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Cooling of a Target by Helium Gas

Abstract

We calculate that the cooling of a primary target of a Neutrino Factory source due to conduction in a static helium atmosphere would be very modest. Convective cooling via flowing helium gas is more favorable, and we estimate that a flow velocity of 1 m/s could provide cooling of about 1/8 of the nominal 40 kW beam energy deposition in the carbon target considered in the Neutrino Factory Feasibility Study 1.

1 Static Helium Atmosphere

We take the target to be a cylinder of radius r_h, length l with surface temperature T_h (h = hot). It is surrounded by helium gas contained inside a cylindrical volume of radius r_c that is maintained at temperature T_c (c = cold). We suppose that $l \gg r_c$ so that end effects may be neglected.

The helium gas has thermal conductivity,

$$\kappa(T) = \kappa_c \sqrt{\frac{T}{T_c}}, \tag{1}$$

where $\kappa_c \approx 0.15$ W/m-K for $T_c \approx 300$K. The dependence of κ on $\sqrt{T}$ follows from its linear dependence on the mean atomic velocity [1].

The rate of radial heat flow at a radius r ($r_h \le r \le r_c$) is,

$$\frac{dU}{dt} = \kappa(T) 2\pi r l \frac{dT}{dr} = \frac{\kappa_c}{\sqrt{T_c}} 2\pi l \frac{r\sqrt{T} dT}{dr}. \tag{2}$$

For a static atmosphere, this rate must be constant, so we can integrate eq. (2) to find the equilibrium radial temperature profile,

$$T^{3/2}(r) = T_c^{3/2} + (T_h^{3/2} - T_c^{3/2}) \frac{\ln r/r_c}{\ln r_h/r_c}. \tag{3}$$

Taking the derivate of this, we find,

$$\frac{r\sqrt{T} dT}{dr} = \frac{2}{3} \frac{T_h^{3/2} - T_c^{3/2}}{\ln r_h/r_c}, \tag{4}$$

so the rate of heat flow is,

$$\frac{dU}{dt} = \frac{4\pi\kappa_c l}{3 \ln r_h/r_c} \left(T_h \sqrt{\frac{T_h}{T_c}} - T_c \right). \tag{5}$$

We illustrate this for parameters of the carbon target given in the Neutrino Factory Feasibility Study 1 [2], namely $r_h = 0.75$ cm, $l = 0.8$ m, $T_h \approx 2150$K, $r_c \approx 10$ cm, and $T_c \approx 350$K. Then, eq. (5) predicts a heat flow of 1.1 kW, compared to the average power of 40 kW deposited in the target by a 1.5-MW, 16-GeV proton beam.

The heat flow due to conduction in static helium gas surrounding the target would reduce the target temperature by only a few percent.

2 Flowing Helium Gas

If helium gas were flowed over the surface of the target, convective heat transfer would occur. An estimate of the convective cooling power due to an axial flow of helium at velocity v can be obtained as follows: multiply the energy storable in a layer of helium gas of radial thickness Δr around the target by the number v/l of volume changes per second. To complete the calculation, we suppose that during time l/v of one volume change the layer of thickness Δr takes up energy due to conduction according to eq. (2).

Assuming the pressure is constant, the storable energy is,

$$U \approx N C_P \langle T \rangle = \frac{5}{2} N R \langle T \rangle = \frac{5}{2} P V \approx 5\pi r_h \Delta r l P \tag{6}$$

using $C_P = 5R/2$ and $PV = NRT$. This energy is also estimated from eq. (2) as,

$$U \approx \kappa_c \sqrt{\frac{\langle T \rangle}{T_c}} 2\pi r_h l \frac{\langle T \rangle - T_c}{\Delta r} \frac{l}{v}. \tag{7}$$

From the two estimates (6)-(7) of the storable energy, we obtain an estimate for the effective thickness Δr involved in convective cooling,

$$\Delta r^2 \approx \kappa_c \sqrt{\frac{\langle T \rangle}{T_c}} \frac{2(\langle T \rangle - T_c)}{5P} \frac{l}{v}. \tag{8}$$

Inserting this in either of eqs. (6) or (7) and multiplying by the number of volume changes per second, we obtain our estimate of the convective cooling power as,

$$\frac{dU}{dt} \approx 2\pi r_h \sqrt{\frac{5}{2} P v l \kappa_c \sqrt{\frac{\langle T \rangle}{T_c}} (\langle T \rangle - T_c)} \approx 5 \text{ kW } \sqrt{v[\text{m/s}]}, \tag{9}$$

where we suppose that $\langle T \rangle \approx 1250$K.

A flow velocity of $v = 64$ m/s might be enough to cool the full 40 kW beam power while keeping T_h near T_c, but this is a relatively high velocity in practice. A flow velocity of 1 m/s might be more practical, and might lead to a reduction of the target temperature by 200C or so.

Cooling via convective flow might be important for higher proton beam power, for which sublimation of the carbon target may be an issue.

We have not considered the interesting issue of the flow path of the helium through the high-field solenoid magnet around the target, nor that of the handling of large volumes of activated helium gas.

References

[1] R.D. Present, *Kinetic Theory of Gases* (McGraw-Hill, New York, 1958), p. 42.

[2] N. Holtkamp and D. Finley, eds., *A Feasibility Study of a Neutrino Source Based on a Muon Storage Ring*, Fermilab-Pub-00-108-E (June, 2000),
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