XVIII Physics in Collision
Frascati, June 18th-19th, 1998
Conference Proceedings, pp. 000-000

HIGHER-ORDER QED EFFECTS AND NONLINEAR QED

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ABSTRACT

After a brief review of higher-order QED effects, a survey is made of novel aspects of QED with emphasis on recent experimental results in strong-field QED: nonlinear Compton scattering and multiphoton pair creation by light.

1 Higher-Order QED Effects

Quantum Electrodynamics (QED) describes the interactions between charged particles that are mediated by quanta (photons) of the electromagnetic field. The simplest QED processes involve only a single photon, as represented by exchange and annihilation diagrams (Fig. 1). These processes involve two vertices of the form γee (where e represents an electric charge, not necessarily an electron). The process that consists only of emission of a real photon by a charged particle is forbidden by energy-momentum conservation. We may then say that higher-order QED is any process that involves more than two γee vertices.

Higher-order QED processes are often divided into two groups: tree and loop. Loop processes involve one or more virtual pairs of charged particles; examples are shown in Fig. 2. Tree processes, such as that of Fig. 3, have no loops.

Higher-order QED tree processes are often called radiative corrections. Any basic interaction can be modified by the emission of one or more photons, and the probability of emission of a soft photon approaches unity. Hence, the empirical measurement of the "basic" processes is dependent on an extrapolation to the improbable case of no radiative

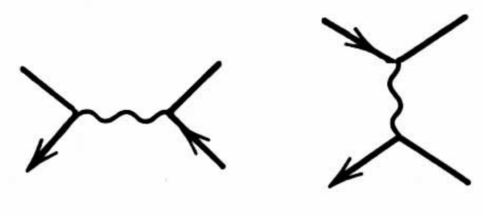

Figure 1: *The simplest QED processes: one-photon exchange and annihilation.*

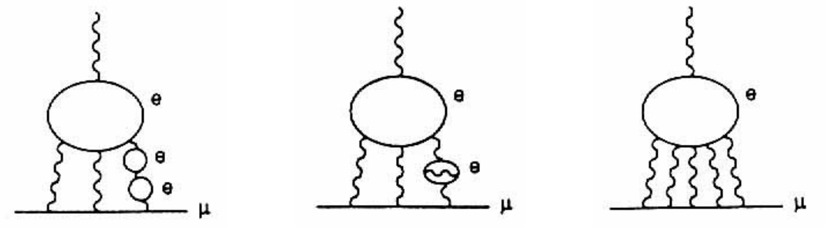

Figure 2: *Examples of 10th-order loop processes (⇔ 10 extra vertices).*

corrections. A contemporary example of the effect of initial-state radiative corrections is shown in Fig. 3.

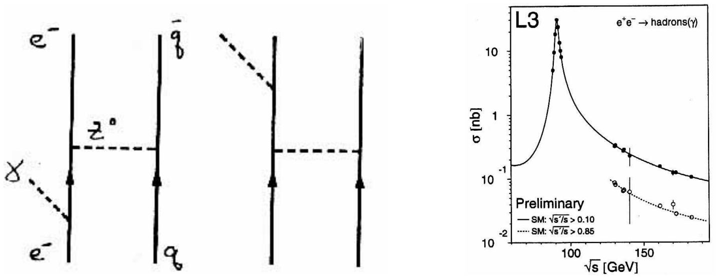

Figure 3: *Initial-state radiative corrections to the reaction* $e^+e^- \rightarrow Z^0$.

The rate for a tree process with n γee vertices is proportional to α^n, where $\alpha = e^2/\hbar c \approx 1/137$ is the QED fine structure constant. Hence, it is usually the case that very high order tree processes are suppressed compared to lower-order processes. However, there are regimes in which this is not the case. Processes in very strong electromagnetic fields can favor larger numbers of photons, as has been shown some years ago in atomic physics 6, 10) (Fig. 4). In sec. 3 we present first evidence for such effects in elementary particle physics.

Higher-order QED loop processes are studied in four classic tests: 17, 28)

- Hydrogen Lamb Shift: $\sigma_{\Delta E}(2S_{1/2} - 2P_{1/2}) = 2$ ppm
  [Theory limited by uncertainty in proton radius].
- Muonium hyperfine splitting: 24) Expt. − Theory ≈ 0.25 ppm
  [muon mass, $\mathcal{O}(\alpha^3)$ terms, hadronic (+ weak) loops].
  New LAMPF data being analyzed; error → 0.1 ppm.

