

**Boom! From Light Comes Matter**



## Positron Production by Laser Light

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*Seminar at U. Maryland*

## Sonoluminescence

In 1850, the Navier-Stokes equation was the “theory of everything”, but it doesn’t predict sonoluminescence. [Erber]

[Sonoluminescence is what makes nitroglycerine explode.]

- Preparata (1998): QED theory of water vapor predicts emission of light when water vapor condenses at density near  $1 \text{ g/cm}^3$ .
- Schwinger (1992): a bubble is an electromagnetic cavity; an imploding bubble will radiate away the changing, trapped zero-point energy.
- Liberati (1998): Imploding bubble  $\Rightarrow$  rapidly changing index  $\Rightarrow$  associated radiation.

This relates to an earlier idea:

- Yablonovitch (1989): An accelerating boundary across which the index of refraction changes is a possible realization of the Hawking-Unruh effect, leading to conversion of QED vacuum fluctuations into real photons.

# The Hawking-Unruh Effect

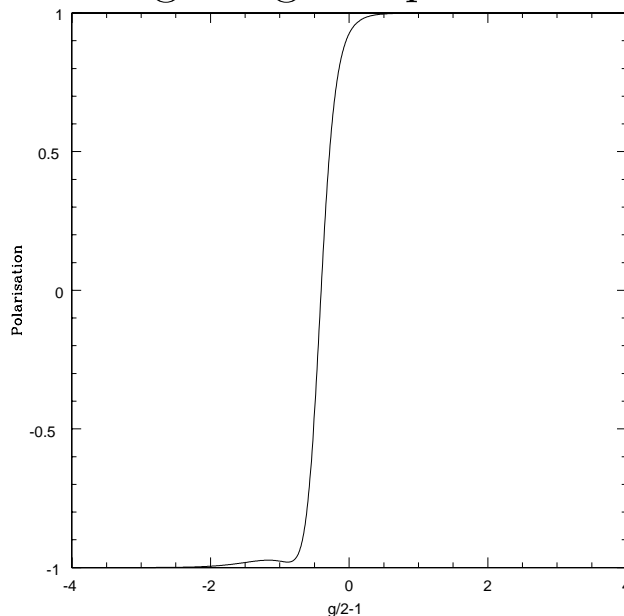
**Hawking** (1974): An observer outside a black hole experiences a bath of thermal radiation of temperature  $T = \frac{\hbar g}{2\pi c k}$ ,

where  $g$  is the local acceleration due to gravity.

**Unruh** (1976): According to the equivalence principle an accelerated observer in a gravity-free region should also experience a thermal bath with:  $T = \frac{\hbar a}{2\pi c k}$ ,

where  $a$  is the acceleration of the observer as measured in his instantaneous rest frame.

Bell (1983), Leinaas (1998), Unruh (1998): Incomplete polarization of electrons in a storage ring is explained in detail by Hawking-Unruh excitation.



## Unruh Radiation (?)

Suppose the observer is an electron, accelerated by a field  $E$ .

Thomson scattering off photons in the apparent thermal bath implies a radiation rate:

$$\frac{dU_{\text{Unruh}}}{dt} = \text{thermal energy flux} \times \text{Thomson cross section} = \frac{\hbar r_e^2 a^4}{90\pi c^6}.$$

[Stefan-Boltzman: flux  $\propto T^4$ , Unruh:  $T^4 \propto a^4$ .]

This equals the Larmor radiation rate,  $dU/dt = 2e^2 a^2/3c^3$ , when the acceleration  $a = eE/m$  is about  $10^{31} g$ , *i.e.*, when

$$E = \sqrt{\frac{60\pi}{\alpha}} \left( \frac{m^2 c^3}{e\hbar} \right) \approx 3 \times 10^{18} \text{ V/cm.}$$

Can we do the experiment?

## Strong-Field QED

For high acceleration, need strong electromagnetic field.

Strongest macroscopic electromagnetic fields are in lasers.

Tabletop teraWatt lasers can be focused to  $> 10^{19}$  W/cm<sup>2</sup>.

$\Rightarrow$  Electric fields  $> 100$  GeV/cm.

[Photon number density  $> 10^{27}$ /cm<sup>3</sup>.]

(Nonperturbative) physics described by two dimensionless measures of field strength:

$$\eta = \frac{e\sqrt{\langle A_\mu A^\mu \rangle}}{mc^2} = \frac{eE_{\text{rms}}}{m\omega_0 c} = \frac{eE_{\text{rms}}\lambda_0}{mc^2},$$

governs the importance of multiple photons in the initial state, and characterizes the “mass shift”:  $\bar{m} = m\sqrt{1 + \eta^2}$ . [Kibble, 1996]

$$\Upsilon = \frac{\sqrt{\langle (F^{\mu\nu} p_\nu)^2 \rangle}}{mc^2 E_{\text{crit}}} = \frac{2p_0 E_{\text{rms}}}{mc^2 E_{\text{crit}}} = \frac{2p_0 \lambda_C}{mc^2 \lambda_0} \eta,$$

governs the importance of “spontaneous” pair creation, where  $E_{\text{crit}} = m^2 c^3 / e\hbar = mc^2 / e\lambda_C = 1.3 \times 10^{16}$  V/cm.

## The QED Critical Field Strength

- O. Klein (Z. Phys. **53**, 157 (1929)) noted that the reflection coefficient exceeds unity when Dirac electrons hit a steep barrier (Klein's paradox).
- F. Sauter (Z. Phys. **69**, 742 (1931)) deduced that the paradox arises only in electric fields exceeding the critical strength:

$$E_{\text{crit}} = \frac{m^2 c^3}{e \hbar} = 1.32 \times 10^{16} \text{ Volts/cm.}$$

- At the critical field, the energy gain across a Compton wavelength is the electron rest energy:

$$e E_{\text{crit}} \cdot \frac{\hbar}{mc} = mc^2.$$

- At the critical field the vacuum 'sparks' into  $e^+e^-$  pairs (Heisenberg and Euler, Z. Phys. **98**, 718 (1936)).

Dimensionless measure of criticality:  $\Upsilon = \frac{E}{E_{\text{crit}}}$ .

## Where to Find Critical Fields

- The magnetic field at the surface of a neutron star approaches the critical field  $B_{\text{crit}} = 4.4 \times 10^{13}$  Gauss.
- During heavy-ion collisions where  $Z_{\text{total}} = 2Z > 1/\alpha$ , the critical field can be exceeded and  $e^+e^-$  production is expected.

$$E_{\text{max}} \approx \frac{2Ze}{\lambda_C^2} = 2Z\alpha E_{\text{crit}}.$$

The line spectrum observed in positron production in heavy-ion collisions (Darmstadt) is not understood.

- Pomeranchuk (1939): The earth's magnetic field appears to be critical strength as seen by a cosmic-ray electron with  $10^{19}$  eV.
- The electric field of a bunch at a future linear collider approaches the critical field in the frame of the oncoming bunch.

## Critical Fields in $e$ -Laser Collisions

- The electric field due to a laser as seen in the rest frame of a high-energy electron is

$$E^* = \gamma(1 + \beta)E_{\text{lab}} \approx 2\gamma E_{\text{lab}},$$

so

$$\Upsilon = \frac{E^*}{E_{\text{crit}}} = \frac{2\gamma E_{\text{lab}}}{E_{\text{crit}}} = \frac{\sqrt{(F_{\mu\nu}p^\nu)^2}}{mc^2 E_{\text{crit}}}.$$

- The critical field is achieved with a laser beam of intensity

$$I = \frac{E_{\text{lab}}^2}{377\Omega} = \frac{E_{\text{crit}}^2}{4\gamma^2 \cdot 377}.$$

Thus for 46-GeV electrons ( $\gamma = 9 \times 10^4$ ) we can achieve  $E_{\text{crit}}$  with a focused laser intensity of  $1.4 \times 10^{19}$  Watts/cm<sup>2</sup> ( $\Rightarrow E_{\text{lab}} = 7 \times 10^{10}$  Volts/cm).

- Such intensities are now attainable in table-top teraWatt ( $\text{T}^3$ ) lasers in which a Joule of energy is compressed into one picosecond and focused into a few square microns.
- At these intensities the photon density is  $\sim 10^{27}/\text{cm}^3$ , and the radiation length of this ‘photon solid’ is  $\sim \lambda_0/\alpha \approx 100 \mu\text{m}$ .



## Another Aspect of Electrons in Strong Wave Fields

Simplest to consider a circularly polarized laser beam incident on an electron at rest. Laser field is  $E$ , frequency is  $\omega_0$ .

Classical response of electron is transverse motion in a circle with angular velocity  $\omega_0$  and velocity described by

$$\frac{v_{\perp}}{c} = \frac{eE}{m\omega_0 c} \equiv \eta.$$

[ $\eta = e\sqrt{\langle A_{\mu}A^{\mu} \rangle}/mc^2$  where  $A_{\mu}$  is the vector potential.]

The accelerating electron emits multipole radiation.

Rate $_n \propto \eta^{2n} \propto I^n$  for  $n$ th-order multipole ( $\eta \lesssim 1$ ).

$n$ th order  $\Leftrightarrow$  absorption of  $n$  photons before emitting a single higher-energy photon  $\Leftrightarrow$  nonlinear Compton scattering.

$\Upsilon$  and  $\eta$  are related by

$$\Upsilon = \frac{2\gamma E_{\text{lab}}}{E_{\text{crit}}} = \frac{2\gamma\hbar\omega_0}{mc^2}\eta \approx \eta \text{ in our experiment.}$$

$\Rightarrow$  Two classes of strong-field effects to be untangled.

## The Mass Shift Effect

An electron propagating in a (periodic) wave field of strength  $\eta = eE/m\omega_0c$  takes on an effective mass

$$\bar{m} = m\sqrt{1 + \eta^2}.$$

Classical view: the transverse oscillations of the electron are relativistic, so it becomes ‘heavier’.

As a result the kinematic limits in nonlinear Compton scattering (and threshold for pair creation) are shifted.

Pedagogic paradox: An electron with 4-momentum  $p_\mu$  in the absence of the field takes on quasimomentum  $q_\mu$  once in a field of 4-momentum  $k_\mu$ :

$$q_\mu = p_\mu + \frac{\eta^2 m^2}{2k \cdot p} k_\mu, \quad q^2 = \bar{m}^2.$$

In our experiment  $\eta^2 m^2 / 2k \cdot p$  takes on a fractional value.

## Summary of Motivation and Goals

- The Higgs mechanism implies that elementary particles have important interactions with strong background fields.
- Only with electromagnetism can intense, controllable, macroscopic fields be created in the laboratory.
- Explore the validity of QED for electromagnetic field strengths in excess of the ‘critical field strength’

$$E_{\text{crit}} = m^2 c^3 / e \hbar = 1.6 \times 10^{16} \text{ V/cm.}$$

- Explore QED in the realm where multiphoton interactions dominate, *i.e.*, when  $\eta \equiv eE/m\omega_0 c \approx 1$ .

*Proposal for a*

**STUDY OF QED AT CRITICAL FIELD STRENGTH**

**IN INTENSE LASER-HIGH ENERGY ELECTRON COLLISIONS**

**AT THE STANFORD LINEAR ACCELERATOR**

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*Full approval on September 30, 1992*

# E-144 Home Page on World Wide Web

<http://www.slac.stanford.edu/exp/e144/e144.html>

# E144

cranked beyond the limit

# E-144 Physics Program

## 1. Compton Polarimetry

- Both the E-144 laser and electron beams are polarized.
- Compton polarimetry provides a basic check of the E-144 apparatus, as well as a confirmation of the SLC beam polarization.

