July 24, 2012

Update on Simulations of Mercury Targets

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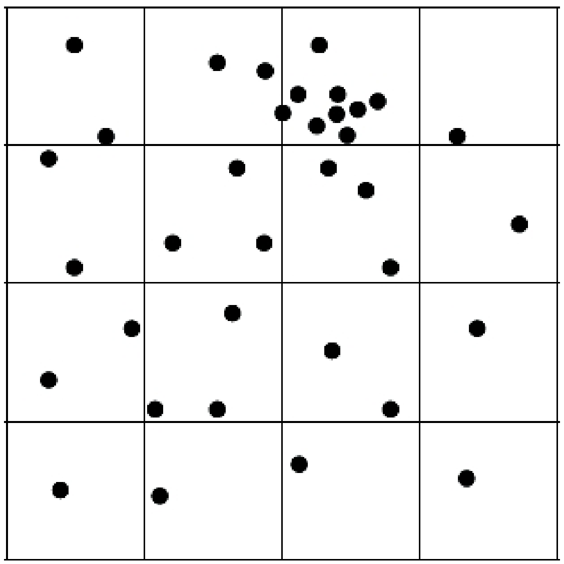
Outline

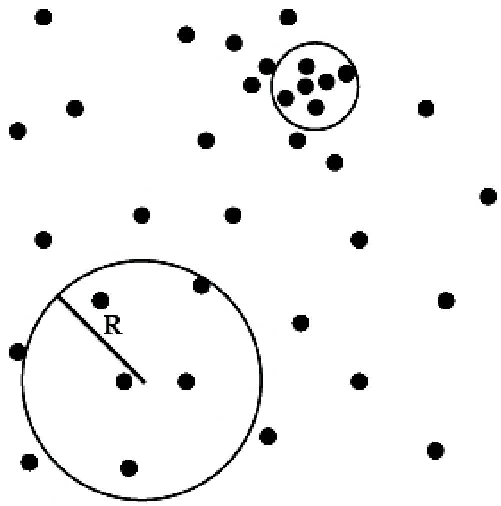
- SPH simulations of jets interacting with proton beams
- Fluid – structure interaction with SPH and mercury thimble simulations
- Front tracking simulations of free surface MHD at large density ratios

Main Idea of SPH

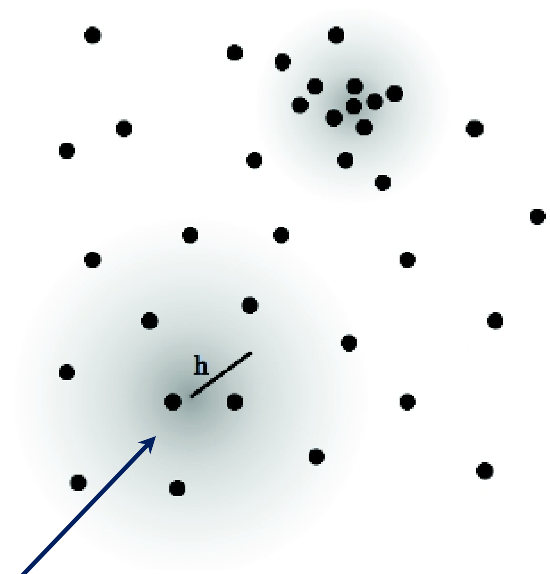
Computing density of continuum using particles



Particle-mesh methods



Sum of particles in disks



SPH:

- Density is weighted sum of particles
- Each particle represents a Lagrangian cell
- No particle connectivity

$$\rho_i = \sum_j m_j W_{ij}(h)$$

Main Approach of SPH

- Kernel approximation: replace the delta-function with a smooth kernel function

$$A(\vec{r}) = \int A(\vec{r}\,')W(\vec{r} - \vec{r}\,', h)\,d\vec{r}\,'$$

- Approximate this integral using some particle distributions

$$A(\vec{r}) = \sum_b m_b \frac{A_b}{\rho_b} W_{ab}$$

- Discretize Navier-Stokes (or MHD) equations in Lagrangian form

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\vec{\nabla}P + \vec{g} + \vec{\Theta}$$

Momentum PDE in Lagrangian system

Discretized Momentum Equation

$$\frac{d\vec{v}_a}{dt} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} + \Pi_{ab} \right) \vec{\nabla}_a W_{ab} + \vec{g}$$

Benefits of SPH

- A parallel SPH hydro / MHD code has been developed
  - Collection of solvers, smooth kernels, EOS and other physics models

- Exact conservation of mass (Lagrangian code)
- Natural (continuously self-adjusting) adaptivity to density changes
- Capable of simulating extremely large non-uniform domains
- Ability to robustly handle material interfaces of any complexity
- Scalability on modern multicore supercomputers

SPH Simulations

- Disruption of mercury targets interacting with proton pulses
- Entrance of spent mercury jets into the mercury pool

Muon Collider vs Neutrino Factory

Beam: 8 GeV, 4 MW, 3.125e15 particles/s, r.m.s. rad = 1.2 mm

Muon Collider:

15 bunches / s

66.7 ms interval

208 teraproton per bunch

Neutrino Factory:

