Mercury storm in the reaction chamber for the Muon Storage Ring and Collider project

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Introduction

During the first E951 test run, the rather dramatic effect of single proton pulses on a mercury jet was investigated [1]. For the final muon storage ring system we must consider the consequences of having a rapid sequence of pulses, such as the one shown in Fig.1 [2]

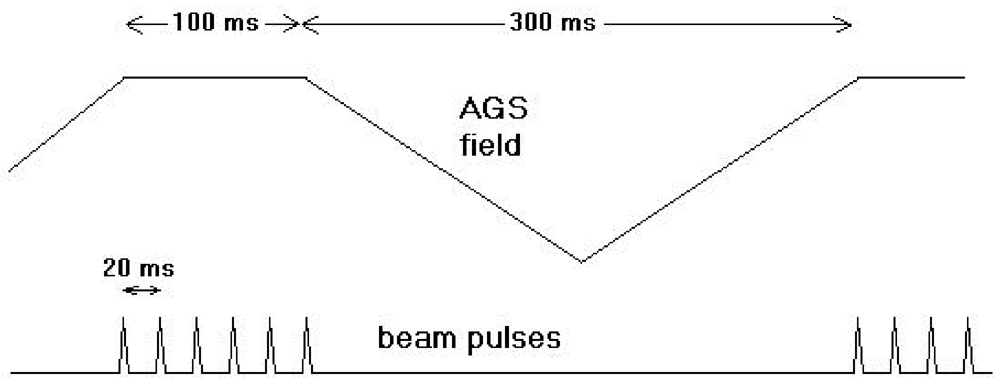

Fig. 1 Proposed beam pulse time sequence

One of the important conclusions of our first E951 run is that, even at the reduced beam intensities that were reached, much of the mercury contained in the jet was dispersed at typical transverse velocities of the order of 10 m/s. We therefore must assume that, at full intensity, most of the mercury contained in a 30 m/s jet entering a 16 cm diameter, 80 cm long cylindrical magnet chamber will be dispersed by the beam and will impact the walls during the 100 ms "beam on" part of the cycle (see fig.1). During the remaining 300 ms the unperturbed jet will exit the chamber and be collected in an appropriate sink.

There are two related reasons why we must be concerned about the mercury hitting the walls and then running along the bottom of the cylindrical cavity to be eventually collected at one or both ends. One is the flying mercury present in the volume causing additional pion absorption and possible perturbation of the jet. The other one is the stream of mercury at the bottom of the cavity in itself obstructing part of the cylindrical pion collection volume, and, unless other precautions are taken, causing additional splashing of mercury when this stream is impacted by fast mercury drops. In what follows we will try to reach rough estimates of these effects and explore some ideas for their mitigation.

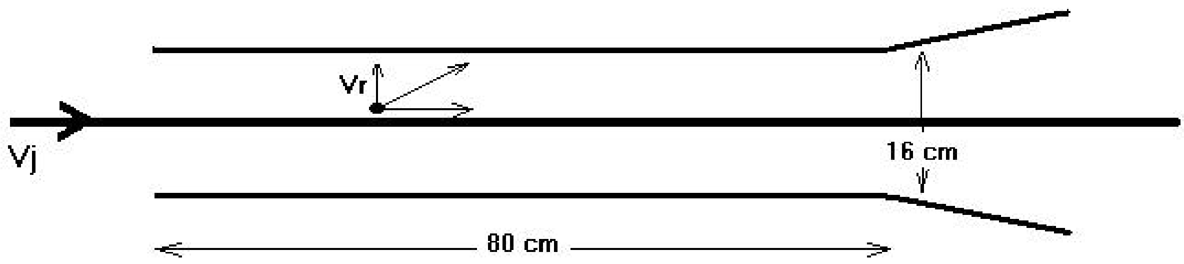

Fig. 2

Fig.2 schematically shows one view of the mercury jet entering the cylindrical reaction chamber at a velocity Vj and a drop ejected at a radial velocity Vr. If the longitudinal velocity of the drop remains Vj = 30 m/s, and Vr = 15 m/s, then the diagonal arrow indicates its trajectory. Of course there will be a wide range of radial velocities, and also the longitudinal velocities of the drops will have contributions (both positive and negative) from the beam-mercury interaction. However, from the geometry and order of magnitude of the velocities one can guess that most of the mercury, say 70%, will not exit the cylindrical chamber before impacting the walls. Most of this liquid will slow down after one or more collisions and eventually end up collecting at the bottom of the cylinder. Let's now consider each of the two issues mentioned above.