## 2. Nonlinear Compton Scattering: $e + n\omega_0 \rightarrow e' + \omega$

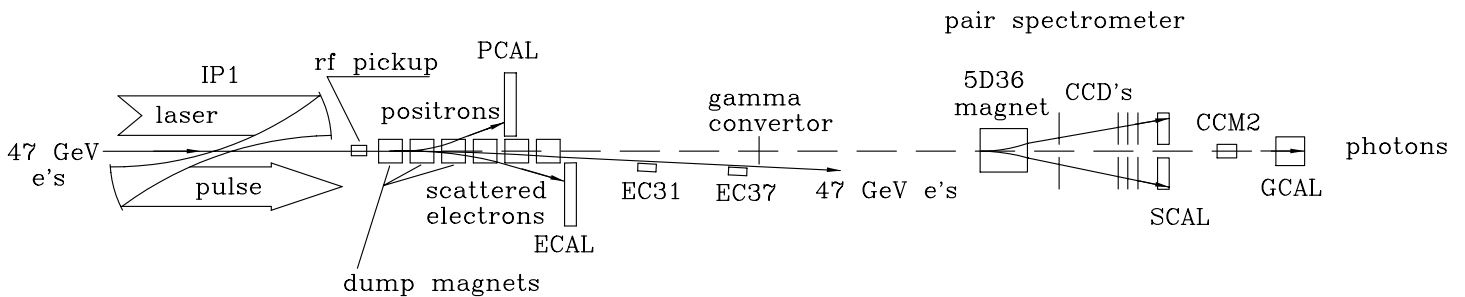
- Semiclassical theory  $\Rightarrow$  data will diagnose laser intensity.
- Provides high-energy-photon beam for light-by-light scattering.

## 3. Multiphoton Breit-Wheeler Process: $\omega + n\omega_0 \rightarrow e^+e^-$

- Might show anomalous structure in  $e^+e^-$  invariant mass when  $E > E_{\text{crit}}$ .

# Experimental Ingredients

- Low-emittance electron beam.
- Terawatt laser.
- Synchronization of  $e$  and laser beams to 1 psec in time, and a few  $\mu\text{m}$  in space.
- Silicon calorimeters for ‘coarse-grain’ detection of  $e^-$ ,  $e^+$  and  $\gamma$ 's.
- CCD pair spectrometer for ‘fine-grain’ measurements.
- Data-acquisition system based on PC's interconnected via a local ethernet.



# **E-144 is at the End of the FFTB**



# TeraWatt Laser Via Chirped-Pulse Amplification

1 Joule in 1 ps  $\Rightarrow$   $10^{12}$  Watt.

Diffraction limited spot area  $\approx \lambda^2(f/D)^2 \approx 10 \mu\text{m}^2$ .

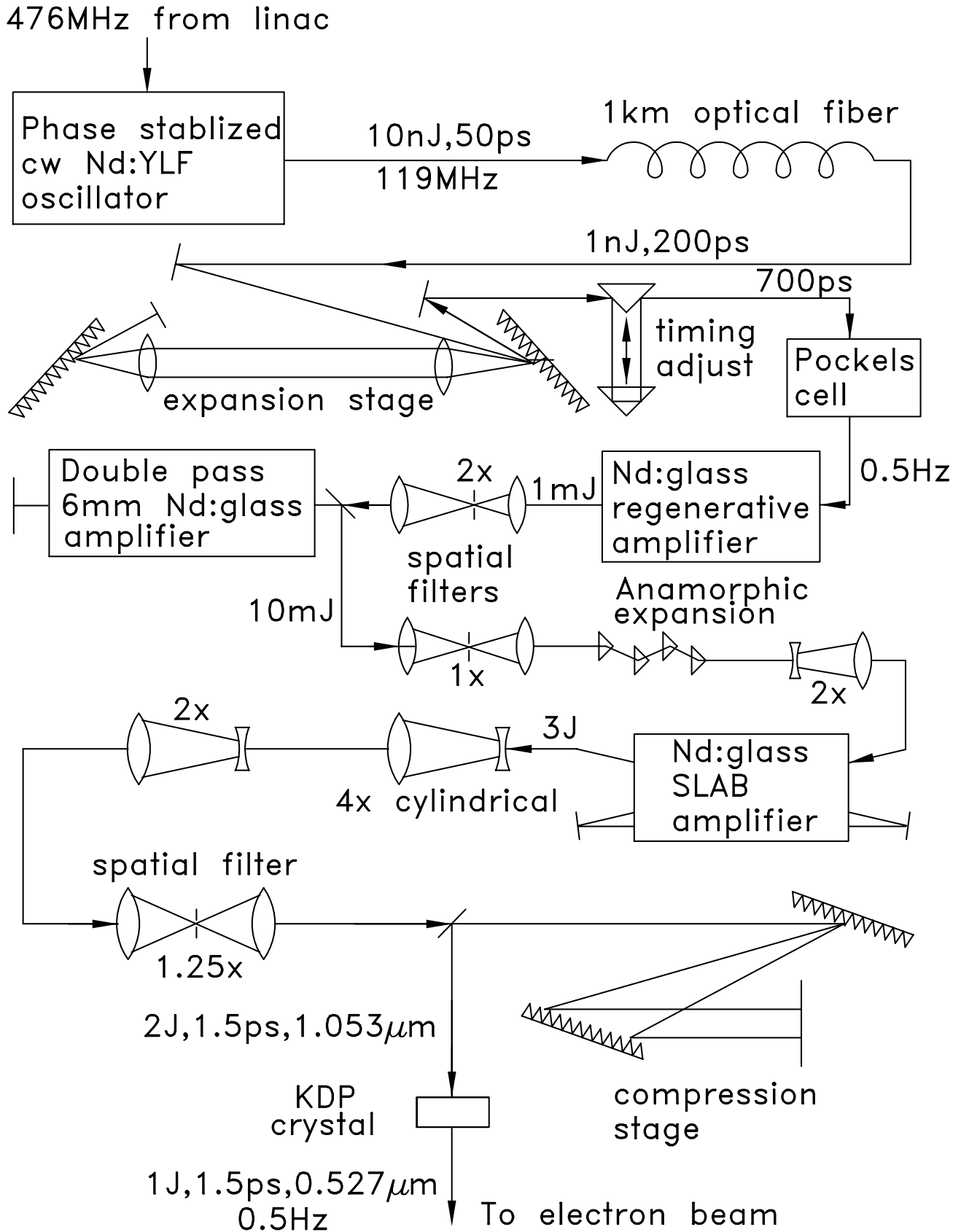
$\Rightarrow I \approx 10^{19}$  W/cm<sup>2</sup>.

High power pulses can damage optics!

$\Rightarrow$  stretch pulse, then amplify and compress.

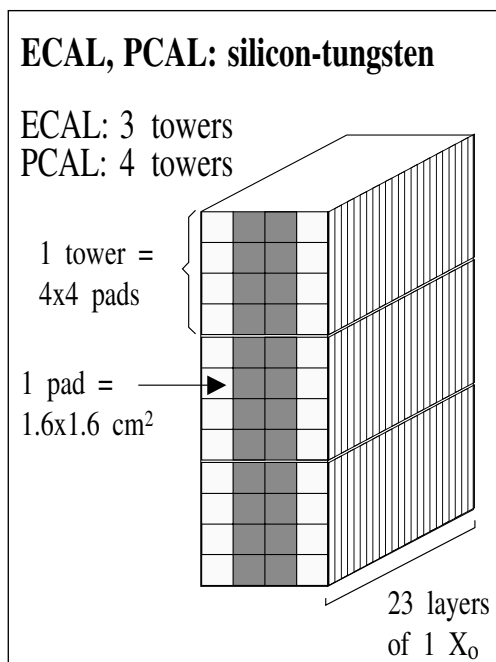
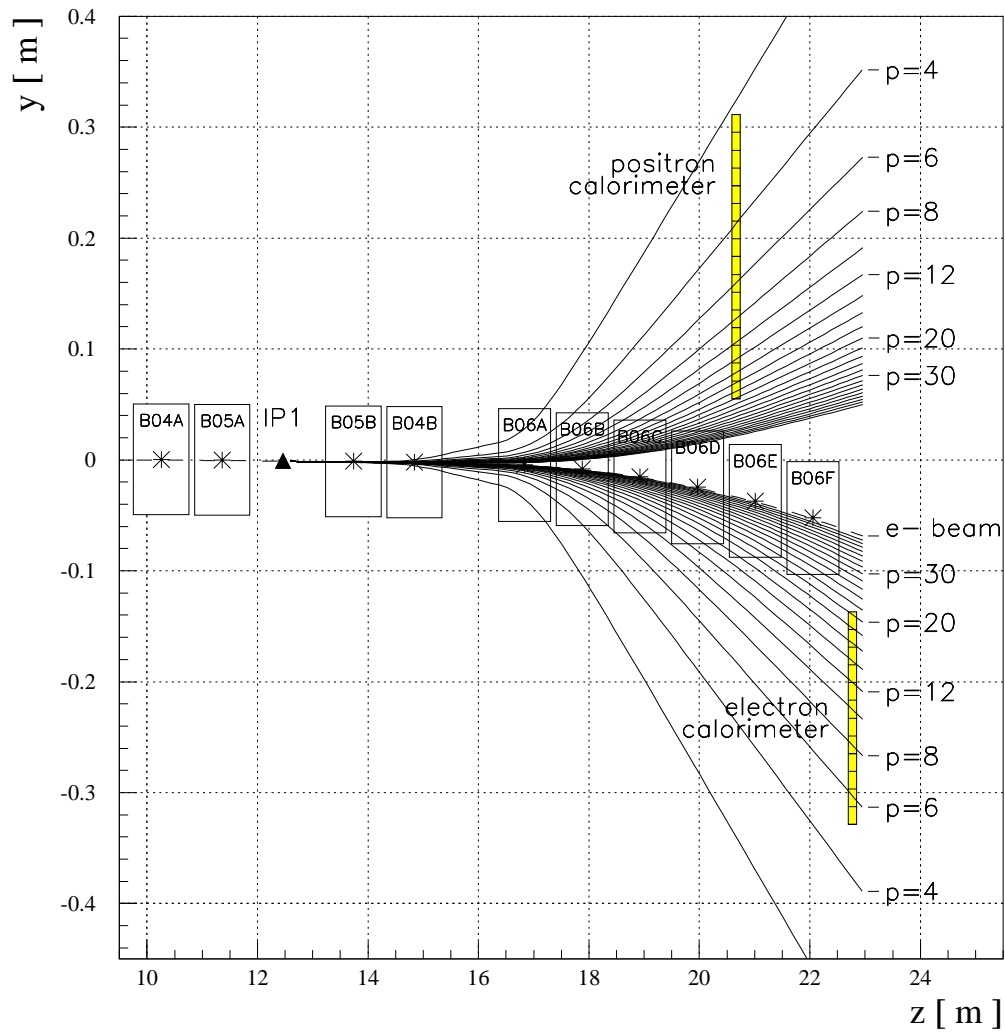
[D. Strickland and G. Mourou, Opt. Comm. **55**, 447 (1985).]

