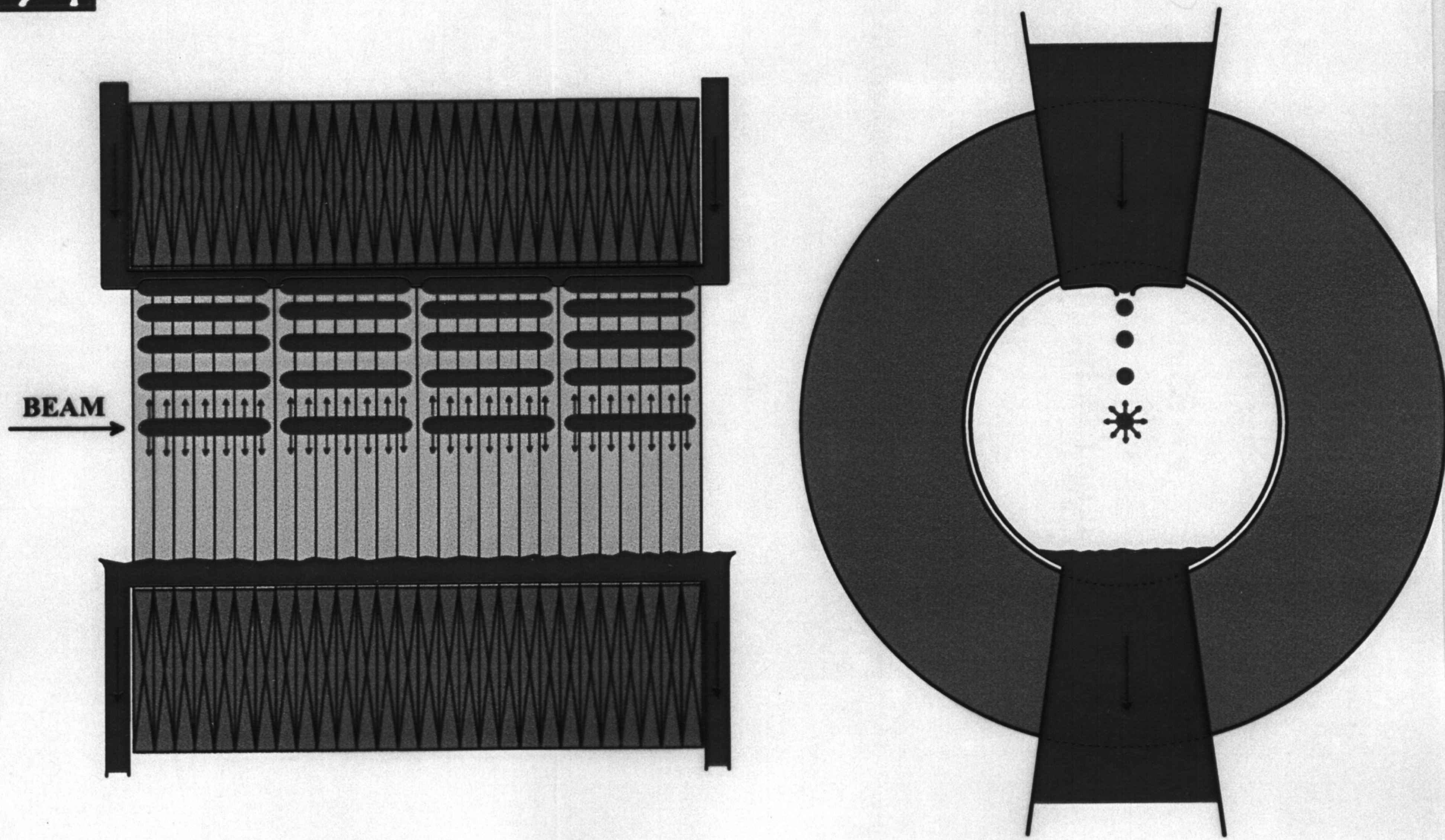
Axial injection $\vec{v} \approx L_{Target} \times f \approx 8 - 15$ m/s

$t = 0$ $R = 10$ cm $v = 0$

$t_1 = 140$ ms $R = 3$ cm $v \approx 1.4$ m/s

$t_2 = (140 + 20)$ ms $R = 0$ cm $v \approx 1.4$ m/s

LIQUID TARGET RADIAL INJECTION INTO SOLENOID



P. SIEVERS 23/05/2000

- Timing at shower head with 50 Hz,
  have to release every 20 ms a target-let
  Precision $\pm 1\,\mathrm{ms}$: Target out of position by ± 1.4 mm

- Electr. Polarization of target-lets ?
  $E = v \times B \approx 1.4\,\mathrm{m/s} \times 20\,\mathrm{T} = 28\,\mathrm{V/m}$

- If target No. n destroys target No. n-1, increase distance between them, increase velocity, drop height, pressure.

- Stored cinetic energy E_c in Target (fast heating)

$$\frac{dE_c}{dV} = \frac{(\alpha_v \Delta T)^2}{4\kappa}$$

$$\frac{E_c}{E_T} \approx \frac{{\alpha_v}^2}{4{c_V}^2 \kappa\rho} \frac{dE_T}{dm}$$

Hg Example: $\frac{dE_T}{dm} = 30 \times 10^3\,\mathrm{J/kg}$

$\alpha_v = 18.1 \times 10^{-5}\,\mathrm{K}^{-1}$ $c_V = 140\,\mathrm{J/kg\,K}$

$\kappa = 0.45 \times 10^{-10}\,\mathrm{m}^2/\mathrm{N}$ $\rho = 13.5 \times 10^3\,\mathrm{kg/m}^3$

$\frac{E_c}{E_T} \approx 0.7 \times 10^{-6} \times \frac{dE_T}{dm} \approx 2.1\%$ $\frac{dE_c}{dm} \approx 0.6\,\mathrm{kJ/kg}$

With this accelerate Hg to

$$v \approx 35 \text{ m/s or } 126 \text{ km/h !}$$

or shoot it up to a height of

$$h = 60 \text{ m !}$$

Average cinetic power to be absorbed inside Solenoid:

$$\bar{P}_c \approx 21\,\text{kW !}$$

Does viscosity (or magnetic field ?) prevent or reduce explosion ?

Conclusion:

Wheel as such do able
Radiation resistance inside target cave is manageable (Hot Lab, Repair, maintenance).
Radioactivity confined to, but also accumulated in solid parts (except water) of wheel.
Target resistance to be verified by beam tests.

Target wheel with forward production:
Target separated from collector. Two separated problems !
The collector, pulsed at 50 Hz ?, and its efficiency may pose a limit.

Target wheel through solenoid: R+D required for target plus cooling system together with Solenoid. Superconducting Solenoid with slots ? Target wheel not cheap, must make it modular to maintain it and recuperate expensive parts.
Production and collection efficiency to be checked.