Heat Leak into Cryostat #1 through 304-SS or G-10 Supports

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My report dated May 18th predicted the electric power needed for refrigeration to cope with the heat leak through mechanical supports of Type 304 stainless steel (SS) with warm end T_{warm} at 300 K and cold end T_{cold} at 4.22 K. This report extends the analysis to G-10 (cryogenic-grade high-pressure epoxy/fiberglass laminate) and finds that a G-10 support of comparable strength (four times the cross section of the SS support) requires only 18% as much power for refrigeration. This report also refines the previous results for a support that is thermally anchored along its length, now optimizing the longitudinal location, L', as well as the temperature, T', of the thermal anchor.

To support the 46-MN axial load on Cryostat #1, its mechanical supports, if of 304 SS, must have an aggregate cross-sectional area, A, of about 0.1 m²—e.g., a tube of wall thickness ~1 cm and diameter ~3.2 m, the mean diameter of the magnets. The thermal conductivity, k, of 304 SS, averaged over 4 K to 300 K, is ~11 W/(m·K), according to Y, Iwasa, *Case Studies in Superconducting Magnets*, p. 307. Therefore, one can expect the heat leak, Q, to 4 K from 300 K by a support of cross section A = 0.1 m² and length, say, L = 0.2 m, to be Q = k ΔT A / L ≈ [11 W/(m·K)] [(300 − 4) K] [0.1 m²] / [0.2 m] = 1.6 kW. This is approximately double the power deposition allowed from radiation. Therefore, one needs to predict the heat leak accurately and to reduce it, ideally without increasing the length of the support, which sacrifices stiffness.

To predict the heat conduction along a support (of constant cross section), I divided it longitudinally into 100 slices of length Δz and calculated the temperature drop between the faces of the slice with longitudinal heat flux density, w [W/m²], and thermal conductivity, k[T(z)], as given by E.D Marquardt, J.P. Le & Ray Radebaugh, "Cryogenic Material Properties Database," 11th International Cryocooler Conference, June 2000. The total temperature drop through the 100 slices is the heat flux density times the sum of the thermal resistances—i.e., w Δz Σ r[T(z)], where r ≡ 1 / k[T(z)]. For 304 SS, k(T) ranges from 0.29 W/(K·m) at 4 K to 15 W/(K·m) at 300 K, so r(T) ranges from 0.065 K·m/W at 300 K to 3.4 K·m/W at 4.22 K.

Parabolic extrapolation from the temperatures at z−Δz and z−2Δz provided estimates of T(z) at the midplane and downstream end of the slice—respectively z+Δz/2 and z+Δz. Simpson's-rule integration of {w r[T(z)] dz} over the longitudinal extent of the slice predicts the temperature at z+Δz to be T(z) + $\frac{1}{12}$ w Δz {23r[T(z)] − 16 r[T(z-Δz)] + 5 r[T(z-2Δz)]}. The temperature T_{100} at the downstream face of the 100th slice could be matched to the intended value of 4.22 K with only half a dozen or so iterative adjustments to w. Optimization was by hand, via interval bisection, because Excel's Solver routine often did not converge. The predicted heat flow is 1.52 kW. If one could achieve 100% Carnot efficiency, the required wall power would be 107 kW. If the actual refrigeration efficiency were 21.3%, the required wall power would be half a megawatt.

One can greatly reduce the required wall power by thermally anchoring the support along its length. Optimization of such a support proceeds by analyzing sequentially the segment of the support upstream of the thermal anchor, and then the segment downstream of it. One selects a location L' and temperature T' for the thermal anchor and iterates the heat flux density, w', to match T_{100} with T'; throughout this segment, Δz = L'/100. Then one sets T_{warm} to T' and iterates a heat flux density, w_{cold}, to match T_{100} with T_{cold}; throughout this segment, Δz = (L-

L')/100. The heat flow $Q_{cold} = A\ w_{cold}$ is the refrigeration required at T_{cold}. The heat flow $Q' = A$ $(w' - w_{cold})$ is the refrigeration required at the thermal anchor temperature T'. Because the Carnot efficiency at T' typically is an order of magnitude better than at 4 K, the wall power required for refrigeration can be much less than without the thermal anchor. For example, Figure 1 shows that if the thermal anchor is at its optimal location of 62% (i.e., 12.4 cm) from the warm end, and its temperature is its optimum value of 42 K (from, say, helium gas from a cryocooler), then the required wall power at 100% Carnot efficiency drops to ~23.5 kW, only 22% as much as without such thermal anchoring.