# E-144 Laser Schematic



synchronization block diagram

# Electron & Positron Spectrometers

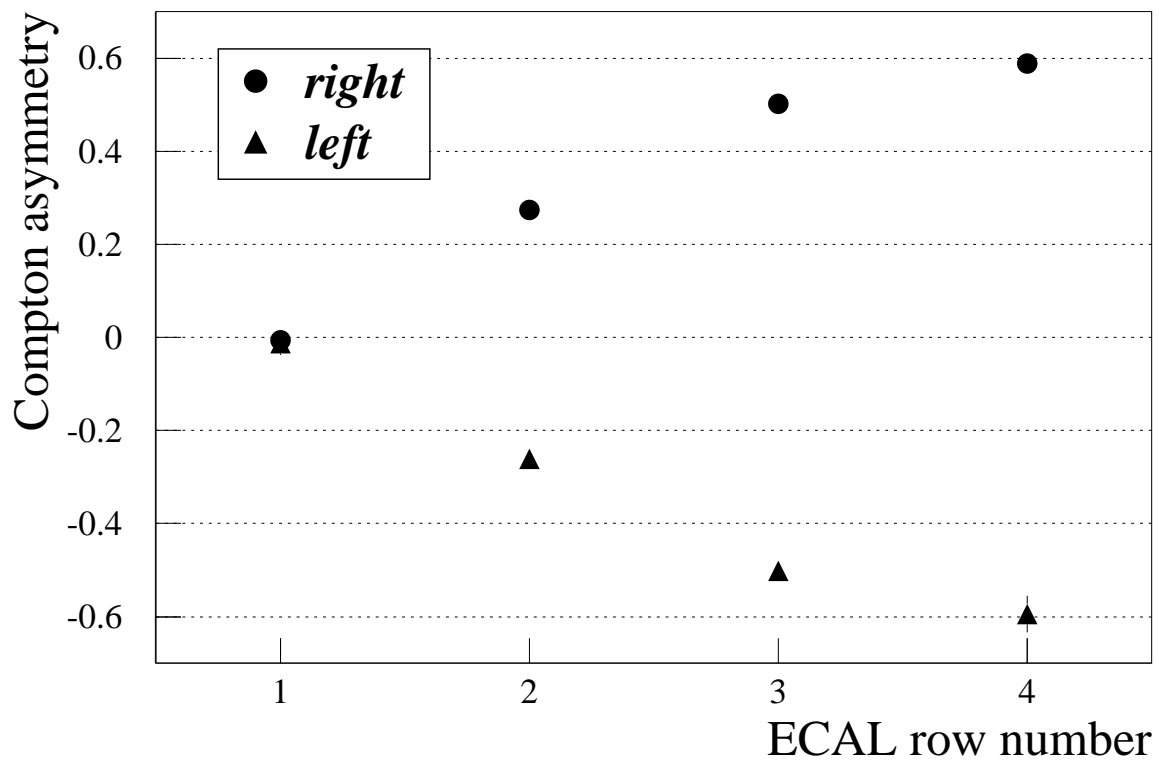


# Measurement of $e$ -Beam Polarization (May '94)

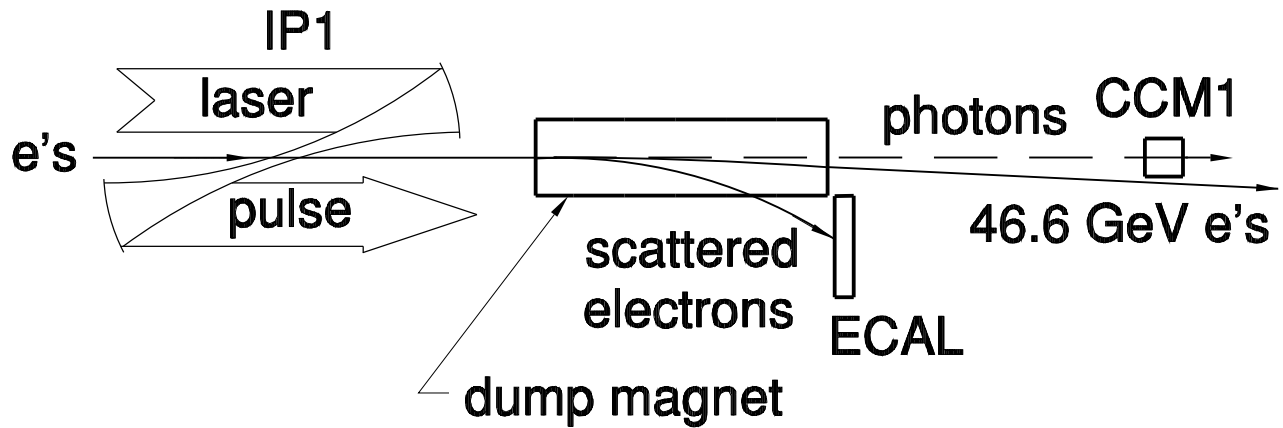
Fit to measured polarization asymmetry in 4 energy bins yields

$$P_e P_{\text{laser}} = 0.81 \pm 0.01.$$

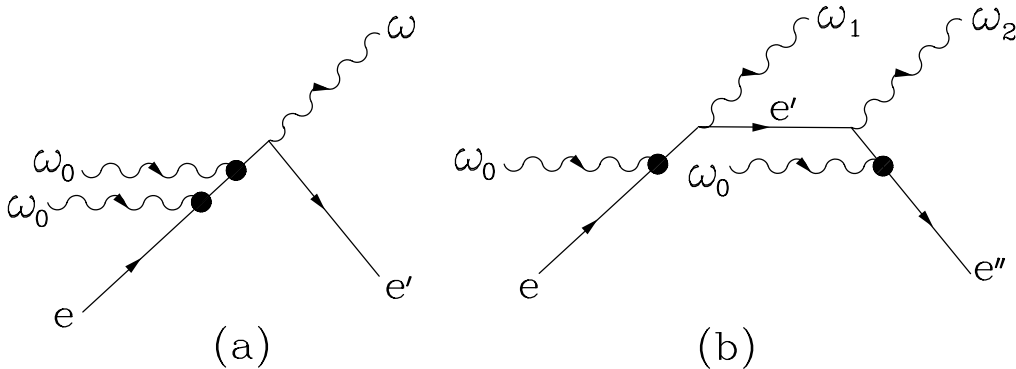
Laser polarization  $> 0.96 \Rightarrow P_e = 0.81^{+0.04}_{-0.01}$ .



# Nonlinear Compton Scattering



$$(a) \quad e + n\omega_0 \rightarrow e' + \omega$$



(b) Background: multiple Compton scattering

Can distinguish process (a) from (b) by detecting scattered photon.

In first experiments, only the scattered electron was detected.

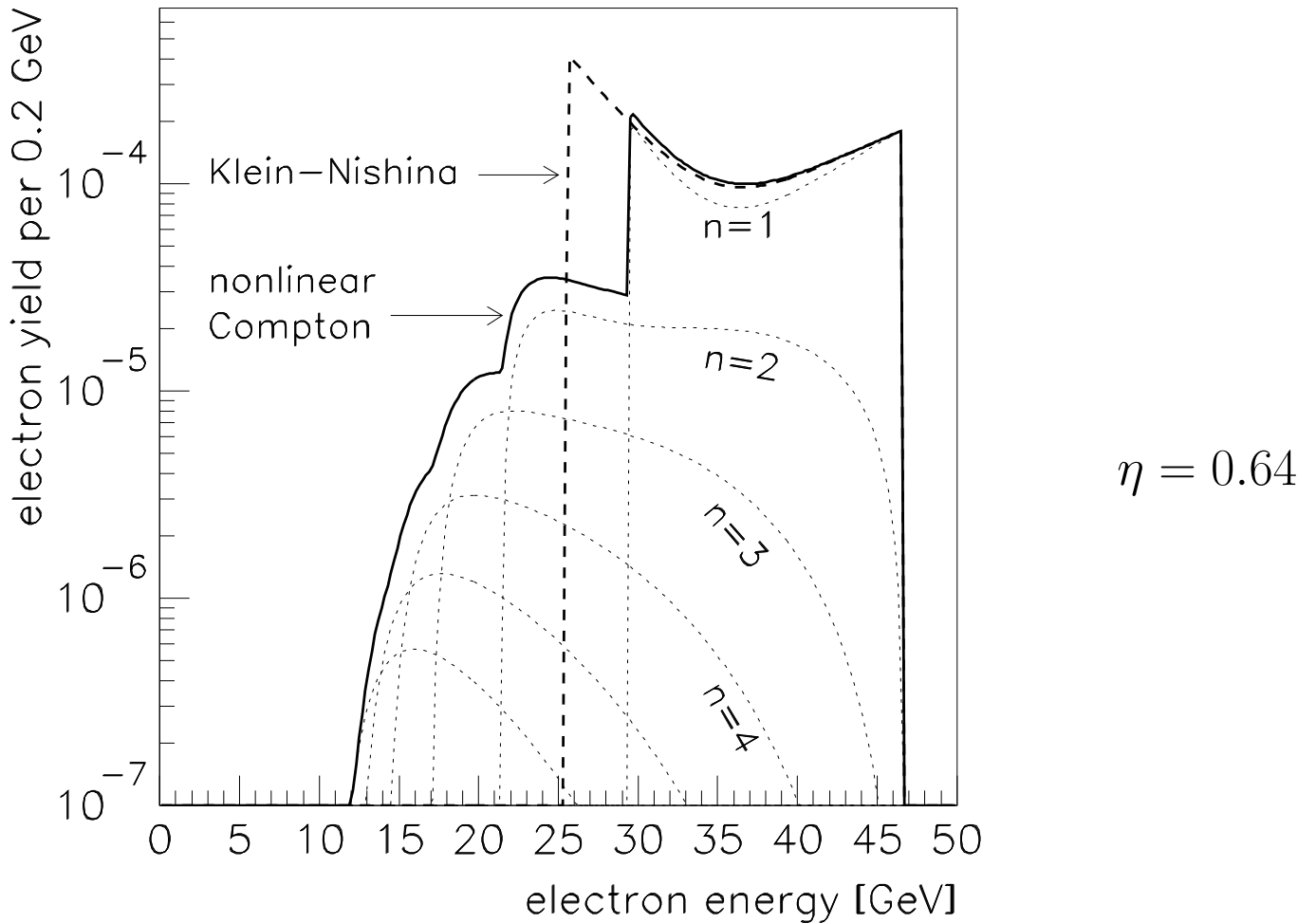
## Theoretical Predictions for $e + n\omega_0 \rightarrow e' + \omega$

Circular polarization (Nikishov *et al.*, JETP **20** (1965) 622).

$$\frac{d\text{Rate}_n}{dE_{e'}} = \frac{4\pi r_0^2 N_{\text{laser}} N_e}{xE_e} \times \left\{ \left( 2 + \frac{u^2}{1+u} \right) [J_{n-1}^2(z) + J_{n+1}^2(z) - 2J_n^2(z)] - \frac{4}{\eta^2} J_n^2(z) \right\},$$

$$x = \frac{4\omega_0 E_e}{m^2}, \quad u \approx \frac{E_e}{E_{e'}} - 1, \quad z = \frac{2\eta}{x} \sqrt{nux - u^2(1 + \eta^2)}.$$

Case of linear polarization is more intricate.

