B(z) from Equations with Only One or Two Adjustable Currents

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Figure 1 plots on-axis field profiles generated by an equation, analytically differentiable to arbitrary order, each of whose terms predicts the on-axis field from a component coil in a Target Magnet. For simplicity, coils typically are modeled by current sheets. Comparison of the blue and black curves of Fig. reveals that even a coil as radially thick as Superconducting Coil #1 can be modeled accurately by a single current sheet at its mean radius.

For simplicity, each of the seven designs of Fig. 1 employs no more than three current elements, all those downstream of z ≈ 5.4 m being consolidated into a single current sheet. Design #1 uses only two elements—a main coil and a current sheet extending from 2.5 m all the way to 20 m. Coil parameters are tabulated below. B∞ is the field that would be generated by a coil of identical current per unit length but of infinite length.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Design Number | #1 | #2 | #3 | #4 | #5 | #6 | #7 |
| Desired field @ z = L = 5 m [T] | 2.5 | 2.5 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 |
| Inner radius a1 of coil #1 [m] | 1.20 | 1.20 | N/A | N/A | N/A | N/A | N/A |
| Outer radius a2 of coil #1 [m] | 2.08 | 2.08 | N/A | N/A | N/A | N/A | N/A |
| Mean radius a0 of coil #1 [m] | 1.64 | 1.64 | 1.64 | 1.64 | 1.64 | 1.64 | 1.64 |
| a0 of downstream coils [m] | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Upstream end of coil #1 [m] | −2.35 | −2.35 | −2.35 | −2.35 | −2.35 | −2.35 | −2.35 |
| Downstream end of coil #1 [m] | 1.34 | 1.34 | 1.34 | 1.34 | 1.34 | 1.34 | 1.34 |
| Upstream end of coil #2 [m] | 2.50 | 2.50 | 2.50 | 2.50 | 2.50 | 2.50 | 2.50 |
| Downstream end of coil #2 [m] | 20.0 | 5.42 | 5.42 | 5.42 | 5.42 | 5.42 | 5.42 |
| Upstream end of coil #3 [m] | N/A | 5.427 | 5.427 | 5.427 | 5.427 | 5.427 | 5.427 |
| Downstream end of coil #3 [m] | N/A | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| jλ of thick-walled coil [A/mm2] | 18.01 | 18.06 | N/A | N/A | N/A | N/A | N/A |
| B∞ of upstream current sheet [T] | N/A | N/A | 20.07 | 20.07 | 20.07 | 20.07 | 20.07 |
| B∞ of middle current sheet [T] | N/A | 1.684 | 0.623 | 1.153 | 1.684 | 2.214 | 2.744 |
| B∞ of downstream sheet [T] | 2.5 | 2.5 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 |

To satisfy constraints on the field magnitude B(z) and slope dB/dz at z =−0.5 m and z = L, one can adjust the energization level of coils. Only coil currents—not coil dimensions—are variables, so that solving for variables requires no iteration. Furthermore, for the designs of Fig. 1 only the interior coil is used to satisfy the requirement that B(L) = Bmin. The upstream coil generates exactly 15 T without help for the downstream coils, which contribute only a few hundredth of a tesla at z = −0.5 m. The downstream current sheet is energized so as to generate exactly Bmin deep in the interior of a coil of infinite length.



Fig. 1. On-axis field profiles generated by equations described in text above.

The field near z = 6 m is higher than desired by as much as 0.17 T (0.5 T for the design with only one downstream coil). The field profiles of Figure 7 reduce this field deviation by means of an additional coil and therefore a second adjustable current. The solution of two simultaneous linear equations suffices to determine the currents in the interior two coils that satisfy the constraints B(z) = Bmin at z = 5 m and at z = 6 m. The latter constraint is more robust than requiring dB/dz = 0 at z = 5 m.