150 bunches / s

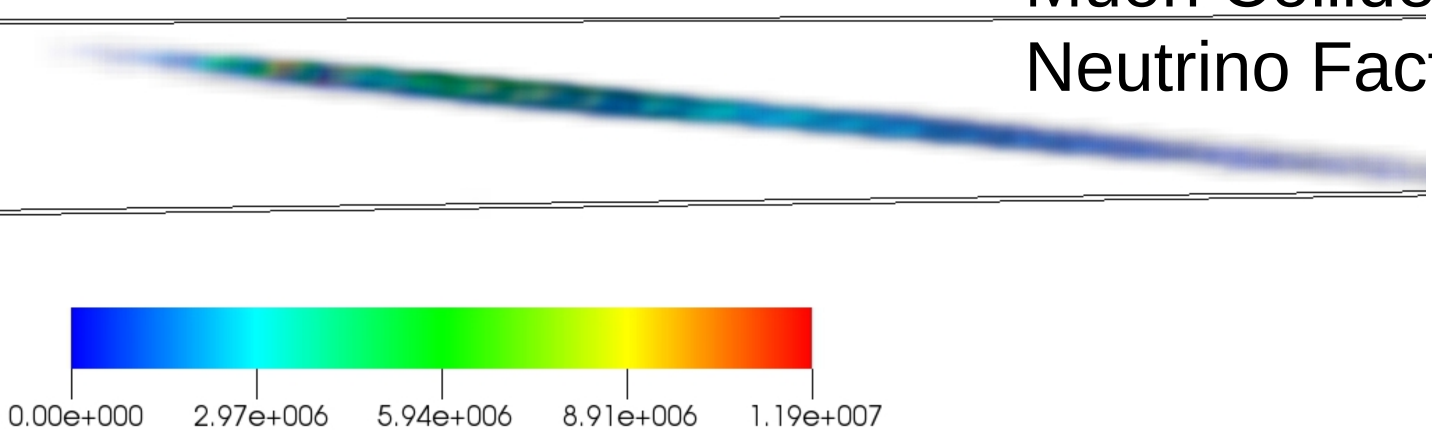
6.67 ms interval

20.8 teraproton per bunch

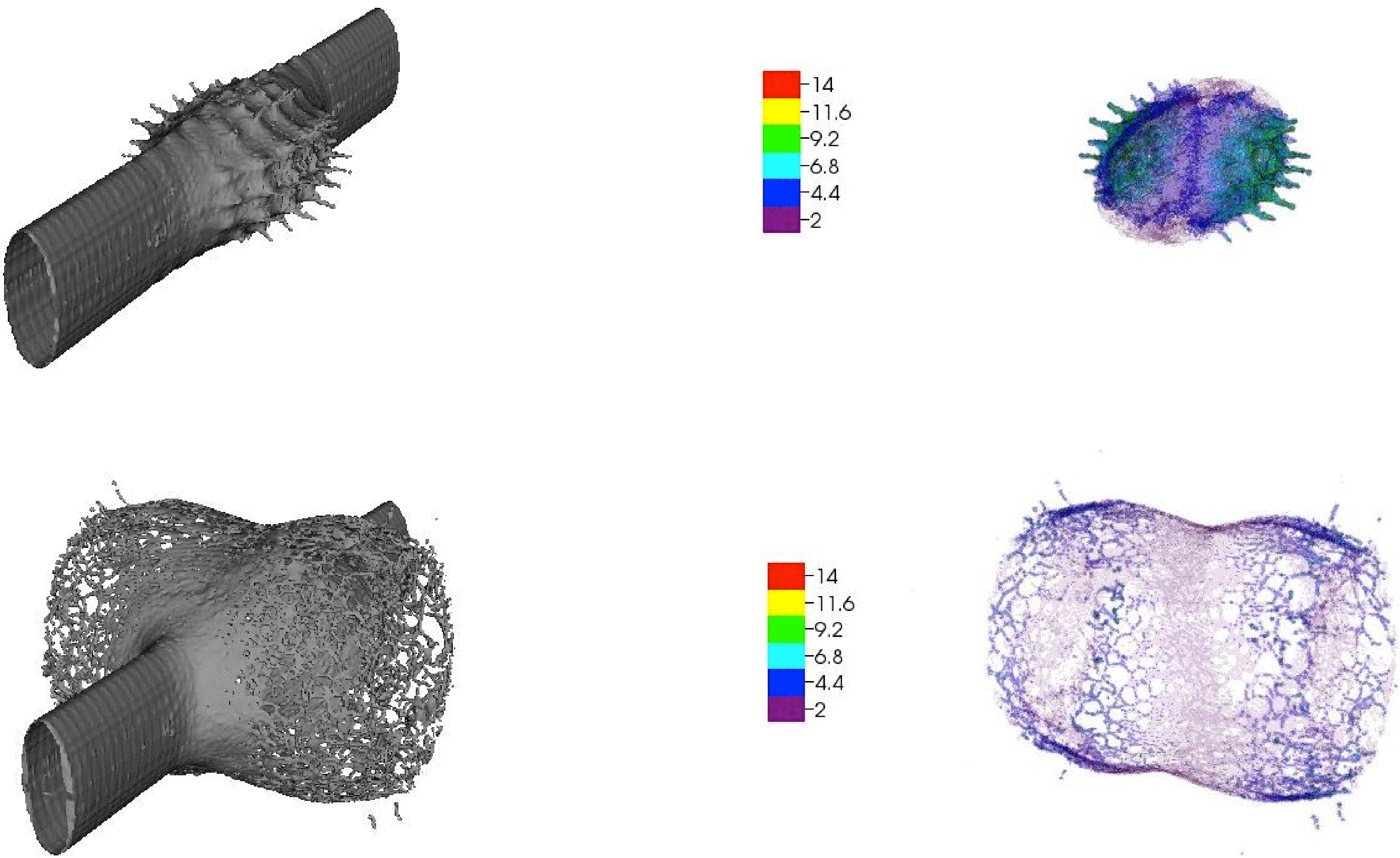
Maximum pressure (estimate):