   With thermal anchoring at an additional axial location one can reduce the power somewhat further. For example, the wall power requirement drops to 19.5 kW if one anchors the temperature at both 77 K and 28 K (the boiling point of liquid neon at atmospheric pressure), with the 77-K thermal anchor at 10 cm and the 28-K anchor at 16 cm from the warm end.
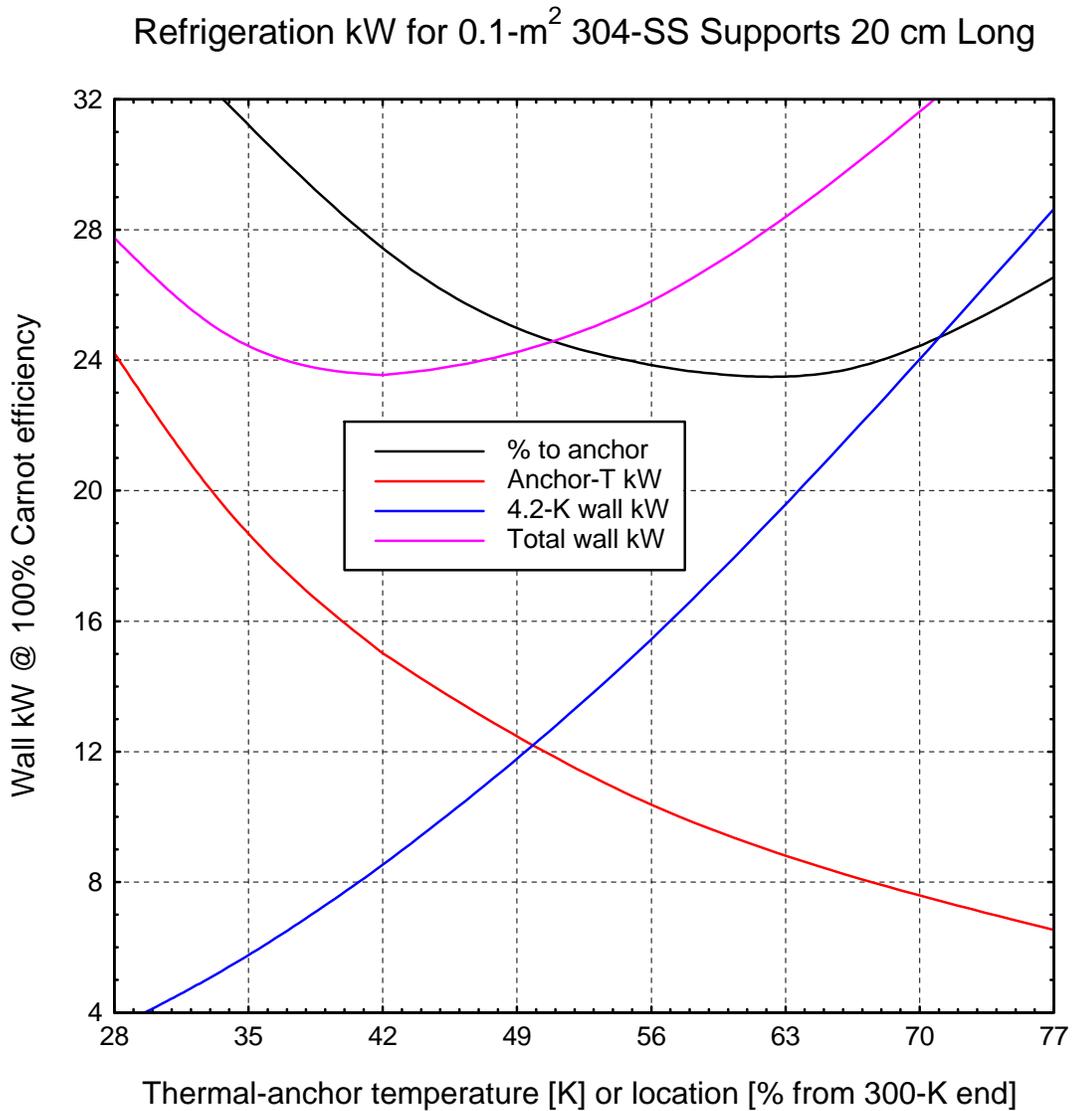

Figure 1: Electrical power (at 100% Carnot efficiency) to refrigerate 304-SS supports for Cryostat #1. Black curve: optimum location of thermal anchor is ~62% from warm end. Magenta curve: optimum temperature of thermal anchor is ~42 K. Wall power ~23.5 kW at 100% Carnot efficiency.