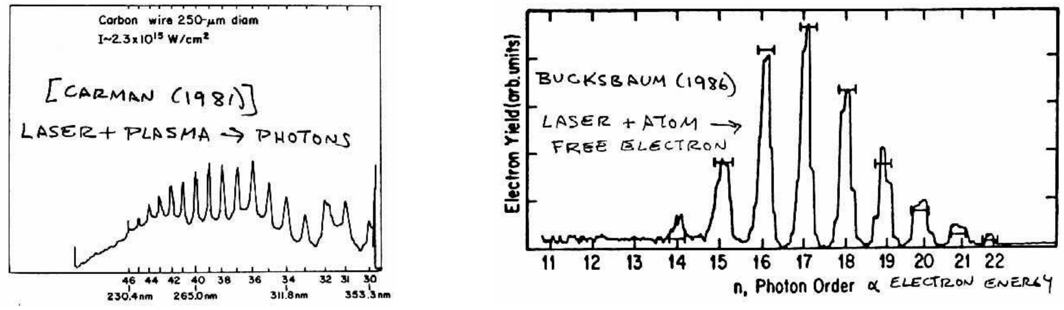

Figure 4: *a) Observation of photon emission at up to the 46th harmonic in laser-plasma interactions. b) Multiphoton ionization of atoms in which up to 22 photons were absorbed.*

- e anomalous magnetic moment: Expt. − Theory $\approx$ 25 ppb
  [α, $\mathcal{O}(\alpha^5)$ terms].
- μ anomalous magnetic moment: 13, 25) Expt. − Theory $\approx$ 10 ppm
  [$\mathcal{O}(\alpha^5)$ terms, hadronic (+ weak) loops].
  New BNL expt. starts in Fall 1998; error → 0.5 ppm. 26)

Trouble spot: the observed orthopositronium decay rate differs from theory by 6 σ; but the theory is incomplete at relative $\mathcal{O}(\alpha^2)$. 30)

Summation an infinite class of loop corrections to the γee vertex results in the "running" of the coupling constant:

$$\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{Q^2}{\Lambda^2}\right)}. \tag{1}$$

For example, when extrapolated to the Z-pole, 12) one obtains $\alpha^{-1}(M_Z^2) = 128.93 \pm 0.02$; half of the change from the value of 137 at low Q^2 is due to hadronic corrections.

Direct evidence for the running of the coupling constant has recently been obtained by the TOPAZ group 34) by comparing $e^+e^- \to \mu^+\mu^-$ to $e^+e^- \to e^+e^-\mu^+\mu^-$; they found $\alpha^{-1}(Q^2 = (57.77\ \mathrm{GeV}/c)^2) = 128.5 \pm 1.8$, while the theory predicts a value of 129.6.

As first remarked by Landau, the form of eq. (1) is such that the coupling α grows arbitrarily large as Q^2 approaches the pole at $\Lambda^2 \exp(3\pi/\alpha_0)$. One way to avoid this singularity is to invoke chiral symmetry breaking. 18) This phenomenon is associated with the possibility of a QED phase transition at strong coupling, as has been suggested by lattice gauge theory calculations. 1, 2, 29)

Could a variant of a QED phase transition occur in strong fields as well as at strong coupling? A possibly relevant measure of electromagnetic field strength is the so-called QED critical field, 20, 43, 45)

$$E_{\mathrm{crit}} = \frac{m^2c^3}{e\hbar} = \frac{mc^2}{e\lambda_C} = 1.3 \times 10^{16}\ \mathrm{V/cm} = 4.3 \times 10^{13}\ \mathrm{Gauss}, \tag{2}$$

above which spontaneous pair creation occurs. No theory of a strong-field QED phase change exists. During the 1980's, the "evidence" for positron peaks in low-energy heavy-ion collisions (Darmstadt) was sometimes associated with a QED phase change, but the evidence is now largely withdrawn. 44, 48)

The Landau pole problem can also be avoided via grand unification and strings. An elegant variant of grand unification invokes supersymmetry to bring the running of the coupling constants α_{QED}, α_{strong} and α_{weak} together at a common energy. 15, 32) This prediction is one of the most far-reaching applications of higher-order QED.

