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Mercury Jet Simulations

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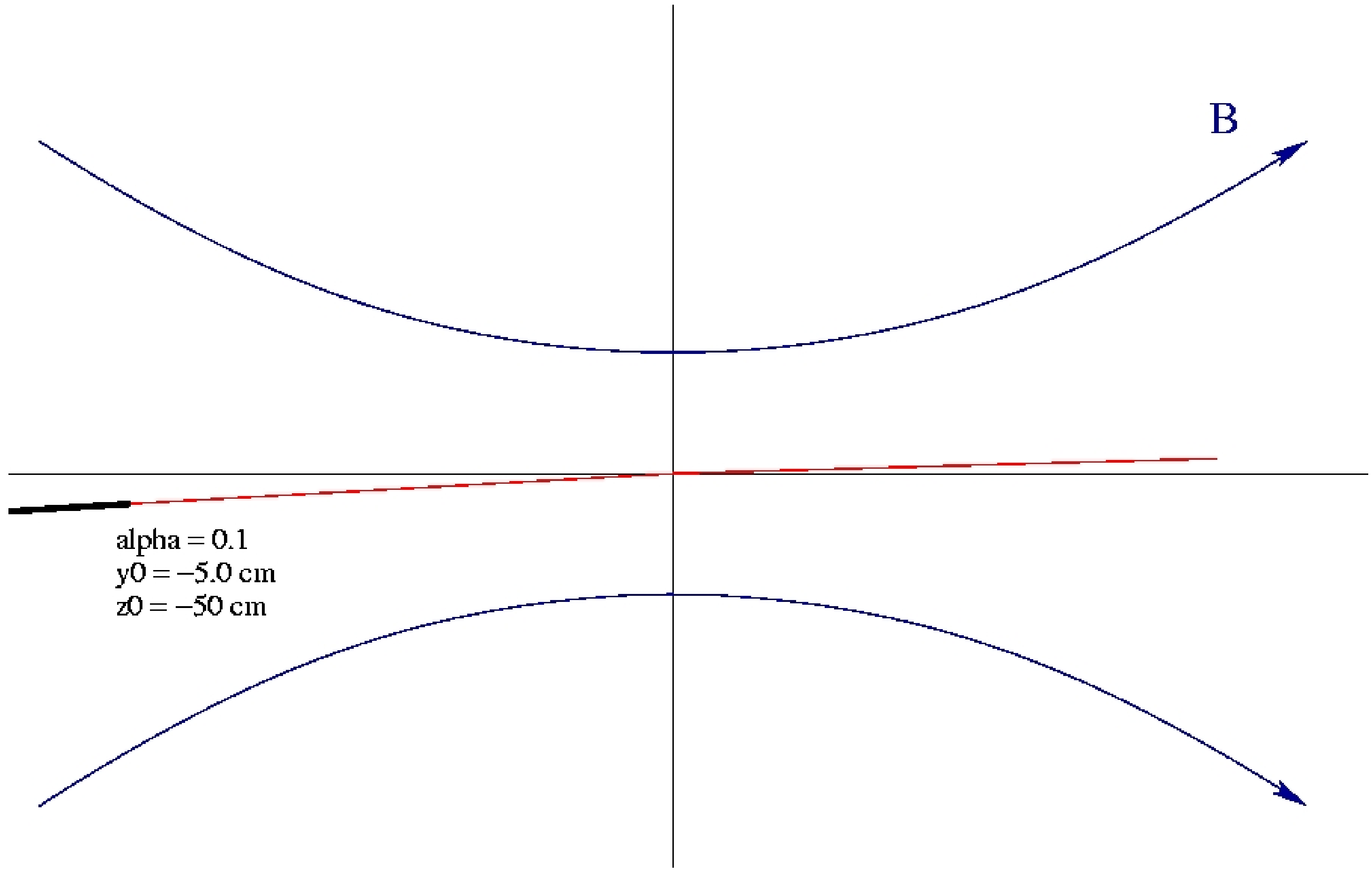
Schematic of the problem



Magnetic field of the 15 T solenoid is given in the tabular format

Equations of compressible MHD implemented in the FronTier code

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)\mathbf{u} = -\nabla P + \mu \Delta \mathbf{u} + \frac{1}{c}(\mathbf{J} \times \mathbf{B})$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)e = -P\nabla \cdot \mathbf{u} + \frac{1}{\sigma}\mathbf{J}^2$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times \left(\frac{c^2}{4\pi\sigma}\nabla \times \mathbf{B}\right)$$

$$P = P(\rho, e), \qquad \nabla \cdot \mathbf{B} = 0$$

$$\mathbf{J} = \sigma\left(-\nabla\phi + \frac{1}{c}\mathbf{u} \times \mathbf{B}\right)$$

$$\Delta\phi = \frac{1}{c}\nabla \cdot (\mathbf{u} \times \mathbf{B}),$$

$$\text{with} \quad \left.\frac{\partial \phi}{\partial \mathbf{n}}\right|_{\Gamma} = \frac{1}{c}(\mathbf{u} \times \mathbf{B}) \cdot \mathbf{n}$$