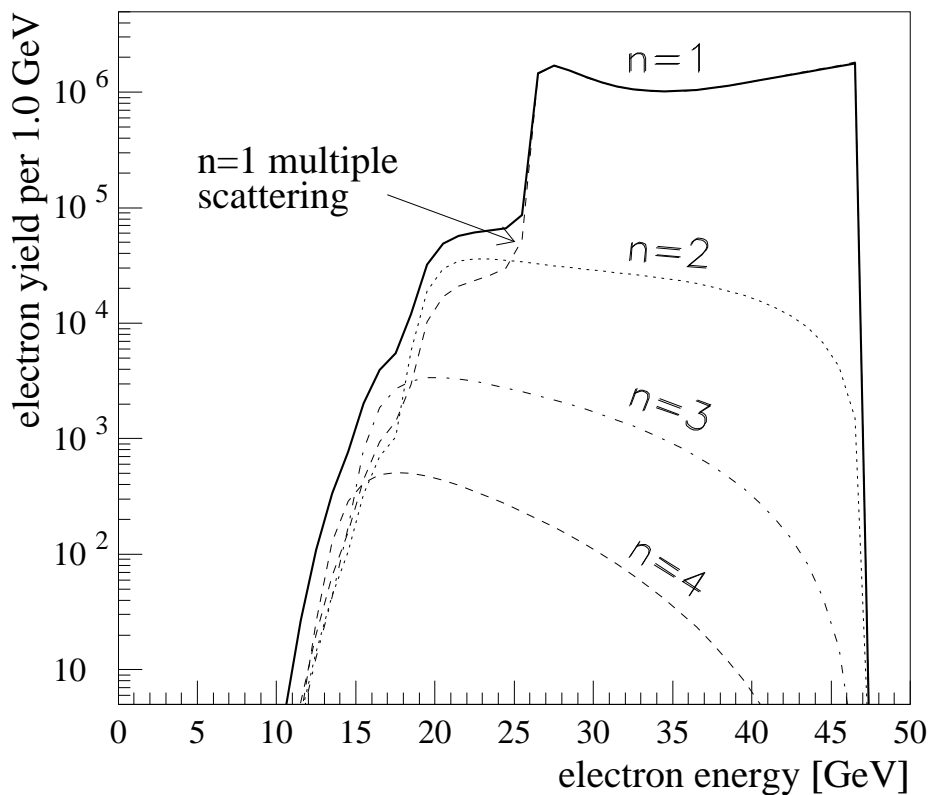
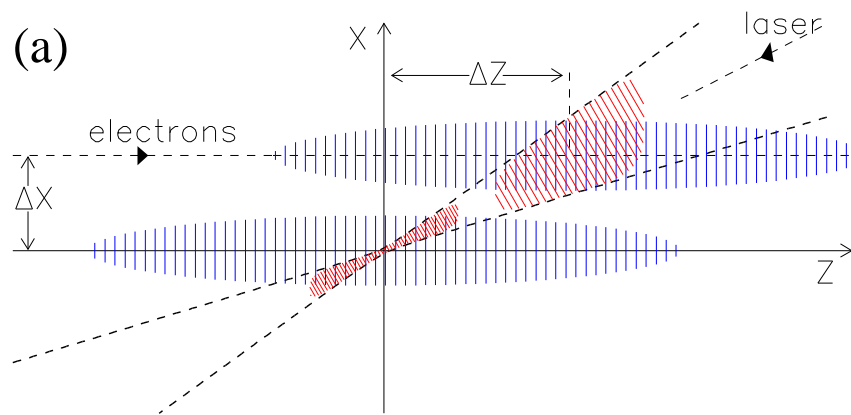


# Effect of Laser and Electron Beam Spots

Beams cross at  $17^\circ$ .

Laser pulse is  $\approx 2$  ps long, focussed to  $\approx 5\lambda_0$ .

Electron pulse is  $\approx 5$  ps long.



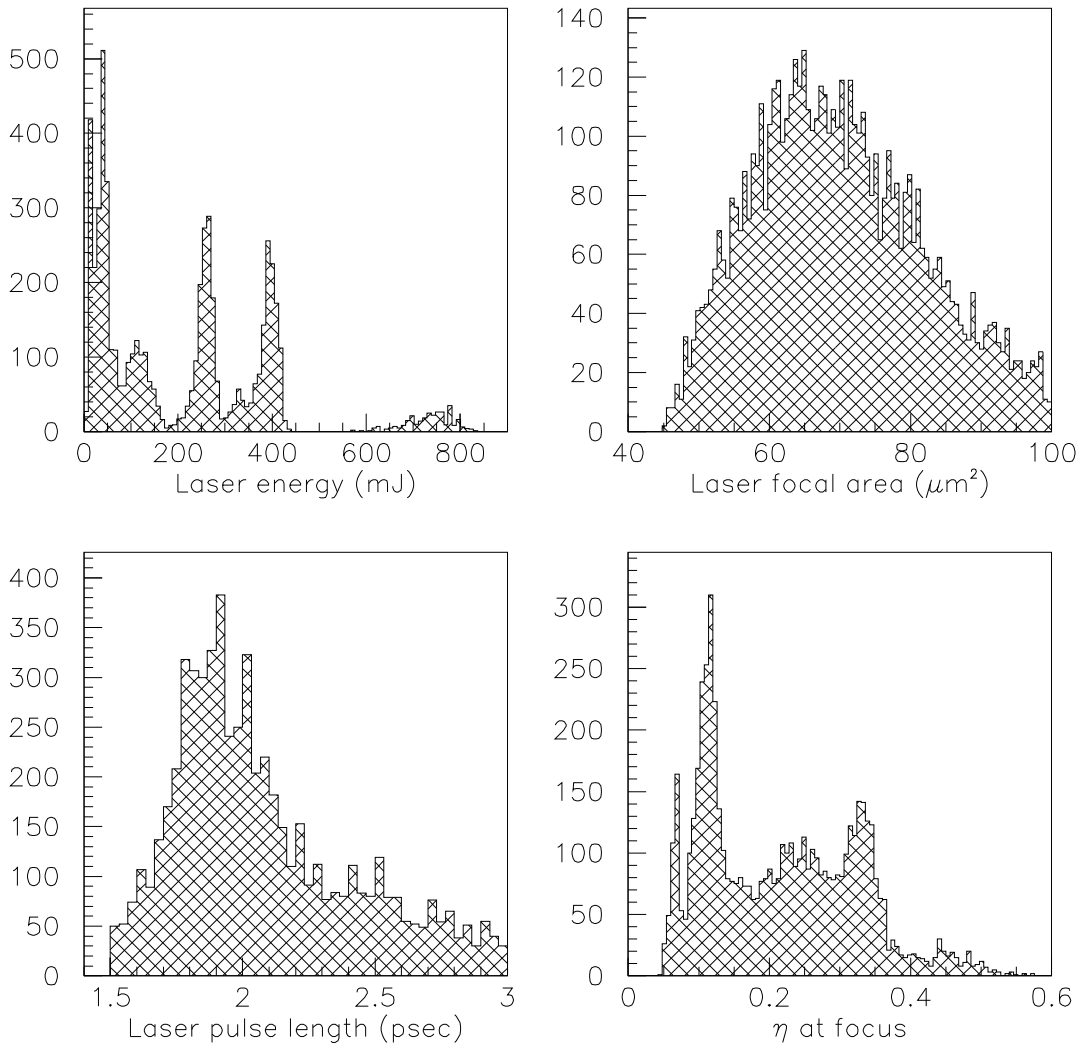
$$\eta_{\max} = 0.64$$



# Measurements of Nonlinear Compton Scattering

[C. Bula *et al.*, Phys. Rev. Lett. **76**, 3116 (1996)]

18,000 infrared laser shots ( $\omega_0 = 1.15$  eV):

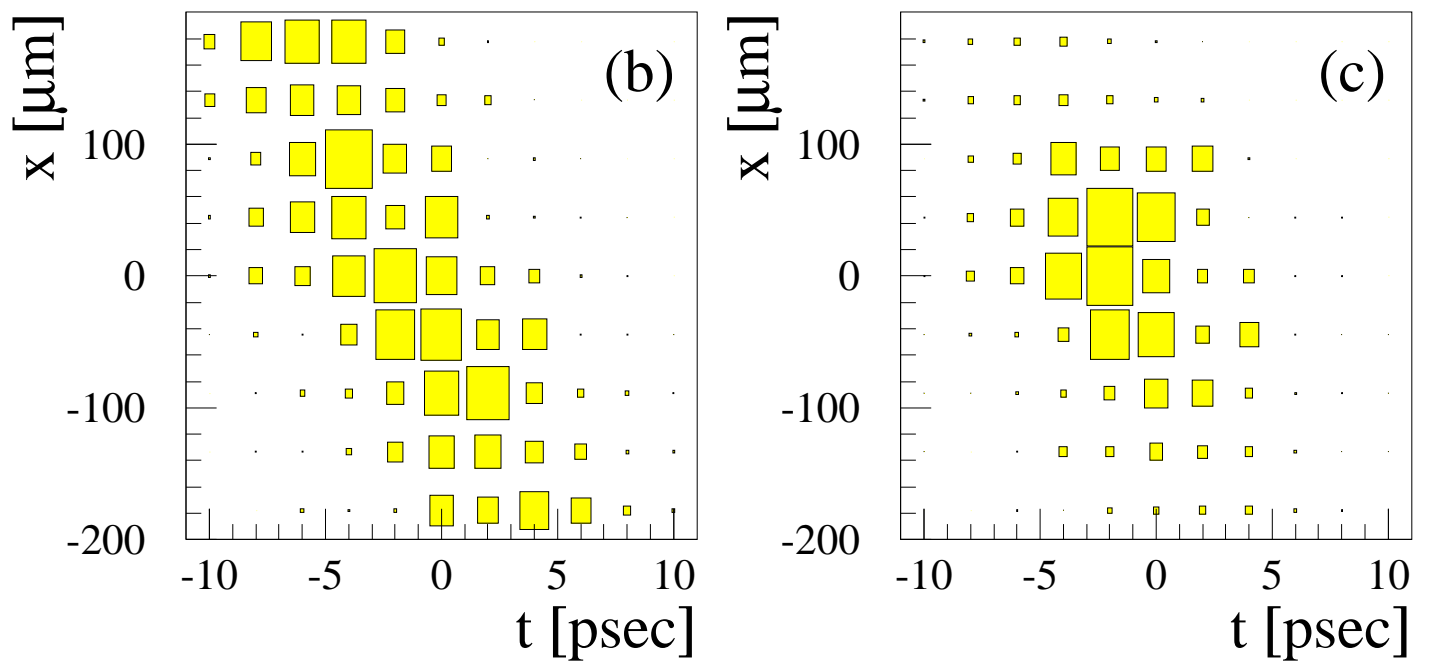


16,000 green laser shots ( $\omega_0 = 2.3$  eV).

## $x-t$ Scans

Vary  $x$  and  $t$  offsets between laser and electron beam.

Detect scattered electrons in (a)  $n = 1$  region and (b)  $n = 2$  region.