Muon Collider: Pmax = 110 kbar

Neutrino Factory: Pmax = 11 kbar

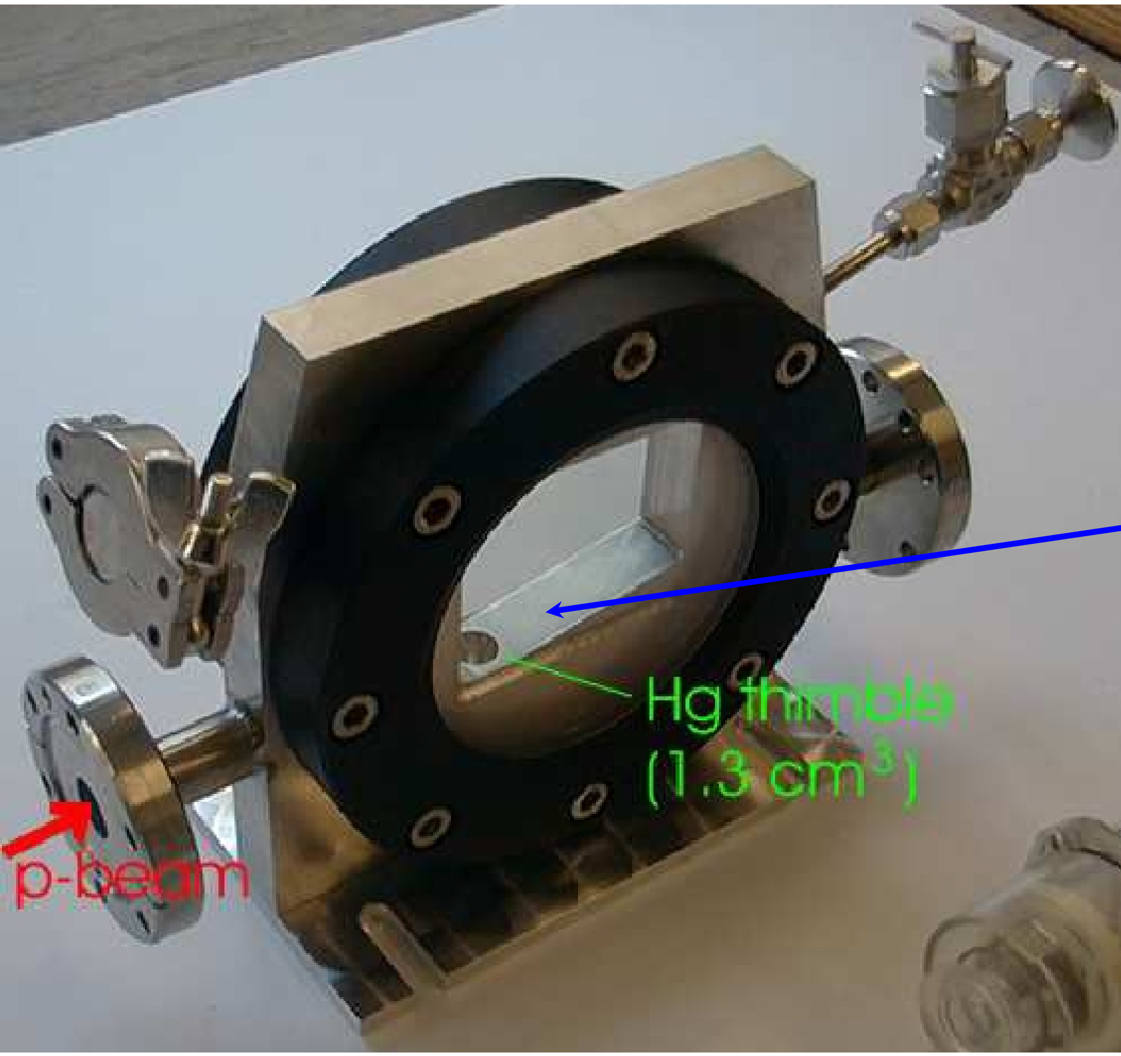


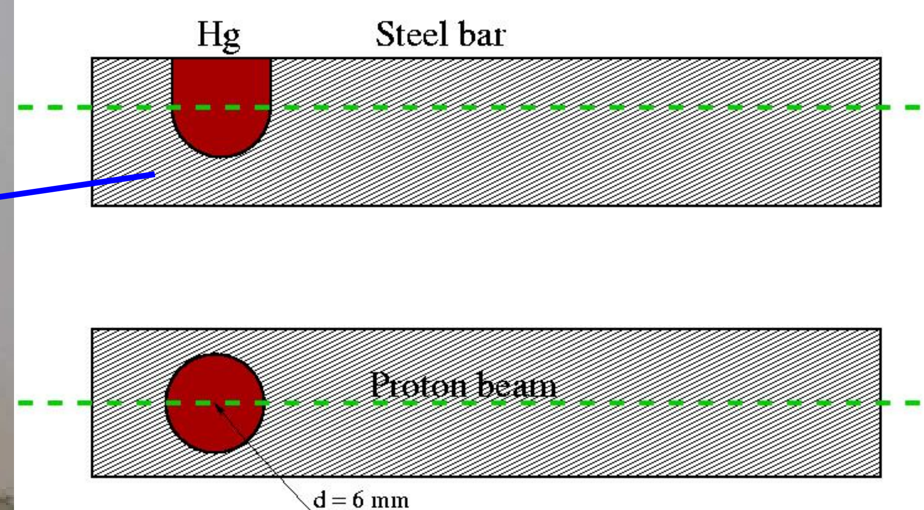
Mercury Jet after Interaction with Proton Pulse