Limitations of DNS with the compressible MHD code

- Very stiff EOS. Low Mach number flow: V_s = 1450 m/s vs. V_{jet} = 25 m/s

- Large aspect ratio of the problem

- Simulation of long time evolution required (40 milliseconds). ~10^5 time steps. Accumulation errors.

- Reduction of the EOS stiffness increases unphysical effects (volume changes due to increased compressibility)

Incompressible steady state formulation of the problem

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \frac{1}{c}\left(\mathbf{J} \times \mathbf{B}_0\right)$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\left.\begin{aligned} \mathbf{J} &= \sigma\left(-\nabla\phi + \frac{1}{c}\mathbf{u} \times \mathbf{B}\right) \\ \nabla \cdot \mathbf{J} &= 0 \end{aligned}\right\} \Rightarrow \Delta\phi = \frac{1}{c}\nabla \cdot \left(\mathbf{u} \times \mathbf{B}\right)$$

$$\nabla \cdot \mathbf{B}_0 = 0$$

$$\nabla \times \mathbf{B} = 0$$

$B.C.:$

$$\left.\frac{\partial \phi}{\partial \mathbf{n}}\right|_{\Gamma} = \frac{1}{c}(\mathbf{u} \times \mathbf{B}) \cdot \mathbf{n}$$

$$p_{\Gamma} - p_a = S\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\mathbf{u}_{\Gamma} \cdot \mathbf{n} = 0$$

Direct numerical simulation approach:

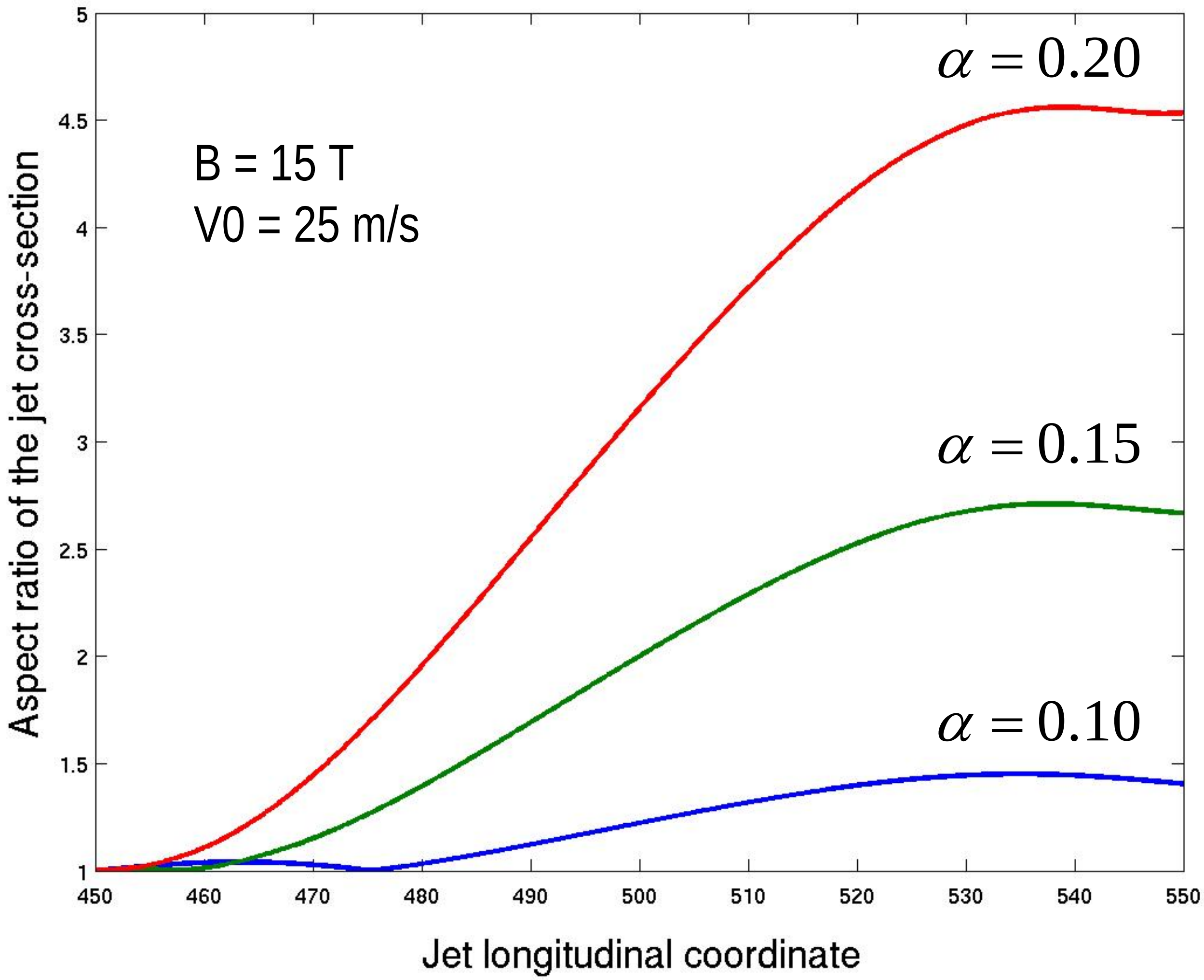
- Construct an initial unperturbed jet along the B=0 trajectory
- Use the time dependent compressible code with a realistic EOS and evolve the jet into the steady state