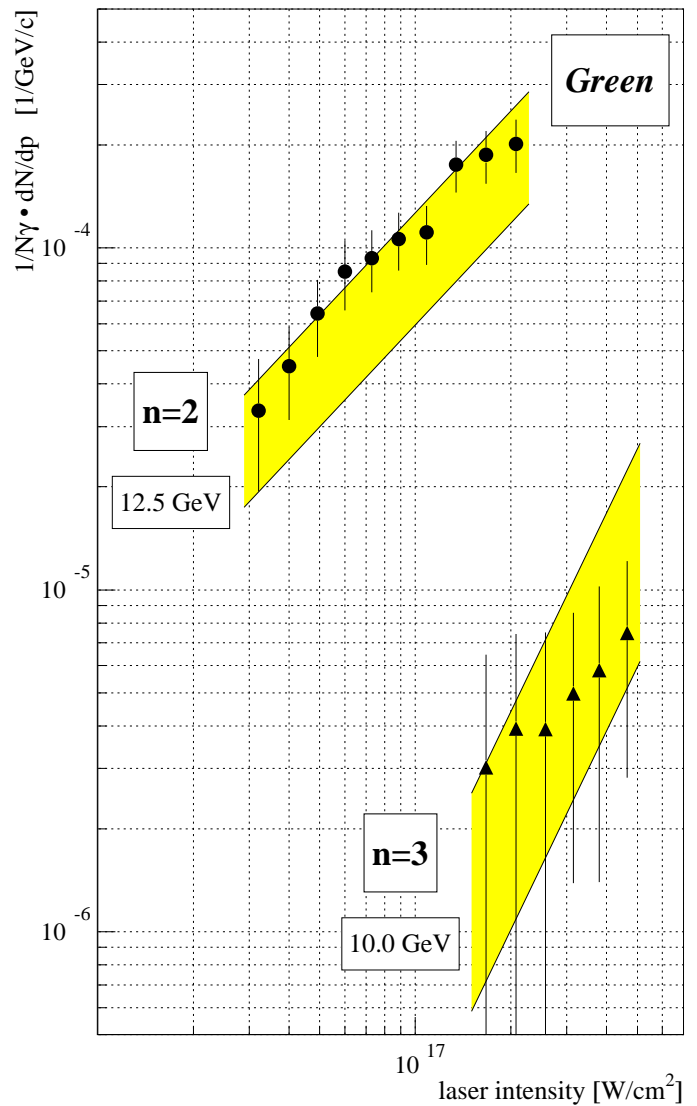
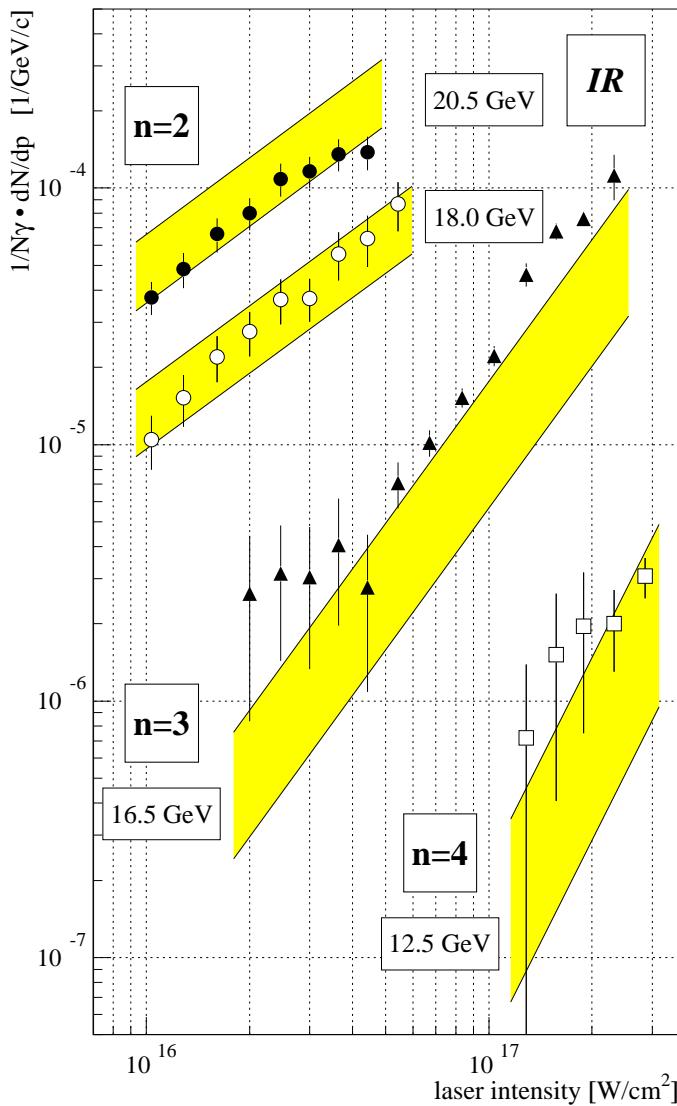
Nonlinear scattering confined to core of laser spot where intensity is the highest.

# Observed Rates *vs.* Laser Intensity

Cross-section not a useful concept for nonlinear scattering.

Plot differential rates normalized to total scattering rate

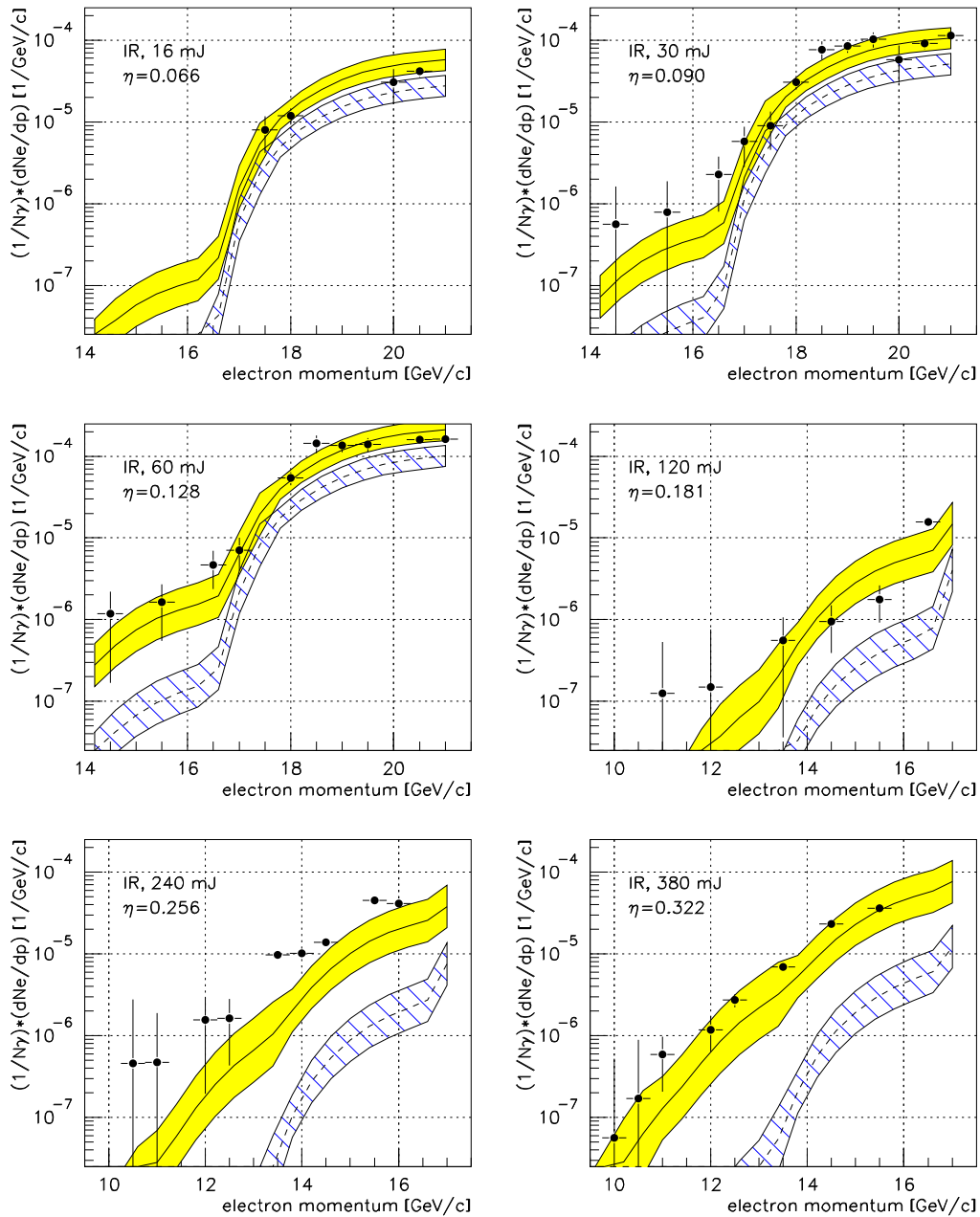
(= total rate of scattered photons).



Normalized to total scattered photon rate

$$\Rightarrow \text{Rate}(\text{order } n) \propto I^{n-1}.$$

# Observed Scattering Rates *vs.* Electron Energy



$n = 2$  edge at 17.6 GeV,  $n = 3$  edge at 13.5 GeV.

Shaded = full simulation.

Striped = multiple- $(n = 1)$  scattering only.

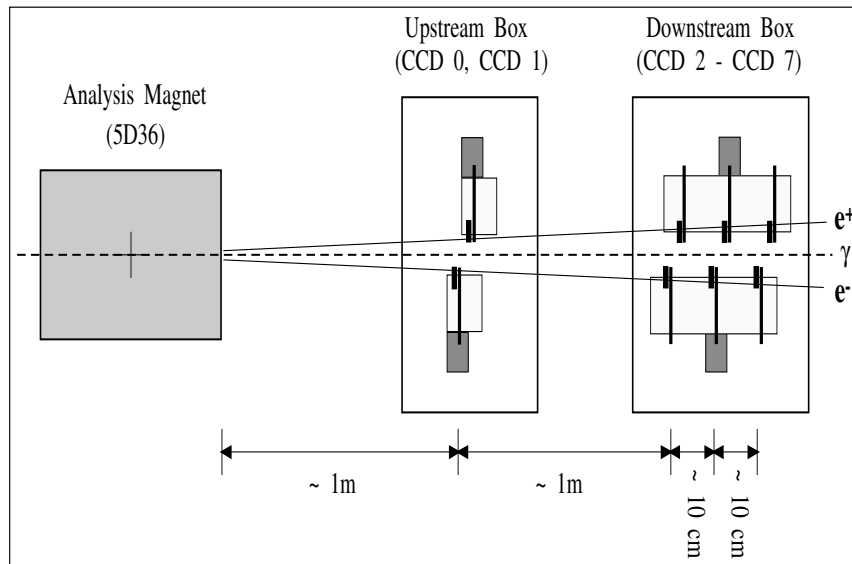
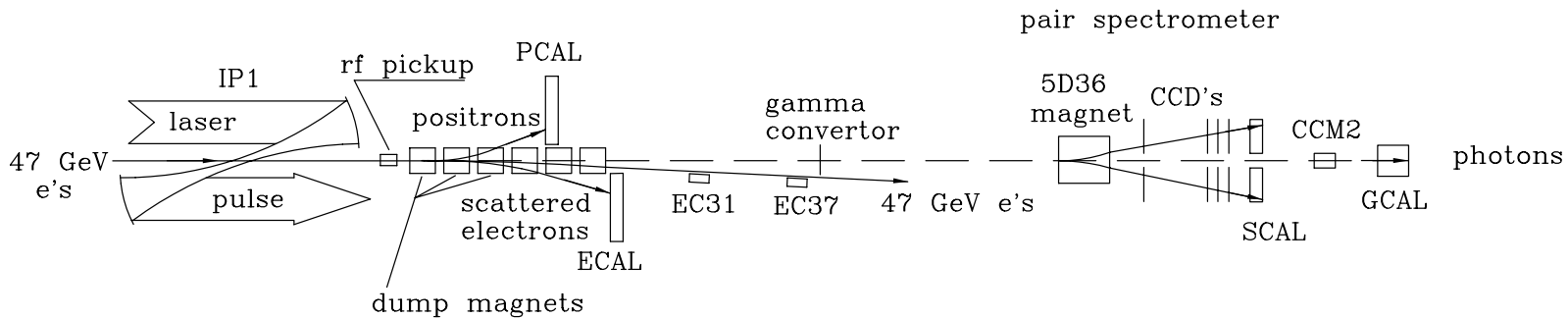
# Measurements of the Scattered Photon

Multiple Compton scatters produce no photons with energy above the maximum for a single scatter.