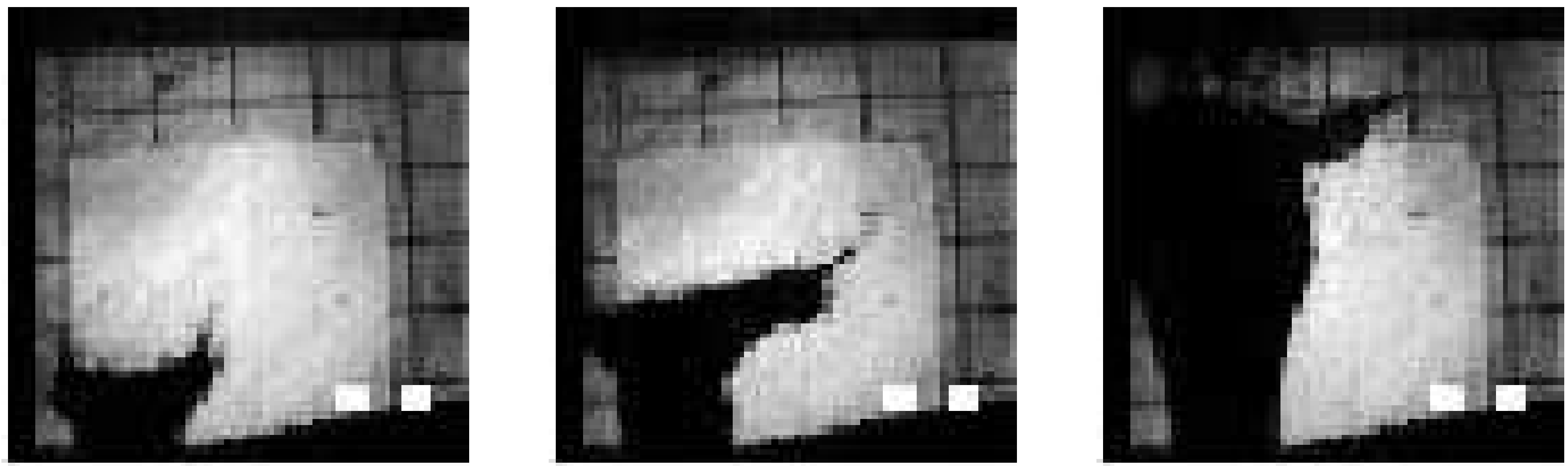
SPH Simulations of mercury thimble experiments