Fig. 2. On-axis field profiles generated by magnets with four current sheets. Currents i1 and i4 are fixed; i2 and i3 are adjustable. B(z) = Bmin at z = 5 m; also B(6m) = Bmin, a more robust constraint than dB/dz = 0 at z = 5m.

All the designs employ four current sheets with consecutive radii of [1.64, 1.00, 0.75, 0.60] m; coil ends are at [−2.35, 2.50, 5.29, 6.28] m upstream and [1.34, 5.17, 6.18, ∞] m downstream. To make the total field 15 T at z = −0.5m), the upstream coil generates 14.97 T. To make the total field approximately Bmin throughout the range 5 m to ~9 m, the downstream coil carries 94% the current that it would need if unassisted by the coils upstream.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Design Number | #1 | #2 | #3 | #4 | #5 |
| Desired field @ z = L = 5 m [T] | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 |
| B∞ of current sheet #2 [T] | 0.918 | 1.536 | 2.153 | 2.770 | 3.387 |
| B∞ of current sheet #3 [T] | 1.226 | 1.802 | 2.377 | 2.953 | 3.528 |

The on-axis field *B* at location *z* generated by a current sheet of radius *a*, upstream end *b*1, downstream end *b*2, and carrying *i*' amperes per meter is the function

*B*(*z*) = ½μ0 *i*' (*b*−*z*) [*a*2+(*b*−*z*2)]−½] teslas

evaluated at *b* = *b*2 minus its evaluation at *b* = *b*1.

Define a function *B*(0)(*x*) ≡ *x*(*a*2+*x*2)−½, where *x* ≡ (*b*−*z*). Similarly, define functions *B*(n)(*x*) ≡ (1/n!) dn*B*(0)/d*x*n. Then the Taylor series expansion of the field an increment ∆ from *z* is the function

½μ0 *i*' [*B*(0)(*x*) + *B*(1)(*x*) ∆ + *B*(2)(*x*) ∆2 + *B*(3)(*x*) ∆3 + *B*(4)(*x*) ∆4 + *B*(5)(*x*) ∆5 + . . .],

evaluated at *x*1 ≡ (*b*1−*z*) and subtracted from its evaluation at *x*2 ≡ (*b*2−*z*).

Define *r* = (*a*2+*x*2)½. The first five terms in the series are:

*B*(1)(*x*) = *a*2 / *r*3 ,

*B*(2)(*x*) = −(3/2) *a*2 *x* / *r*5 ,

*B*(3)(*x*) = −(1/2) (*a*2 – 4 *x*2) / *r*7 ,

*B*(4)(*x*) = (5/8) *a*2 *x* (3 *a*2 – 4 *x*2) / *r*9 ,

*B*(5)(*x*) = (3/8) *a*2 (*a*4 – 12 *a*2 *x*2 + 8 *x*4) / *r*11 .

Figure 3 plots these functions with *a* = 1. To devote most of the graph to values of *x* less than unity, values greater than unity are transformed as *x* = (2–*u*)–1, where *u* is the abscissa. The right-hand border of the graph, *u* = 1.8, transforms to *x* = 5.

The figure also plots the power-series coefficients for the function *B*(0)(*x*) = tanh (*x*), which behaves much like (1+*x*2)–½. Its coefficients are:

*B*(1)(*x*) = 1 / cosh2(*x*) ,

*B*(2)(*x*) = −sinh(*x*) / cosh3(*x*) ,

*B*(3)(*x*) = –(1/3) [1 – 2 sinh2(*x*)] / cosh4(*x*) ,

*B*(4)(*x*) = (1/3) [sinh(*x*)] [2 – sinh2(*x*)] / cosh5(*x*) ,

*B*(5)(*x*) = (2/15) [2 – 11 sinh2(*x*) + 2 sinh4(*x*)] / cosh6(*x*) .



Fig. 3. Field and power-series coefficients of current sheet and function tanh(*x*), which behaves much like (1+*x*2)−½.