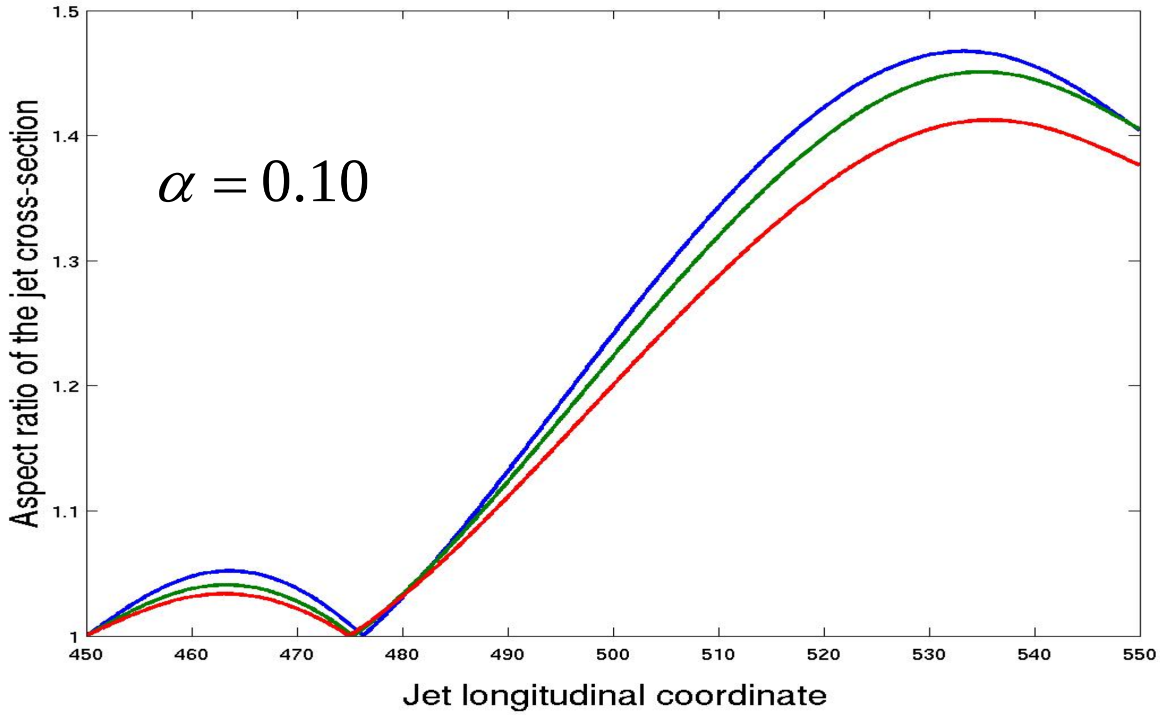
Semi-analytical / semi-numerical approach:

- Seek for a solution of the incompressible steady state system of equations in form of expansion series
- Reduce the system to a series of ODE’s for leading order terms
- Solve numerically ODE’s

Ref.: S. Oshima, R. Yamane, Y. Mochimary, T. Matsuoka, JSME International Journal, Vol. 30, No. 261, 1987

Results: Aspect ratio of the jet cross-section





Blue: V0 = 20 m/s
Green: V0 = 25 m/s
Red: V0 = 30 m/s

B = 15 T
V0 = 25 m/s
$\alpha = 0.10$