Height of the mercury stream at the bottom of the pipe

A 30 m/s, 1 cm diameter jet injects 2.36×10^3 cm^3 /s of mercury into the volume, and 25% of it will be subjected to the beam. With the above assumption then $.25 \times 2.36 \times 10^3 \times 0.7 \times 1/2$ 207 cm^3 /s on average will run to each end of the cylinder under the action of gravity. That is an average of 3.3 gallons/minute at each end. In reality there will be somewhat more at the downstream than at the upstream end, due to the part of the forward jet momentum that may survive the Hg collisions with the walls. We will neglect this difference as the first of several approximations we'll need to make to estimate the height of the mercury stream at the bottom of the cylinder.

Part of the mercury reaching the bottom of the cylinder will get there directly from the jet; other portions will run down the walls; others will rain down from the upper portions of the cylinder; and the rest will be secondary drops and spray resulting from collisions of primary drops with the walls, with other drops and with the mercury existent at the bottom. Since this is much too complicated to model, we will simply assume an appropriate flux φ [cm/s] of gentle uniform rain of mercury reaching the surface of the stream. As a zero-order calculation let's take a square channel of equal cross section instead of the cylinder, and neglect viscosity Both of these last assumptions are clearly optimistic and will result in underestimating the height of the stream.

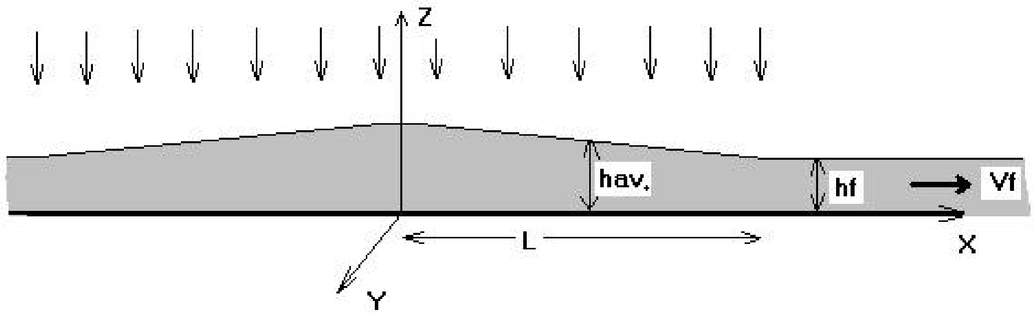

Fig. 3

In fact this reduces the problem to a two dimensional one as indicated in Fig.3. In other words we can solve the problem for a slice 1 cm thick in the Y direction. For that slice we have the uniform flux φ depositing a mass per second:

$M/s = \varphi \times L \times 1\ \text{cm} \times \rho$, 1)

where ρ is the density of mercury. This incoming mass flow must be, in the steady state, equal to the outgoing flow, which we assume occurs at a constant velocity V_f (fig. 3). (A more realistic final velocity distribution only makes matters worse). What drives the flow is the hydrostatic pressure due to the height difference $h_{av} - h_f$, and the potential energy per second deposited by the incoming material $g \times M/s \times (h_{av} - h_f)$ must, again in the steady state, be equal to the kinetic energy per second leaving the system in the outgoing flow. We can thus write the following equations:

$g \times M/s \times (h_{av} - h_f) = 1/2\ M/s \times V_f^2$ or $g \times (h_{av} - h_f) = 1/2\ V_f^2$, 2)

$\varphi \times L \times 1\text{cm} = V_f \times h_f \times 1\text{cm}$ or $\varphi \times L = V_f \times h_f$. 3)

We can't solve two equations with three unknowns (h_{av}, h_f and V_f), but we can find the minimum value h_{av} (min) which h_{av} could conceivably have. Eliminating V_f between 2) and 3) we get:

$h_{av} = \varphi^2 \times L^2 / (2g \times h_f^2) + h_f$, 4)

which is minimum for

$h_f = (\varphi^2 \times L^2 / g)^{1/3}$. 5)

With the above example of 207 cm^3 /s, L = 30 cm and a 14 cm wide square channel, we get φ = 0.49 cm/s, h_f (for min h_{av}) = 0.6 cm, and h_{av}(min) = 0.9 cm. The largest height of mercury in the pipe would then be ~1.8 cm (Fig. 3). Due to all the optimistic assumptions made along the way, we can expect at least 3 or 4 cm in the real cylindrical pipe. This is obviously not a very good estimate, but it is good enough to show that the problem is significant.