⇒ No backgrounds for  $n > 1$  nonlinear Compton Scattering

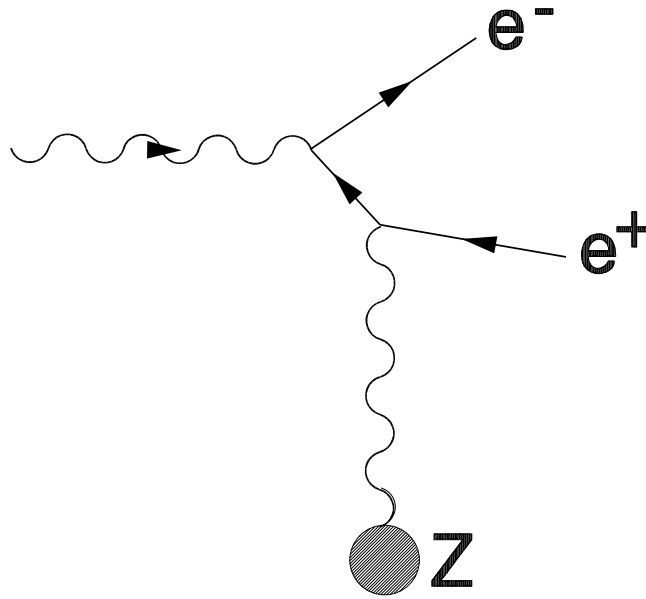
However, no simple spectrometer for high-energy photons:

We convert the photons to  $e^+e^-$  pairs and analyze the latter in a magnetic spectrometer with CCD detectors



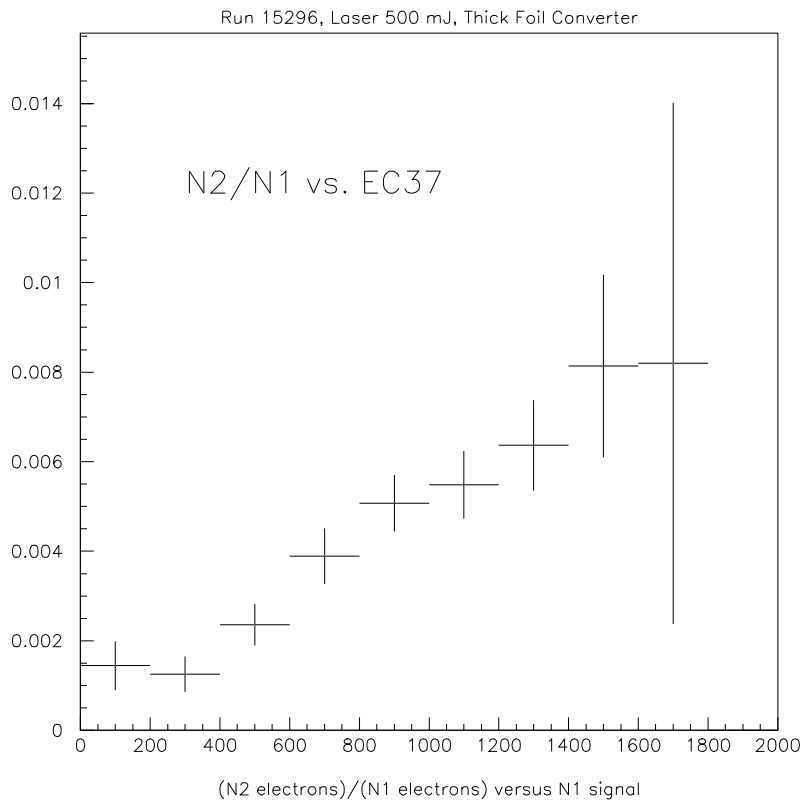
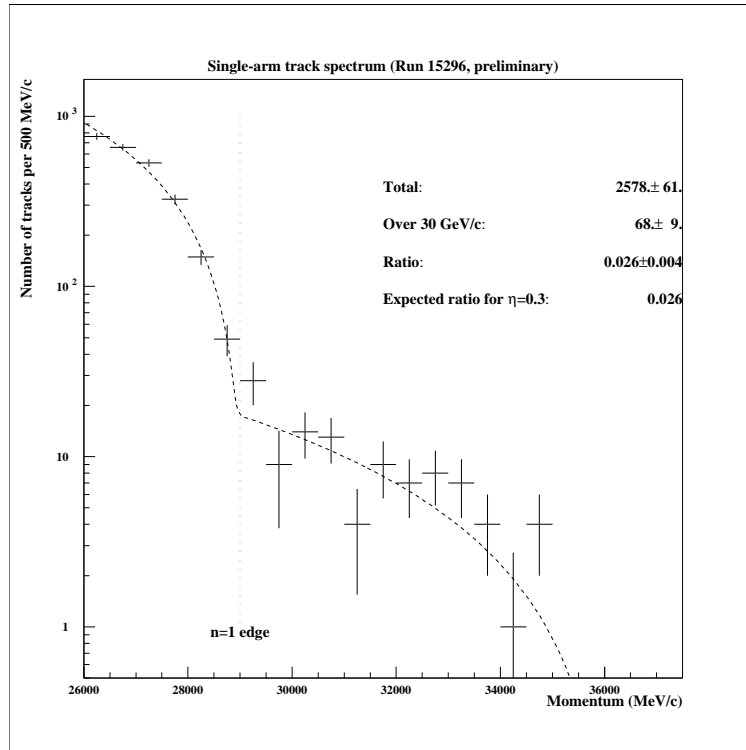
## Making Positrons the Old-Fashioned Way

Bethe-Heitler (1934): A real photon combines with a virtual photon from the field of a nucleus to create an  $e^+e^-$  pair.



Nuclear electric fields are strong but not critical; Bethe-Heitler pair creation is well described in perturbation theory involving a single virtual photon.

# $e^+$ and $e^-$ from Converted Compton Photons



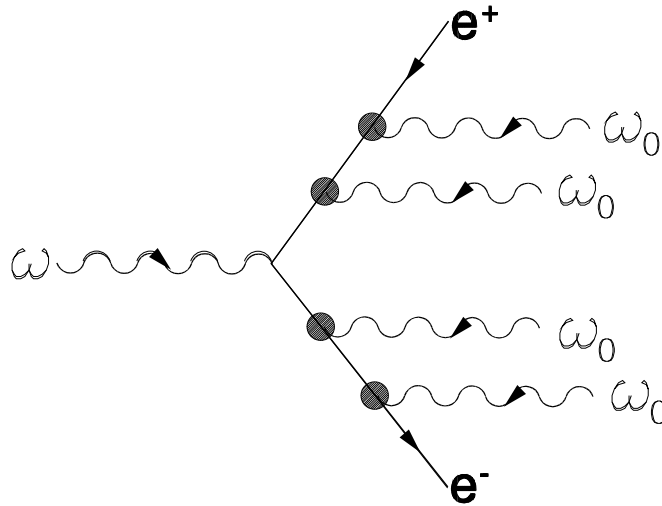
Threshold:  $\hbar\omega_1 \hbar\omega_2 = (mc^2)^2$

Cross section near threshold :  $\sigma_{\text{B-W}} \approx \pi r_e^2 \sqrt{1 - \frac{m^2 c^4}{\hbar\omega_1 \hbar\omega_2}}$ .



## Pair Creation by Light

Two step process:  $e + \omega_0 \rightarrow e' + \omega$ , then  $\omega + n\omega_0 \rightarrow e^+e^-$ .



Multiphoton pair creation is cross-channel process to nonlinear Compton scattering.

$\Rightarrow$  Similar theories [sums of Bessel functions whose arguments depend on  $\eta^2$ ].

$\Rightarrow$  Breit-Wheeler cross section in weak-field limit.

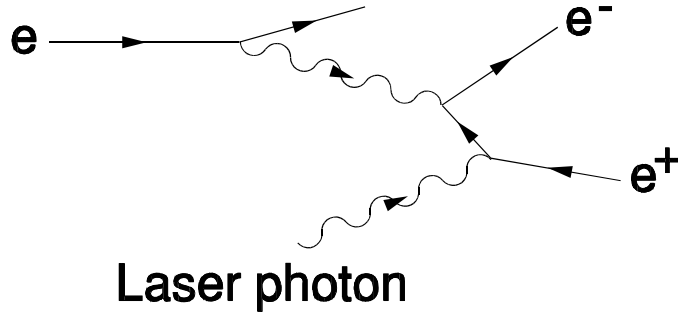
$\omega_{\max} \approx 29$  GeV for 46.6-GeV electrons + ( $n = 1$ ) green laser.

Then need at least  $n = 4$  laser photons to produce a pair.

$\Leftrightarrow$  Below threshold for 2-photon pair creation.

# Trident Production

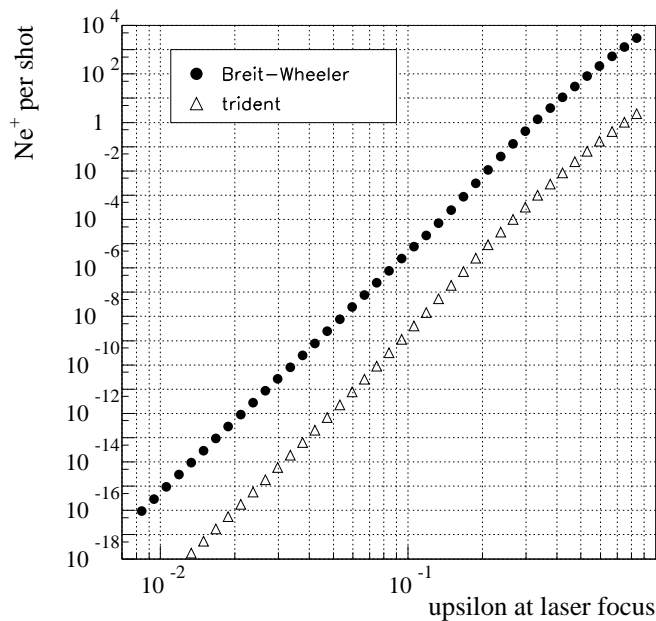
$$e + n\omega_0 \rightarrow e'e^+e^-$$



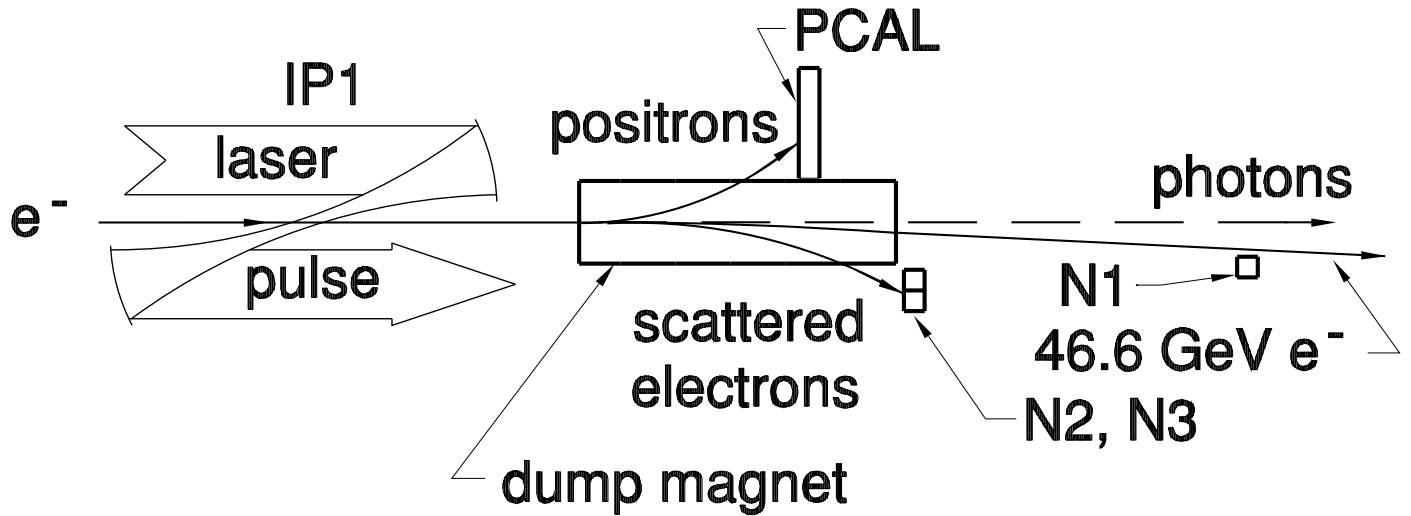
Background when scattering occurs in presence of electron beam.

Theory only approximate: Weizsäcker-Williams + multiphoton Breit-Wheeler.

Predicted to have rate only 1% that of the two-step process.

