Ph 205 FINAL EXAM

8:30 – 11:30 A.M.

JAN. 26, 1990

PALMER 306A

- THIS EXAM CONSISTS OF 4 QUESTIONS, EACH WORTH 10 POINTS.
- THE EXAM IS CLOSED BOOK, CLOSED NOTES.
- PLEASE DO ALL WORK YOU WISH GRADED IN THE EXAM BOOKLETS PROVIDED.

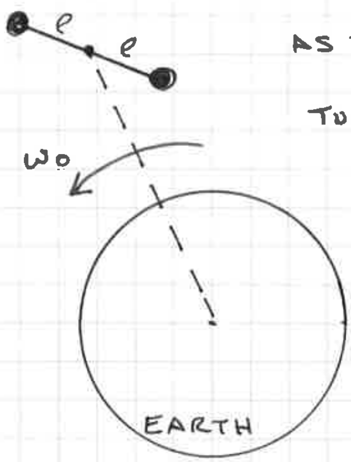
1) CLASSICAL NUCLEAR FORCES. THE FORCE THAT HOLDS PROTONS AND NUCLEONS TOGETHER IN A NUCLEUS IS THOUGHT TO FALL OFF QUICKER THAN $1/r^2$. A SIMPLE MODEL IS TO SUPPOSE THAT THE FORCE IS CENTRAL, AND OBEYS

$$\bar{F} = -\frac{\alpha}{r^2} e^{-\beta r} \hat{r}.$$

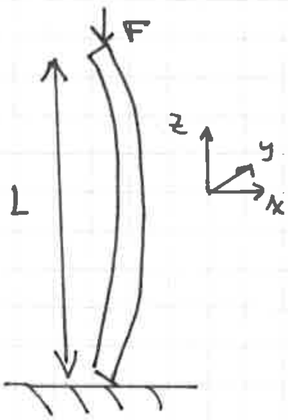
FIND A CONDITION FOR STABILITY OF CIRCULAR ORBITS OF A PARTICLE MOVING ABOUT A FIXED FORCE CENTER UNDER THIS FORCE. EXPRESS THE FREQUENCY OF SMALL OSCILLATIONS ABOUT A CIRCULAR ORBIT IN TERMS OF THE ROTATION FREQUENCY OF THE EQUILIBRIUM ORBIT.

DEDUCE A QUALITATIVE DISTINCTION BETWEEN THE STABLE CIRCULAR ORBITS UNDER THIS FORCE AND THOSE OF A $1/r^2$ FORCE (BASED ON YOUR ANSWER TO THE 2 PREVIOUS PARTS OF THE QUESTION).

(2) A SATELLITE CONSISTS OF 2 POINT MASSES ON THE ENDS OF A MASSLESS ROD OF LENGTH 2ℓ. THE SATELLITE IS IN A CIRCULAR ORBIT ABOUT THE EARTH WITH ω_0 AS THE ANGULAR FREQUENCY OF ROTATION OF THE CENTER OF MASS OF THE SATELLITE ABOUT THE EARTH. FIND THE FREQUENCY OF SMALL OSCILLATIONS OF THE ORIENTATION OF THE ROD, SUPPOSING THE ROD ALWAYS LIES IN THE PLANE OF THE ORBIT.



③ CHARLIE CHAPLIN'S CANE. WHEN CHARLIE LEANS ON HIS CANE IT POPS INTO A BOW.

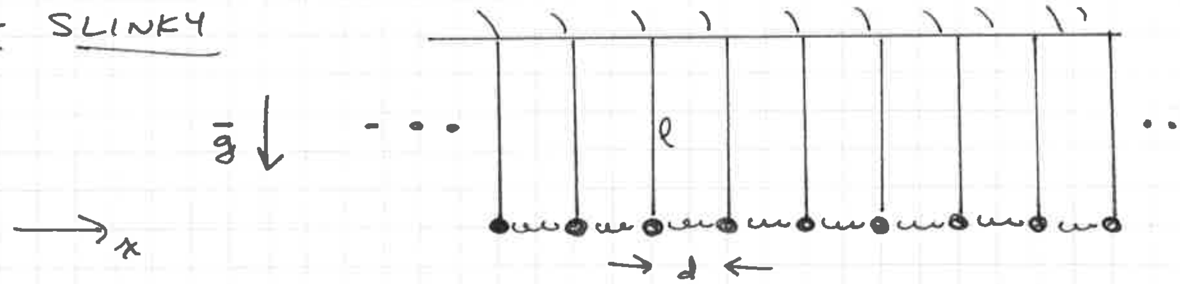


CONSIDER A TALL, SLENDER BEAM (THE CANE) OF LENGTH L SUBJECT TO AN APPLIED VERTICAL FORCE F. DERIVE A RELATION FOR THE MINIMUM FORCE F FOR BOWING (OR 'BUCKLING') TO OCCUR IN TERMS OF L, THE YOUNG'S MODULUS E, AND THE BENDING MOMENT OF INERTIA DEFINED BY $I = \int x^2 \, dx \, dy$.

YOU MAY IGNORE GRAVITY AND THE COMPRESSION OF THE BEAM'S LENGTH L. THE ENDS OF THE BEAM ARE FREE TO ROTATE.

[POSSIBLE HINT: CONSIDER THE STABILITY OF A BENT BEAM.]

④ SLUNG SLINKY



CONSIDER A LONG SERIES OF MASSES m EACH HUNG FROM A (MASSLESS) STRING OF LENGTH ℓ. EACH ADJACENT PAIR OF MASSES IS CONNECTED BY A SPRING OF SPRING CONSTANT A AND REST LENGTH d, THE EQUILIBRIUM SPACING BETWEEN MASSES.

IF THE LEFTMOST MASS IS SHAKEN HORIZONTALLY, WAVE MOTION MAY ENSUE.

DERIVE AN EQUATION OF MOTION FOR THE (SMALL) HORIZONTAL DISPLACEMENT, s_i, OF MASS i. THEN TAKE THE CONTINUUM LIMIT OF THIS EQUATION, INTRODUCING $\rho = m/d$ AND ANY OTHER RELEVANT PARAMETERS.

BY EXAMINING THE DISPERSION RELATION, $\omega = \omega(k)$, FOR WAVES IN THIS SYSTEM, DEDUCE A <u>MINIMUM</u> POSSIBLE FREQUENCY. WHAT HAPPENS TO THE SYSTEM IF WE ATTEMPT TO DRIVE IT WITH A FREQUENCY LOWER THAN THE CRITICAL VALUE?