It should be possible to use water to make relevant measurements of the heights of a stream in a cylindrical channel. As we can see from the above equations, the lower density of the water as compared to mercury will not affect the result. To the extent that viscosity may play a significant role we must compare the values for mercury and water:

Table 1

		Water		Mercury	
Temperature (^oK)	Temperature (^oC)	η 10^{-3} kg/(ms)	ν 10^{-3} m^2/s	η 10^{-3} kg/(ms)	ν 10^{-3} m^2/s
280	7	1.42	1.42		
300	27	0.82	0.82	1.6	0.118
320	47	0.56	0.56		
340	67	0.41	0.41		
360	87	0.32	0.32		

The kinematic viscosity, $\nu = \eta/\rho$, is the relevant parameter for this problem, since it reflects the ratio of shear forces to gravitational or inertial forces. We can see from Table 1 that the values of ν for practical water temperatures are at least three times larger than for 27 ^oC mercury. Therefore, if viscosity plays any significant role (which I doubt), the experiment with water will yield larger values for the height of the liquid stream than would be the case with mercury. On the other hand, the strong temperature dependence of the water viscosity will make it possible to vary the viscosity sufficiently to investigate the viscosity dependence of the result.

Flying mercury drops

The questions are how much mercury from previous beam pulses may there still be present traversing the volume of the cylinder when a subsequent pulse arrives, and what will the effect be of this flying mercury on the pion collection efficiency and on the mercury jet stability. The phenomena involved are very complicated, and many of the relevant factors such as drop size and velocity distributions are poorly known. We shall, somewhat artificially, consider four types of drops: "primary drops" originating from the beam jet interaction, "falling drops" from mercury stopping at the top of the pipe and then dropping under the action of gravity, "splashed drops" from primary drops hitting the walls and being partly reflected back into the volume, and "splashed stream" caused by primary drops impacting the mercury stream at the bottom of the pipe.

1) Primary drops

The undisturbed jet will be aimed towards a mercury collecting pool, and there will be no drops present in the pion collection volume when the first beam pulse arrives. We guessed above that 70% of the irradiated part of the jet will impact the walls. The remaining 30% will thus exit the interaction volume, but part of this 30% will not be aimed towards the pool. Depending on where we place a mercury containment barrier, an appreciable fraction of this 30% will still be in flight when the following pulse arrives. Also some of the mercury collecting on the barrier will remain in the path of the spiraling pions for an even longer time.

2) Falling drops

To evaluate this contribution we shall again adopt the "square pipe" approximation (fig. 4, case 2).

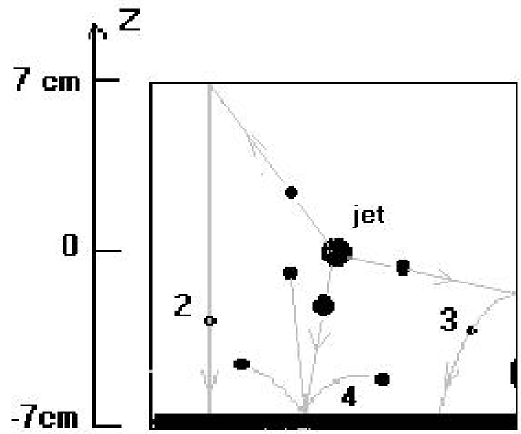

Fig. 4

For primary drop radial velocities > ~1 m/s , the dominant times involved for secondary drops to clear the collection volume after a beam pulse are the free-fall times for these drops. This is illustrated in fig. 5 for primary drops traveling up with initial velocities of 2.5 m/s, 5 m/s and 10 m/s, stopping at the top and then dropping (case 2, fig. 4).