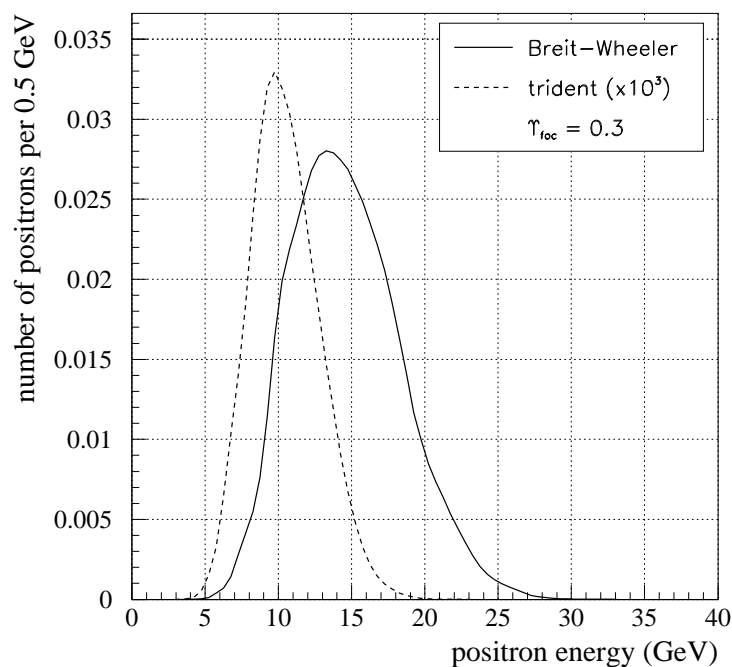


# Positrons from $e$ -Laser Interaction Region



$\approx 10^7$  electrons per laser shot from Compton scattering,  
 $\Rightarrow$  Only detect  $e^+$  from  $e^+e^-$  pair.

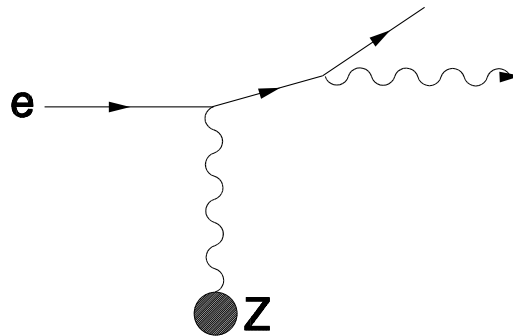
Predicted positron spectra:



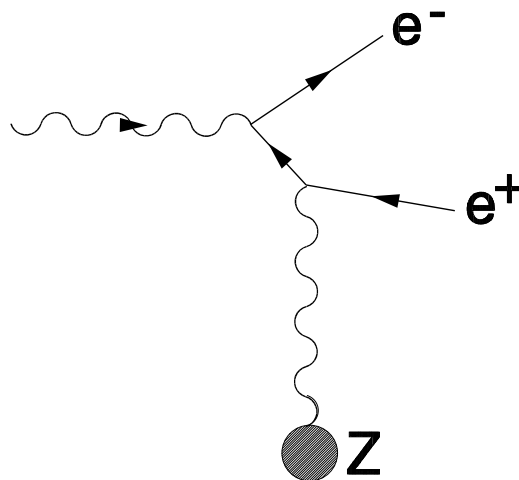
# Laser-Off Positron Backgrounds

Laser-off positrons are from showers caused by electrons that have fallen out of the beam.

## 1. Bremsstrahlung.



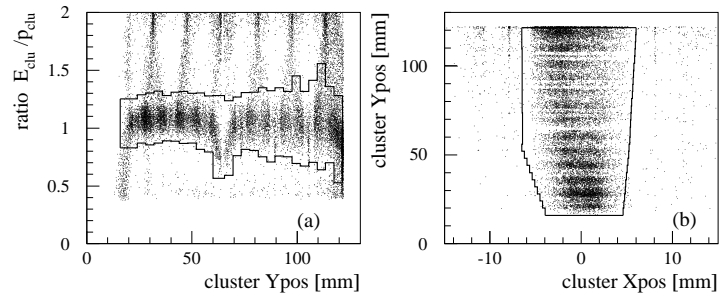
## 2. Bethe-Heitler pair creation.



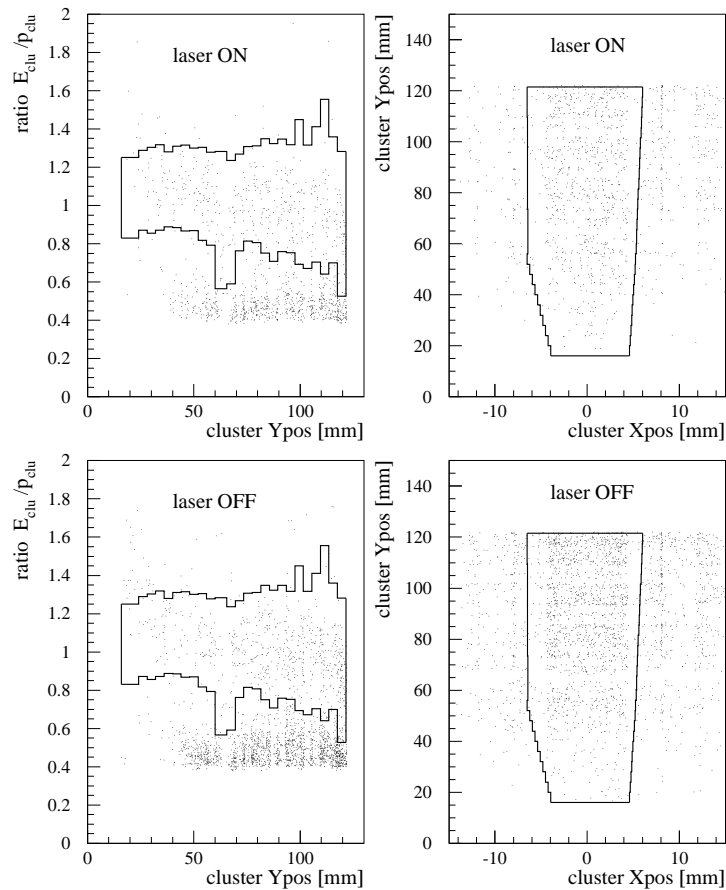
Study with data collected with laser off but electron beam on.

# Signal Processing

1. 'Signal' positrons from a wire at IP1 (no laser)



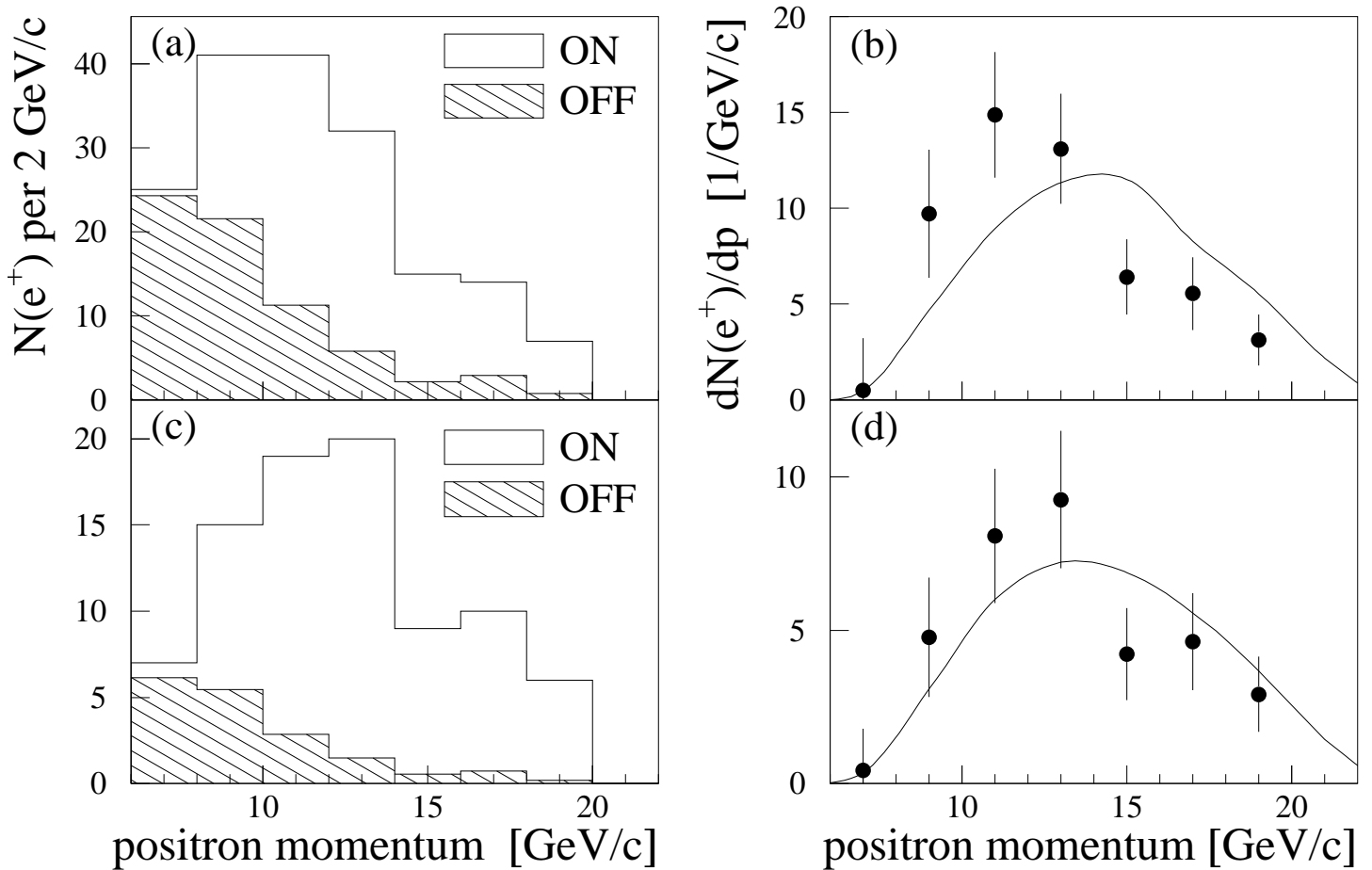
2. Define signal region for laser-on and -off data.



# Evidence for Positron Production (August '96)

178 laser-on candidates -  $0.175 \times 398$  laser-off candidates,

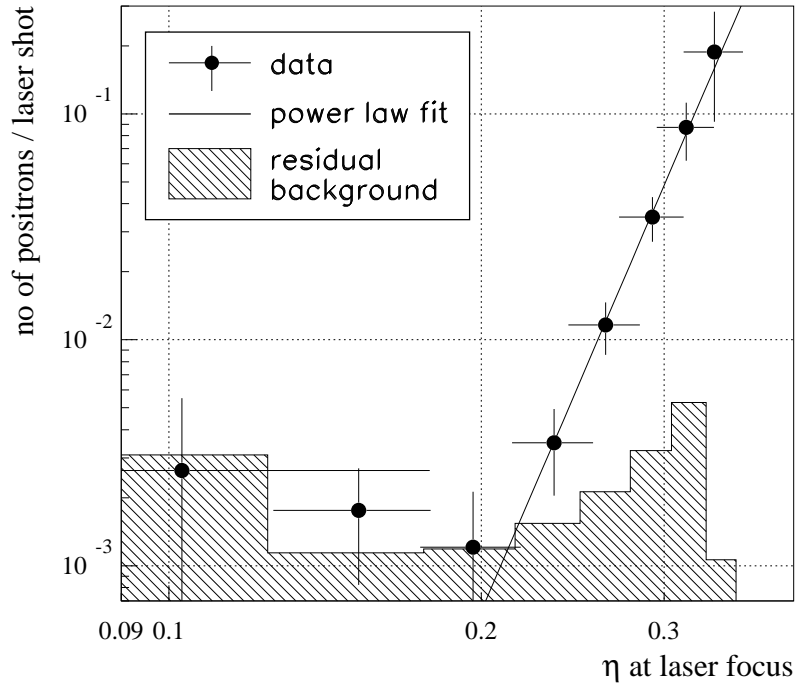
$\Rightarrow 106 \pm 14$  signal positrons (upper plots, no  $\eta$  cut)