Experimental Setup



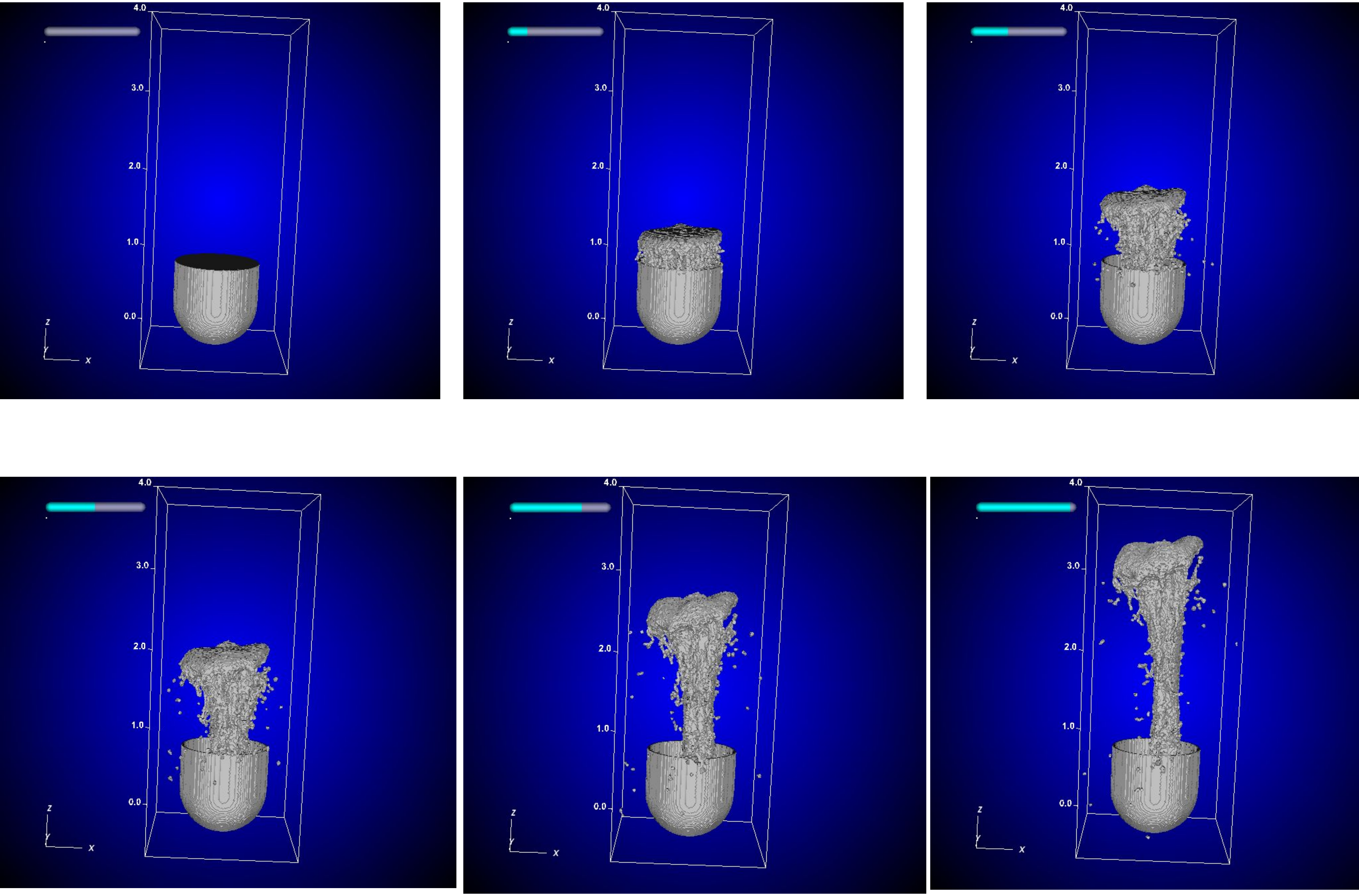


Typical Experimental Results

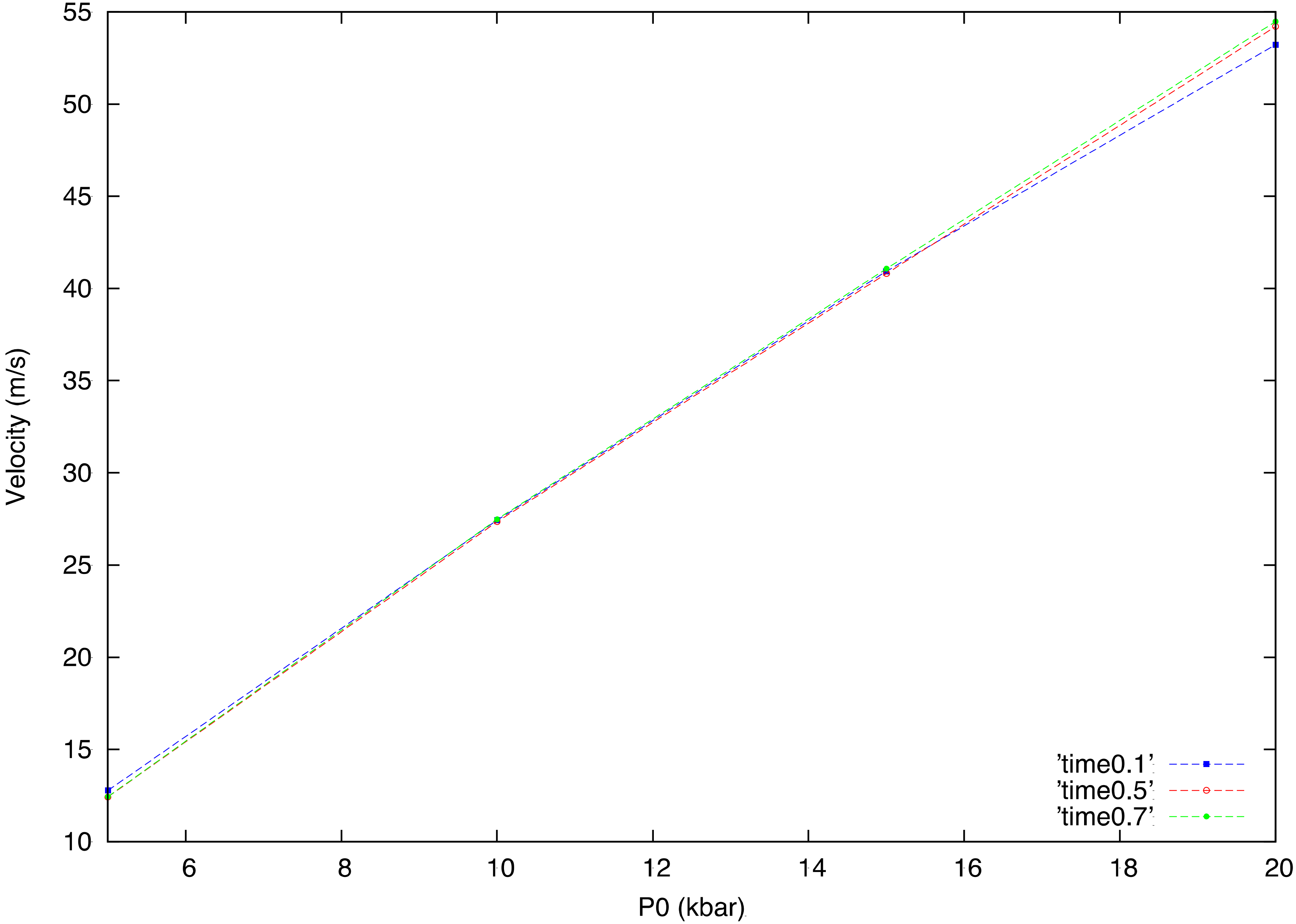


Mercury splash at t = 0.88, 1.25 and 7 ms after proton impact of 3.7 teraprotons

Simulation results



Simulation results



FronTier Simulations of Incompressible MHD at Large Density Ratios

Equations of Incompressible MHD at Low Magnetic Reynolds Numbers

$$
\begin{aligned}
\rho\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla\right)\mathbf{u} &= \mu\Delta\mathbf{u}-\nabla P+\rho g+\frac{1}{c}(\mathbf{J}\times\mathbf{B})\\
\nabla\cdot\mathbf{u} &= 0\\
\mathbf{J} &= \sigma\left(-\nabla\varphi+\frac{1}{c}\mathbf{u}\times\mathbf{B}\right)\\
\nabla\cdot\mathbf{J} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\nabla\cdot(\nabla\varphi) &= \frac{1}{c}\nabla\cdot(\mathbf{u}\times\mathbf{B})\\
\left.\frac{\partial\varphi}{\partial\mathbf{n}}\right|_{\Gamma} &= \left.\frac{1}{c}(\mathbf{u}\times\mathbf{B})\cdot\mathbf{n}\right|_{\Gamma}