K. Marquardt *et al.* also gives curve fits for cryogenic thermal conductivity of six other alloys and five insulators. Among these is G-10 which, according to Iwasa, should have only one-fifth the heat leak of a SS support of comparable strength. My calculations—based on G-10 with four times the cross section of a comparable support of Type 304 SS—predict that G-10 is better than 304 SS by a factor of 6.8, if neither is thermally anchored. Figure 2 shows that G-10 thermally anchored at ~43 K at z/L ≈ 54% requires only 4.3 kW of wall power, a factor of 5.5 times less than the ~23.5 kW required with 304 stainless steel.

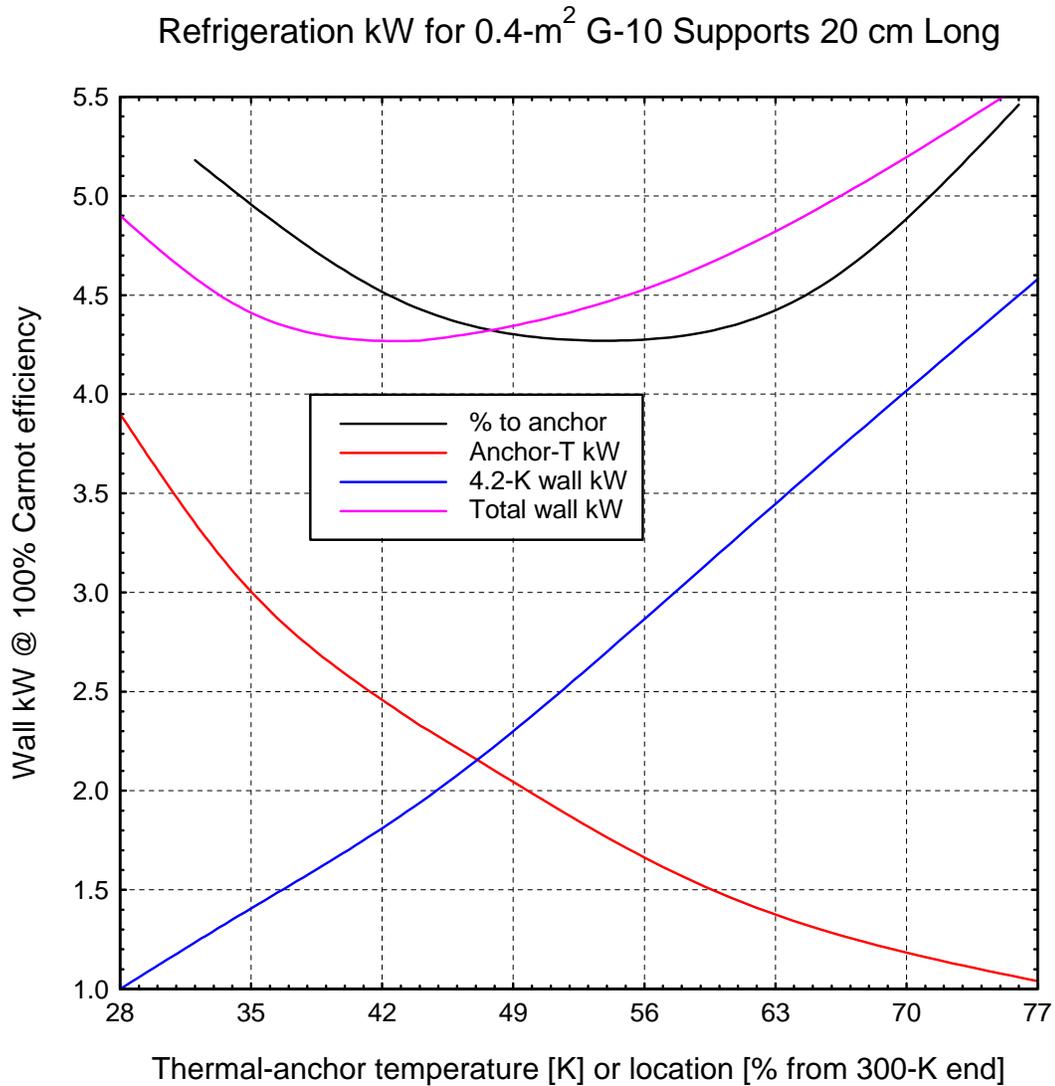### Refrigeration kW for 0.4-m^2 G-10 Supports 20 cm Long

Figure 2: Electrical power (at 100% Carnot efficiency) to refrigerate G-10 supports for Cryostat #1. Black curve: optimum location of thermal anchor is ~54% from warm end. Magenta curve: optimum temperature of thermal anchor is ~43 K. Wall power ~4.3 KW at 100% Carnot efficiency.

The improvement afforded by twofold thermal anchoring is negligible: to 4.2 kW with thermal anchoring at both [77 K, 10 cm] and [28 K, 16 cm].