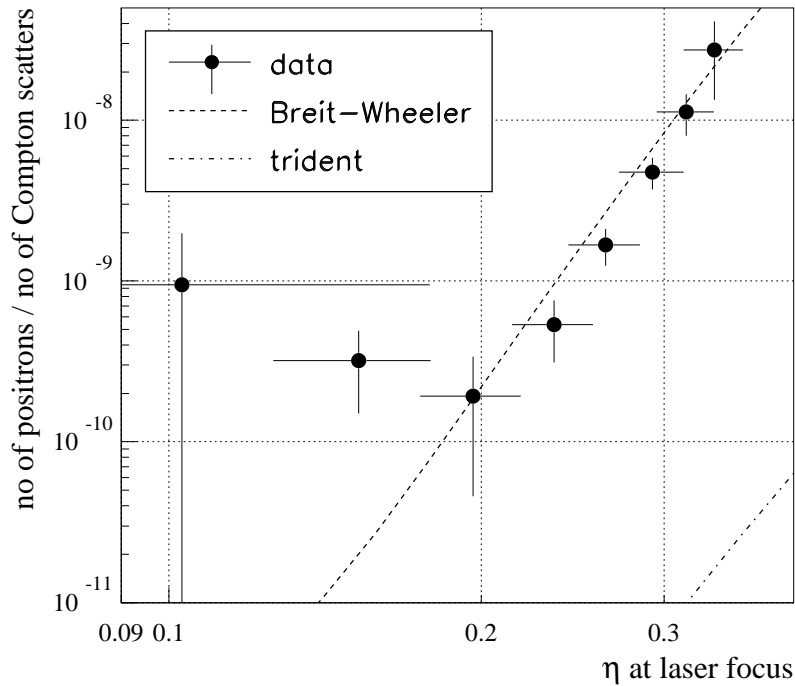
Lower plots:  $\eta > 0.216 \Rightarrow 69 \pm 9$  signal positrons.

# Positron Rate *vs.* $\eta$

Rate  $\propto \eta^{2n}$  where  $n = 5.1 \pm 0.2$  (stat.)  $^{+0.5}_{-0.8}$  (syst.):



Normalized to Compton scattering rate:



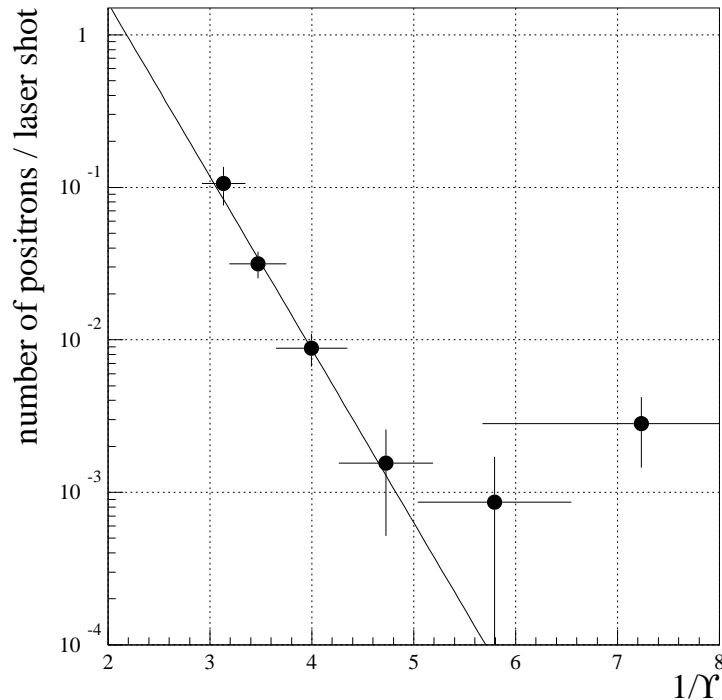
# Strong Field Pair Creation as Barrier Penetration

For a virtual  $e^+e^-$  pair to materialize in a field  $E$  the electron and positron must separate by distance  $d$  sufficient to extract energy  $2mc^2$  from the field:

$$eEd = 2mc^2.$$

The probability of a separation  $d$  arising as a quantum fluctuation is related to penetration through a barrier of thickness  $d$ :

$$P \propto \exp\left(-\frac{2d}{\lambda_C}\right) = \exp\left(-\frac{4m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{4E_{\text{crit}}}{E}\right) = \exp\left(-\frac{4}{\Upsilon}\right).$$



$$R_{e^+} \propto \exp[(-2.8 \pm 0.2 \text{ (stat.)} \pm 0.2 \text{ (syst.)})/\Upsilon].$$

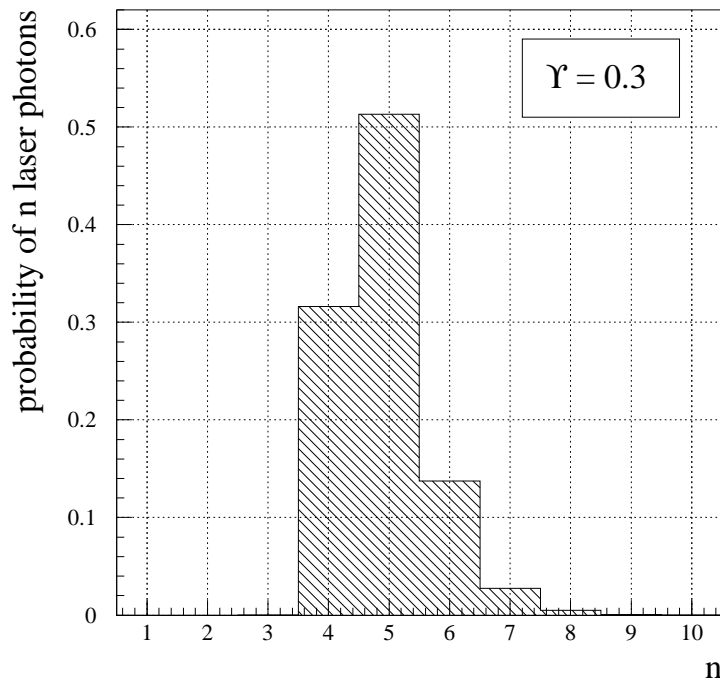


# Comments on Positron Observations

Signal rate  $\approx 1$  positron per 10  $e$ -laser collisions at highest  $\Upsilon$ .

The laser-induced positrons are  $> 99\%$  from light-by-light scattering and  $< 1\%$  from trident production.

In  $n\omega_0 + \omega \rightarrow e^+e^-$  the average number  $n$  of laser photons is 5 (plus 1 more to produce the high-energy photon by Compton backscattering).



This is the first observation of positron production in light-by-light scattering with only real photons.

## Reports in the Popular Media

<http://www.slac.stanford.edu/exp/e144/popular.html/>

*Presto! Light Creates Matter*, Science Now Online, Aug. 20, 1997.

*Conjuring Matter from Light*, Science magazine, 29 Aug. 1997, p. 1202.

*Scientists Use Light to Create Matter*, N.Y. Times, Sept. 16, 1997.

*Real Photons Create Matter*, AIP Physics News, Sept. 18, 1997.

*Dalla luce e nata la materia*, Corriere della Sera, Sept. 21, 1997.

*Materie aus Licht*, Neue Zuricher Zeitung, Sept. 26, 1997;  
*Materie aus Licht erschaffen*, Oct. 1, 1997,

*Light Work*, New Scientist magazine, Sept. 27, 1997.

*Amerikanische Physiker schufen erstmals Materie aus reinem Licht*, Die Zeit, Oct. 17, 1997.

*Matter Created from Pure Light*, OE Reports, No. 167, Nov. 1997.

*Boom! from Light comes Matter*, Photonics Spectra online, Nov. 1997, also, p. 31 of Photonics Spectra, Nov. 1997.

*Matter from Light*, CERN Courier, Nov. 1997, p. 4.

*E = mc<sup>2</sup>, Really*, Scientific American, Dec. 1997.

*Let There Be Matter*, Discover magazine, Dec. 1997, p. 40.

*Researchers Create Matter from Light*, Laser Focus World, Dec. 1997, p. 29.

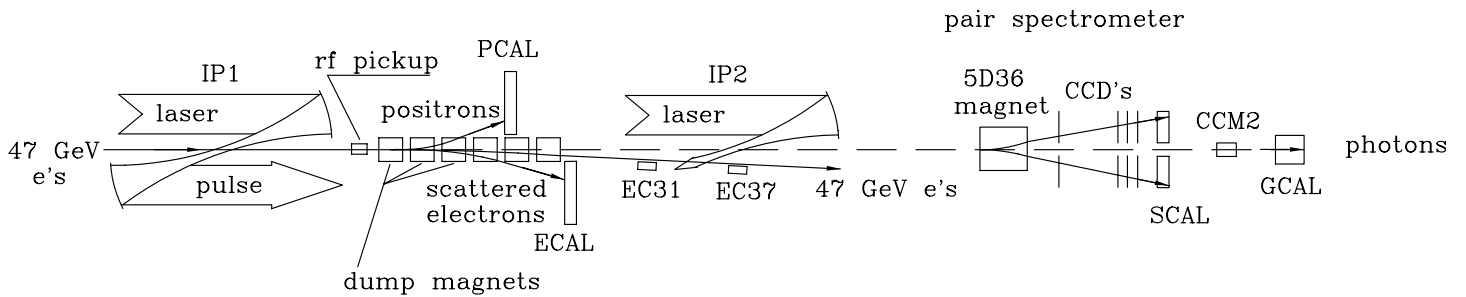
*Out of Pure Light, Physicists Create Particles of Matter*, SLAC Interaction Point magazine, Dec. 1997.

*Let There Be Matter*, a 2:04 min. interview by Karen Fox for the AIP Science Report

*Gamma Rays Create Matter Just by Plowing into Laser Light*, Physics Today, Feb. 1998, p. 17.

## To Do: Basic Physics

1. Study the mass-shift effect in nonlinear Compton scattering.
  - Continue at SLAC, or use 50-Mev electrons and CO<sub>2</sub> laser at BNL.
2. Study pair creation in a pure light-by-light scattering situation:



- No trident production.
- Search for structure in the  $e^+e^-$  invariant-mass spectrum.
- Upgrade laser to 10-Hz, 100-femtosecond pulses with  $\Upsilon_{\max} \approx 5$ .

## To Do: Applied Physics

### 1. Copious $e^+e^-$ Production.

- $e^+e^-$  pairs from  $e$ -laser collisions could be best low-emittance source of positrons.
- No Coulomb scattering in laser ‘target.’
- Positrons largely preserve the geometric emittance of the electron beam  $\Rightarrow$  ‘cooling’ of invariant emittance.
- Can produce 1 positron per electron if  $E^* > E_{\text{crit}}$ .
- Production with visible laser is optimal for  $\sim 500$  GeV electrons.

[Or use a 50-nm FEL with 50-GeV electrons.]

### 2. High-energy $e$ - $\gamma$ and $\gamma$ - $\gamma$ colliders.

- $e$ -laser scattering can convert essentially all of an electron beam to a photon beam.

### 3. Picosecond/femtosecond pulsed- $\gamma$ sources from Compton backscattering.

## Vacuum Laser Acceleration?

- A Maxwellian view: acceleration (energy gain) of a charge is due to interference between the drive field and the “spontaneous radiation” of the charge.
- Ex: The energy gained by a electron as it moves across the gap of a capacitor is compensated by a loss of field energy due to the interference between the DC field and the dipole field of charge + image.
- Ex: The energy gained by an electron in an rf cavity is compensated by the energy of interference between the cavity field and transition radiation of the charge.
- In these examples, the energy gain is linear in the strength of the external field.
- $\Rightarrow$  No vacuum linear acceleration (*i.e.*, linear in the laser field strength).
- Weak quadratic acceleration is possible in vacuum.
- <http://puhep1.princeton.edu/~mcdonald/accel/>