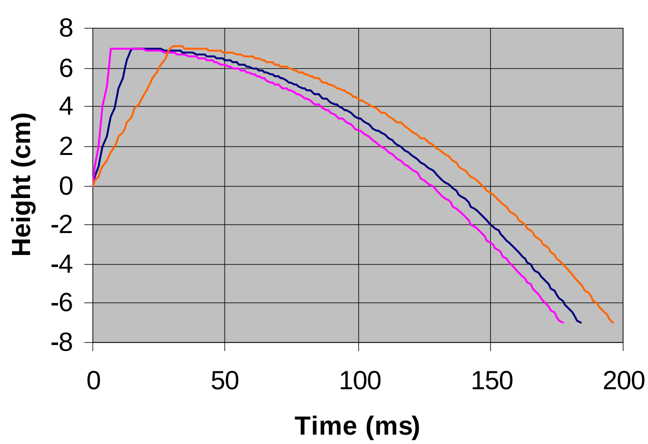

Fig. 5

Comparing figs.1 and 5 we see that this type of drops will remain in flight in the chamber during the entire 6-bunch sequence, but they will be gone by the time the next bunch sequence arrives. To estimate how much mercury may be contained in these drops we return to the previous guess of 70% of the irradiated jet being dispersed. We further assume that ~1/4 of these drops impacts the upper surface. By the time the sixth bunch arrives there will be an estimated 70% × 5/4 = 87.5% more flying mercury due to just this type of drops in addition to the mercury irradiated by that bunch.

3) Splashed drops

The contribution of these drops (case 3, fig. 4) is even more difficult to estimate because we don't know the fraction of mercury returning to the volume after impact of a drop, and that fraction will depend of the size and velocity distributions of the drops, the knowledge of which is very limited. One thing we can say is that the time distributions will on average be shorter as compared to the ones shown in fig. 5, but not short enough to be of much help. We now have two surfaces instead of the one in the previous case. Therefore if the backsplash factor were 50% we would have roughly another 80% contribution to the flying mercury by the arrival time of the last bunch in each series of six.

4) Splashed stream

Estimating the effects of a drop impacting a liquid surface at different angles is a complicated fluid dynamics free boundary value problem which would be difficult to solve even for a drop of known size, shape and velocity. Not only don't we know the size, shape and velocity distribution of the individual drops, but there may be additional collective effects of a large number of almost simultaneous impacts. We can therefore not make any credible estimates of this effect. One thing we can say is that, for drops with appropriate upward velocity components, the total flight times can be up to twice as long as typical times indicated in fig. 5, i.e. up to ~360 ms. From fig. 1 we see that in this case some of the drops from one bunch sequence may still be present during the next one.

CONCLUSIONS

From the "falling drop" and "splashed drop" rough estimate above, and neglecting "primary drops" and "splashed stream", we see that for the last bunch of each series of six bunches an additional 1.6 times as much mercury could be randomly flying around in the volume as there is in the 60 cm long irradiated portion of the jet. For a more typical "central bunch" that mass would be something like: 0.8 × ð × 0.5^2 cm^2 × 60 cm × 13.6 grams/cm^3 = 513 grams.
Over a 60 cm long cylindrical volume of radius 8 cm this corresponds to an average density of 42.5 mg/cm^3, or correspondingly less if we consider that a fraction of these drops will migrate out and mainly downstream of this cylindrical volume. In any case there will be about as much dispersed mercury in the path of the spiraling pions as there is "undisturbed" mercury in that part of the primary jet that interacts with the protons. Since we know that the mercury in the jet

causes considerable pion absorption we must conclude that the additional losses due to the dispersed mercury will also be significant.

Regarding the highly uncertain contribution to splashing from the flowing stream, we may have to cover this stream with some sort of chevron shaped splash-guard that would intercept most drops before they can hit the liquid surface. Such a splash guard would of course take up some additional vertical space, but its height can probably be kept well under 1 cm.

The mercury stream at the bottom of the pipe will most likely prove to be a problem anyway if experiments with water confirm that its height will be several cm. Such an encroachment on the opening of a 8 cm radius chamber would be significant from the point of view of pion transport. To mitigate this problem one would need to find a way to increase the average velocity of the flowing mercury. One (probably not very practical) possibility would be to tilt the magnet and early part of the pion transport. Another idea would be to provide suction pipes with strategically located openings at the bottom of the chamber. Or we may generate a high velocity gas flow in the chamber to sweep out much of the dispersed mercury (hurricane-force wind, the only missing ingredient for a true mercury storm). A gas flow of a velocity matched to the jet velocity may also be desirable to minimize gas-induced jet instabilities.

Finally, the potentially most serious problem associated with flying drops is their likely destabilizing effects upon the mercury jet. This may further complicate the difficult problem of creating a 30 m/s, 1 cm diameter jet which must be reasonably coherent over 1 m.

REFERENCES

1) http://pubweb.bnl.gov/users/kirk/www/e951/targets/hg_jet/

2) U.S. Neutrino Factory Studies, S. Ozaki, R. B. Palmer, M. S. Zisman, June 2001