A subclass of loop processes involves "boxes", fermion loops coupled to four photons (Fig. 5). Delbrück scattering and photon splitting in the field of a nucleus have been measured, 22) but light-by-light scattering of real photons has not yet been observed in the laboratory.

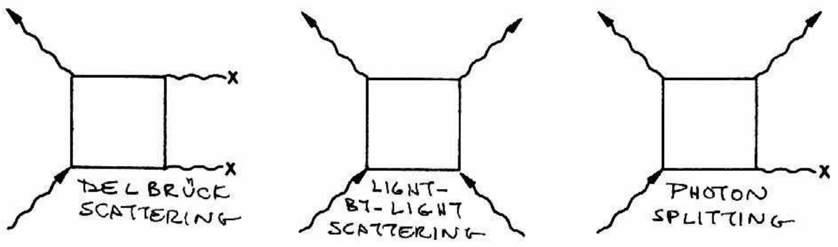

Figure 5: *QED box processes: Delbrück scattering, light-by-light scattering and photon splitting.*

An application of light-by-light scattering is finite-temperature QED, in which the frequency of the Planck spectrum is slightly shifted. 3, 41) The effect, however, is small:

$$\frac{\Delta\lambda}{\lambda} \propto \alpha^2 \left(\frac{kT}{mc^2}\right)^4 \approx 10^{-35} \left(\frac{T}{300\text{K}}\right)^4 . \qquad (3)$$

A recent experiment 14) can be said to have observed an effect of finite-temperature QED, as shown in Fig. 6.

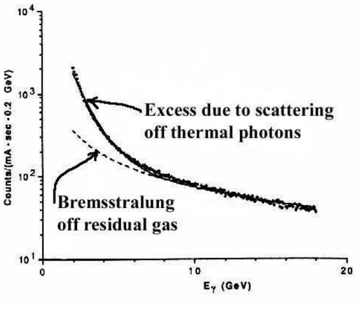

Figure 6: *Evidence for Compton scattering of the LEP electron beam off the thermal photons inside the beam pipe.*

The well-known success of QED in describing higher-order effects remains an inspiration for theories of other interactions, both elementary and complex. 21)

2 Novel Aspects of QED

Prior to the Maxwellian synthesis of electromagnetism, the Navier-Stokes equation of hydrodynamics was a candidate for the "theory of everything"; but it didn't predict sonoluminescence. 16) The latter is the phenomenon in which an imploding bubble of gas inside a liquid converts a large fraction of the initial acoustic energy into visible light by a process that is not well understood. 40) [It is likely that the eV-scale photons of sonoluminescence are what makes liquid nitroglycerine explode when dropped. 23)]

Several recent speculations relate sonoluminescence to QED process:

- Preparata *et al.*: 9) a QED theory of water vapor predicts emission of light when water vapor condenses at density near 1 g/cm^3.
- Schwinger: 46) a bubble is an electromagnetic cavity; an imploding bubble will radiate away the changing, trapped zero-point energy. This is a dynamic manifestation of the Casimir effect. 31)
- Liberati *et al.*: 35) an imploding bubble ⇒ rapidly changing index of refraction ⇒ associated radiation. This relates to an earlier idea:
- Yablonovitch: 52) an accelerating boundary across which the index of refraction changes is a realization of the Hawking-Unruh effect, leading to conversion of QED vacuum fluctuations into real photons.

In 1974, Hawking noted that an observer outside a black hole experiences a bath of thermal radiation of temperature

$$T = \frac{\hbar g}{2\pi c k} , \qquad (4)$$

where g is the local acceleration due to gravity. 19) In some manner, the interaction of the gravitational field with QED vacuum fluctuations produces measurable effects on an observer. Shortly thereafter, Unruh remarked that according to the equivalence principle an accelerated observer in a gravity-free region should also experience a thermal bath with temperature

$$T = \frac{\hbar a}{2\pi c k} , \qquad (5)$$

where a is the acceleration of the observer as measured in his instantaneous rest frame. 49)

The Hawking-Unruh effect is perhaps novel, but it should be contained within the standard QED theory. Indeed, Bell noted that the incomplete polarization of electrons in a storage ring is explained in detail by Hawking-Unruh excitations. 4) Recently, Leinaas 33) and Unruh 50) have extended that argument to arbitrary g-factors for the electron, fully reproducing the intricate dependence of the polarization on g (Fig. 7).