\end{aligned}
$$

Equations of Incompressible MHD at Low Magnetic Reynolds Numbers

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\rho \mathbf{u} \right)$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla P + \mu \Delta \mathbf{u} + \frac{1}{c} \left(\mathbf{J} \times \mathbf{B} \right)$$

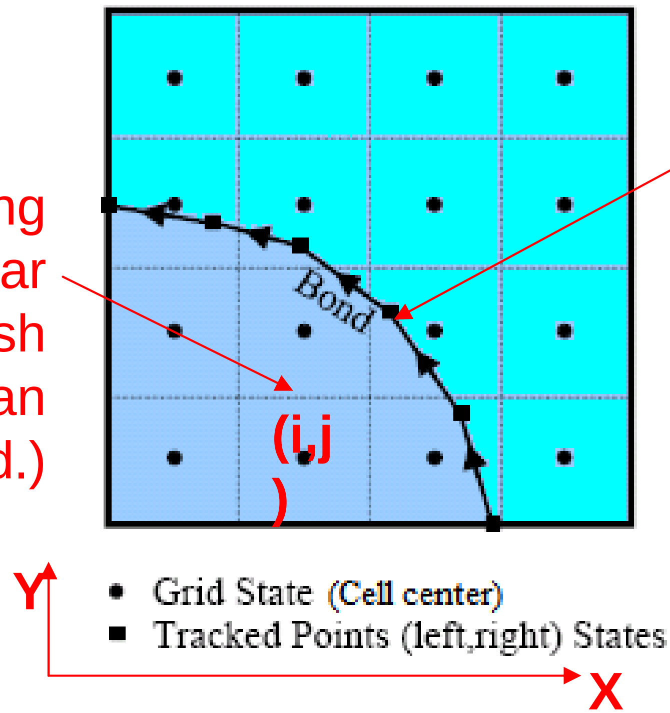
$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) e = -P \nabla \cdot \mathbf{u} + \frac{1}{\sigma} \mathbf{J}^2$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} \right) - \nabla \times \left(\frac{c^2}{4\pi\sigma} \nabla \times \mathbf{B} \right)$$

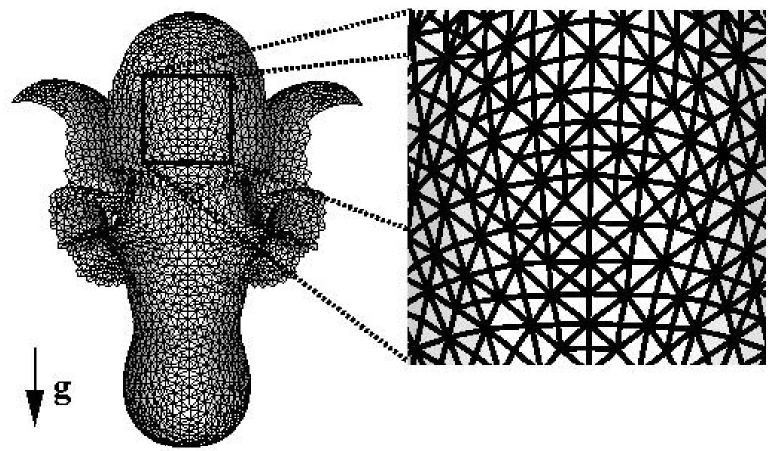
$$P = P(\rho, e), \qquad \nabla \cdot \mathbf{B} = 0$$

Main Idea of Front Tracking

- Front tracking is a hybrid Lagrangian-Eulerian method for systems with sharp discontinuities in solutions or material properties