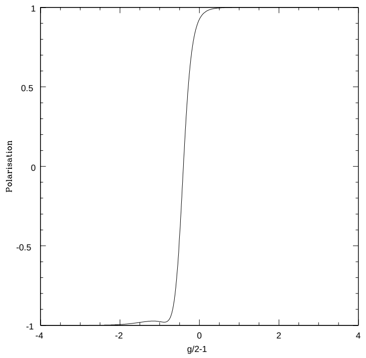

Figure 7: *Calculated dependence of the residual polarization in an electron storage ring on the g-factor of the electron.*

3 Nonlinear QED of Strong Fields

The advice of Hawking and Unruh is that novel QED phenomena may be found in highly accelerated systems. For example, if the energy kT in eq. (5) is to correspond to the electron rest energy, then the acceleration a must be $2\pi mc^3/\hbar$. If this acceleration is caused by an electric field E, then $a = eE/m$ and we find that $E = 2\pi E_{\rm crit}$, in terms of the QED critical field strength introduced in eq. (2).

Thus we are led to consider physics opportunities in very strong macroscopic electromagnetic fields. The strongest laboratory fields are found in lasers. Tabletop teraWatt lasers 47) can be focused to $> 10^{19}$ W/cm^2, $\Rightarrow E > 100$ GeV/cm, for which the photon number density exceeds 10^{27}/cm^3.

In such strong fields, the (nonperturbative) physics is described by two dimensionless measures of field strength, η and Υ:

$$\eta = \frac{e\sqrt{\langle A_\mu A^\mu \rangle}}{mc^2} = \frac{eE_{\rm rms}}{m\omega_0 c} = \frac{eE_{\rm rms}\lambda_0}{mc^2}, \tag{6}$$

governs the importance of multiple photons in the initial state, and characterizes the "mass shift": 27) $\overline{m} = m\sqrt{1+\eta^2}$.

The second parameter,

$$\Upsilon = \frac{\sqrt{\langle (F^{\mu\nu}p_\nu)^2 \rangle}}{mc^2 E_{\rm crit}} = \frac{2p_0}{mc^2}\frac{E_{\rm rms}}{E_{\rm crit}} = \frac{2p_0}{mc^2}\frac{\lambda_C}{\lambda_0}\eta, \tag{7}$$

governs the importance of "spontaneous" pair creation, where $E_{\rm crit}$ is the QED critical field strength (2).

Where can critical fields be found?

- The magnetic field at the surface of a neutron star approaches the critical field $B_{\rm crit} = 4.4 \times 10^{13}$ Gauss. 38)
- During heavy-ion collisions where $Z_{\rm total} = 2Z > 1/\alpha$, the critical field can be exceeded and e^+e^- production is expected: 44, 48) $E_{\rm max} \approx 2Ze/\lambda_C^2 = 2Z\alpha E_{\rm crit}$.

- The earth's magnetic field appears to be critical strength as seen by a cosmic-ray electron with 10^{19} eV. 39)
- The effective field between planes of a crystal can appear critical to a highly relativistic electron. 44)
- The electric field of a bunch at a future linear collider approaches the critical field in the frame of the oncoming bunch. 11)
- The electric field of a focused teraWatt laser appears critical to a counterpropagating 50-GeV electron. 42)

The remainder of this paper presents results obtained in the latter circumstance.

3.1 Physics at High η: Nonlinear Compton Scattering

The process of nonlinear Compton scattering,

$$e + n\omega_0 \to e' + \omega, \tag{8}$$

has recently been studied for values of η up to 0.6 with apparatus as sketched in Fig. 8. 7)

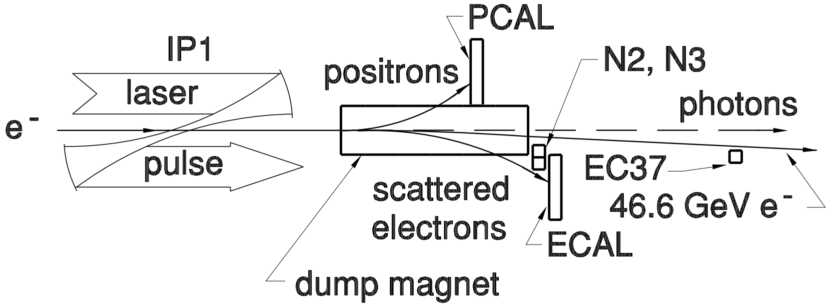

Figure 8: *Layout of SLAC E-144 in which a teraWatt laser collides at 17° with a 46.6 GeV electron beam to produce backscattered photons, electrons and positrons via nonlinear QED processes.*

The key results are summarized in Fig. 9, in which the differential rate of scattered electrons is normalized to the total rate of scattered photon electrons and plotted against laser intensity I. The minimum number n of laser photons that interacted with the electron is a simple function of the energy of the scattered electron; lower energy $\Leftrightarrow$ more photons, as labelled on the figure. The scattering rate for n-photon absorption is expected to vary as I^n, so the normalized rate varies as I^{n-1}.

The results are in good agreement with the theory based on Volkov states 51) of Dirac electrons in a plane wave, 36, 37) given the fairly large systematic uncertainty in the normalization of the data, shown by the bands in Fig. 9.

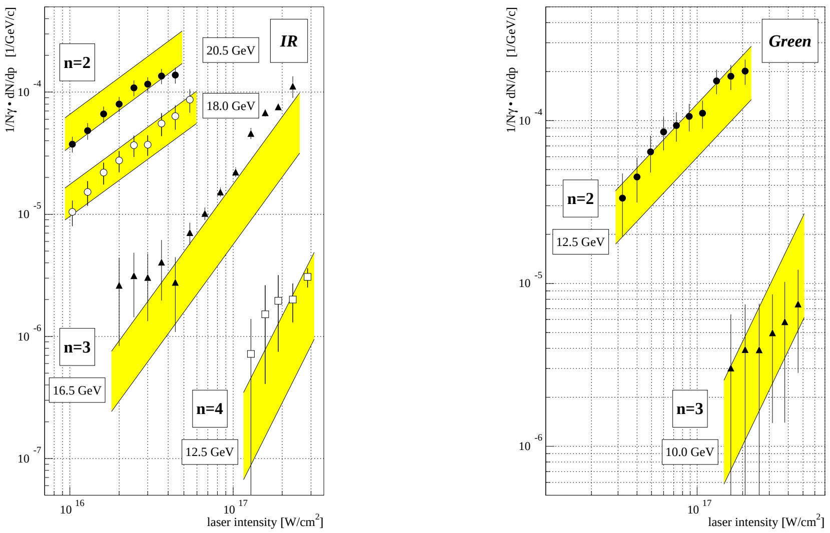

Figure 9: *Normalized rate of nonlinear Compton scattering, reaction (8), as a function of laser intensity for various electron energies corresponding to different numbers of interacting laser photons. Up to four laser photons participated in a single scattering event.*

3.2 Physics at High Υ: Pair Creation by Light

Multiphoton pair creation by light can arise from a two-step process in which a high-energy photon from reaction (8) interacts with the laser beam:

$$\omega + n\omega_0 \rightarrow e^+e^-. \tag{9}$$

This process is the strong-field variant of Breit-Wheeler pair creation. 5)

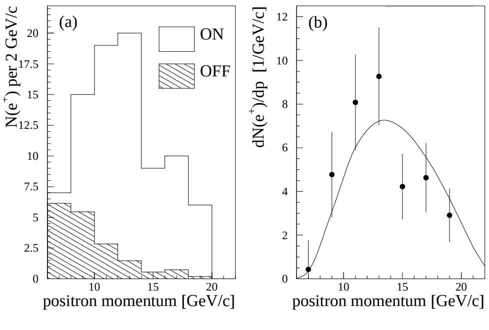

Figure 10: *(a) Laser-on and -off spectra of positrons from reaction (9). (b) Subtracted spectrum. The solid line is a model calculation.*

In the experiment described above, a signal of 106 ± 14 laser-induced positrons was observed after subtracting a small background of positrons from showers of lost beam electrons, as shown in Fig. 10. 8) The rate of positron production, normalized to the Compton scattering rate, is again in good agreement with strong-field QED, 36, 37, 42) as shown in Fig. 11. The observed rate varied as η^{2n} where $n = 5.1 \pm 0.2$ (stat.) $^{+0.5}_{-0.8}$ (syst.), ⇒ 5 laser photons involved. In this experiment, reaction (9) was below threshold for 1-3 photons.