2D Interface

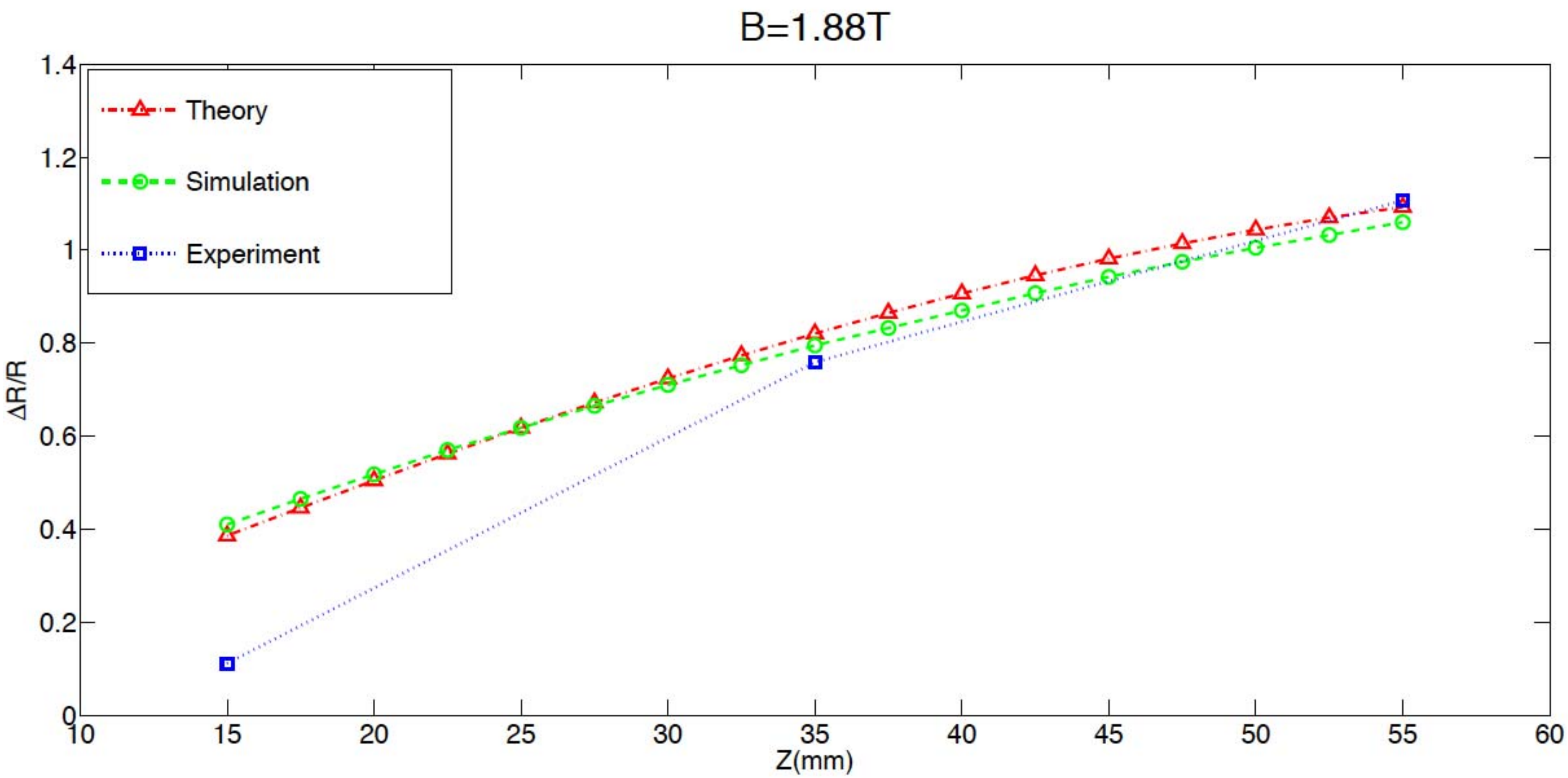


3D Interface

Verification and Validation:

Mercury Jet in Transverse Magnetic Field

Real density ratio, sharp interface



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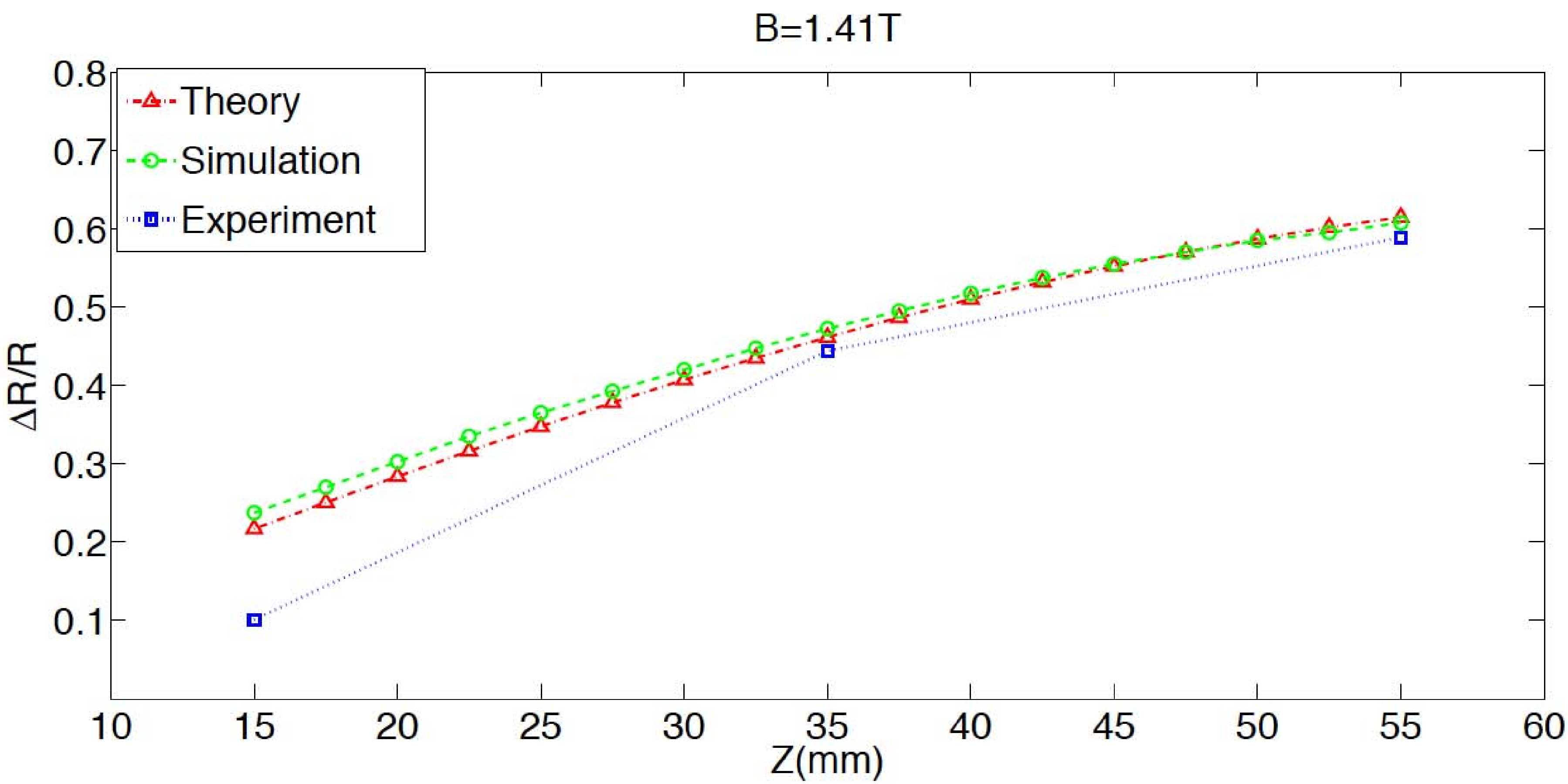
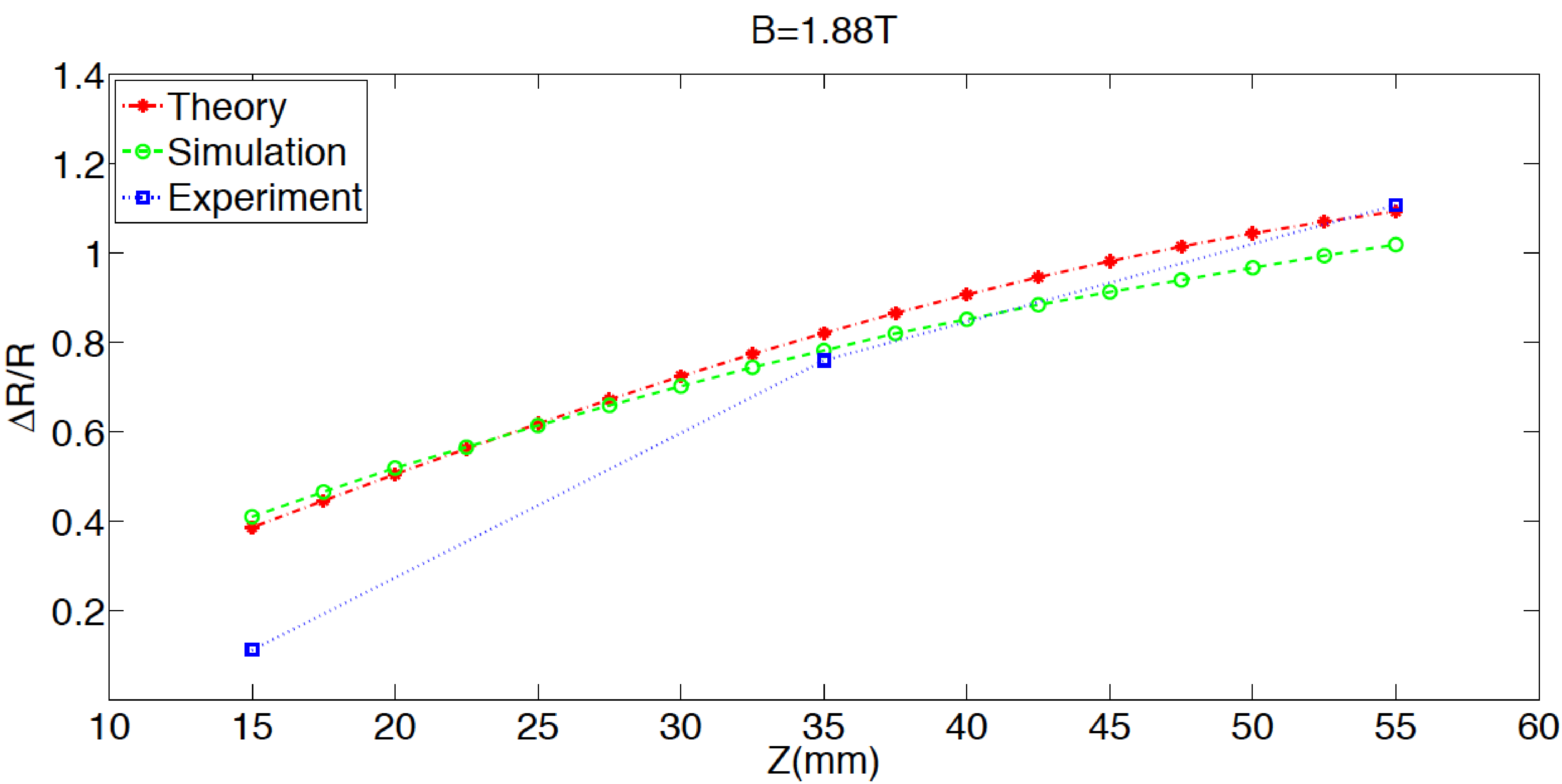


Verification and Validation:

Mercury Jet in Transverse Magnetic Field

Density ratio 10, smoothed interface



Verification and Validation:

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Density ratio 10, smoothed interface