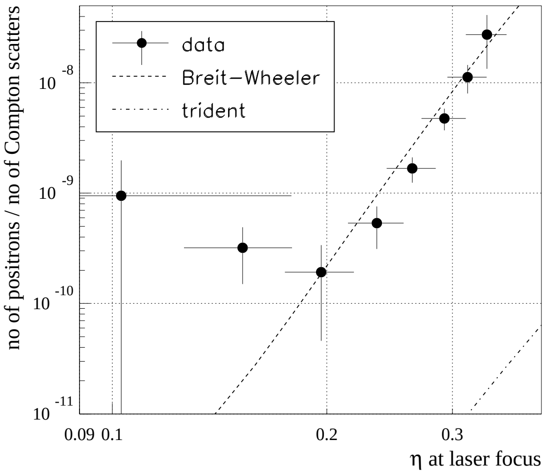

Figure 11: *Rate of positron production in reaction (9), normalized to the total Compton scattering rate, as a function of laser field strength. Background from the trident process,* $e + n\omega_0 \to e'e^+e^-$, *is calculated to be negligible. About 2-3* $\times 10^{-10}$ *positrons per Compton scatter arise from interactions of backscattered laser photons with beam gas.*

The preceding discussion emphasized a particle viewpoint. It is instructive to consider a field viewpoint as well. For a virtual e^+e^- pair to materialize in a field E, the electron and positron must separate by distance d sufficient to extract energy $2mc^2$ from the field; hence, $eEd \geq 2mc^2$. The probability of a separation d arising as a quantum fluctuation is related to penetration through a barrier of thickness d:

$$P \propto \exp\left(-\frac{d}{\lambda_C}\right) = \exp\left(-\frac{2m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{2E_{\text{crit}}}{E}\right) = \exp\left(-\frac{2}{\Upsilon}\right). \qquad (10)$$

In a detailed analysis, $2/\Upsilon$ becomes π/Υ. 20, 43, 45)

When the results of Fig. 11 are plotted against $1/\Upsilon$, one finds that $R_{e^+} \propto \exp[(-1.8 \pm 0.2(\text{stat.}) \pm 0.2(\text{syst.}))/\Upsilon]$. This agreement with the barrier-penetration model justifies the interpretation of the data as "sparking the vacuum".

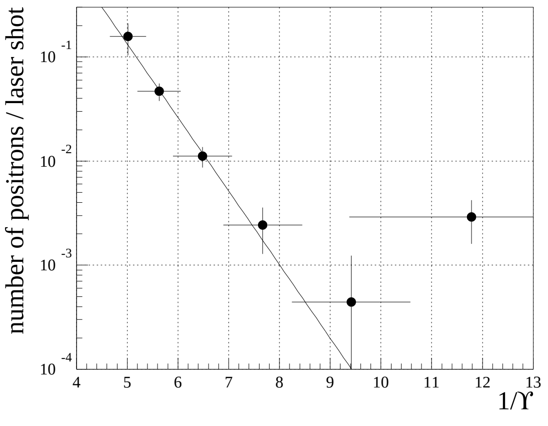

Figure 12: *The observed rate of positron production vs.* $1/\Upsilon$.

4 Summary

- Higher-order QED (physics depending on high powers of $\alpha_{\rm QED}$) is very mature both experimentally and theoretically.

  New results will probe strong and electroweak corrections rather than yet-higher orders of QED.

- Nonperturbative (strong-field) QED is still relatively young.

  New experiments involving intense laser beams at $\eta \approx 1$ and $\Upsilon \approx 1$ agree with existing theories.

  The frontier is at $\eta,\ \Upsilon \gg 1$.

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