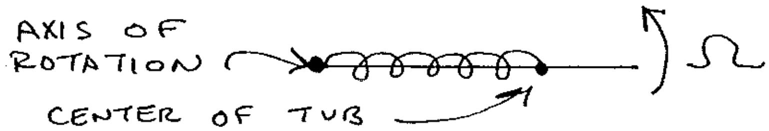
Ph 205 Set 9

Due: Thursday, Nov. 29, 1990   Maximum Recorded Score = 60 points

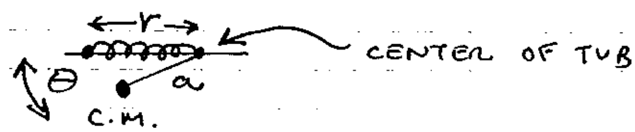
① <u>Physics in the Laundromat</u>. If a washing machine is unevenly loaded, violent motions can occur at the start of the spin cycle. Nonetheless as the spin velocity increases a stable motion (usually) results.

The mechanism of the washing machine is something like the following. The center of the wash tub is not necessarily on the axis of rotation, but is tied to the axis by a spring of constant K (rest length zero)



The center of the tub is constrained to rotate about the axis of rotation with angular velocity Ω = constant. For example, the center of the tub might slide along a rod which rotates with angular velocity Ω.

Because of the uneven load, the C.M. of the tub is not at the center of the tub, but a distance a away.



The tub can rotate about its center, so there is no constraint on θ. Meanwhile the center of the tub still rotates about the axis. All rotations take place in the same plane.

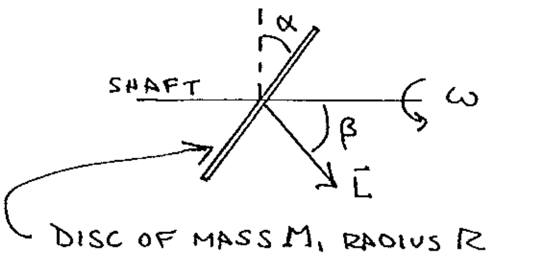
a) What are the equilibrium values of θ and r = distance from axis to the center of the tub? Let m = mass of tub and load. Suppose $\Omega > \omega_0 = \sqrt{K/m}$. For simplicity, you may imagine all mass is concentrated at the C.M. Your answer should have a simple physical interpretation.

b) Show that the equilibrium in r is unstable if $\theta = \theta_{\text{equilibrium}}$ always.

c) Show that the equilibrium is stable if θ is free to vary. You may restrict your argument to the case $\Omega \gg \omega_0$ for simplicity. Sketch the motion of the C.M. in this limit.

Hint: Use the method of accelerated coord systems, rather than Lagrange. But note that the 'true' force on the C.M. must lie along the line from the C.M. to the center of the tub.

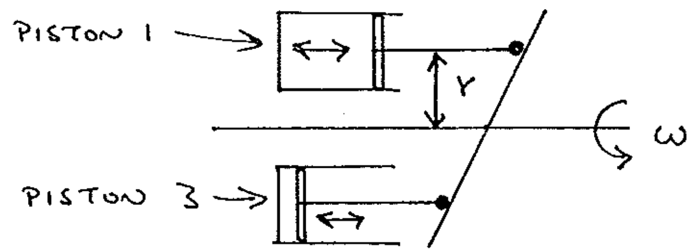
② THE WOBBLE-PLATE ENGINE



A DISC OF MASS M, RADIUS R IS MOUNTED ON A SHAFT THRU ITS C.M. AT AN ANGLE α AS SHOWN. THE DISC ROTATES ABOUT THE SHAFT WITH CONSTANT ANGULAR VELOCITY ω.

a) IDENTIFY THE PRINCIPAL AXES AND THE PRINCIPAL MOMENTS OF INERTIA TO OBTAIN ANGULAR MOMENTUM $\bar{L}$ IN THE PRINCIPAL AXIS FRAME. SHOW THAT $\bar{L}$ MAKES ANGLE β TO THE SHAFT, WHERE

$$\tan\beta = \frac{\tan\alpha}{2+\tan^2\alpha}.$$



b) FOUR PISTONS HAVE PISTON RODS WHICH ARE PARALLEL TO THE MAIN SHAFT, AND PRESS AGAINST THE ROTATING DISC. HENCE THE PISTONS ARE DRIVEN BACK AND FORTH BY THE ROTATING DISC (OR VICE VERSA, TO MAKE AN ENGINE!)

THE 4 PISTONS ARE MOUNTED 90° APART IN AZIMUTH, 2 IN THE PLANE OF THE PAPER AS SHOWN, 2 IN A PLANE $\perp$ TO THE PAPER.

THE PISTONS EACH HAVE MASS m AND ARE A DISTANCE r AWAY FROM THE SHAFT.

SHOW THAT THE PISTONS EXECUTE SIMPLE HARMONIC MOTION.

CALCULATE THE TOTAL ANGULAR MOMENTUM OF THE 4 PISTONS ABOUT THE C.M. OF THE DISC. THEN SHOW THAT THE ENGINE WILL BE 'BALANCED' ($\bar{L}_{TOTAL}$ PARALLEL TO THE SHAFT) IF THE PISTONS EACH HAVE MASS $m = \dfrac{MR^2\cos^2\alpha}{8r^2}$.

③ a) B & O PROBLEM 13, P 259. [SEE P. 8 OF THIS SET]

b) A SPACE STATION IN THE FORM OF A DISC OF MASS M, RADIUS a IS ROTATING ABOUT ITS SYMMETRY AXIS WITH ANGULAR VELOCITY $\bar{\omega}_0$. IT IS STRUCK BY A METEORITE WHICH IMPARTS AN IMPULSE $\bar{P}$ PARALLEL TO $\bar{\omega}_0$ AT A POINT ON THE RIM OF THE DISC. WHAT IS THE ANGLE BETWEEN $\bar{\omega}_0$ AND THE NEW INSTANTANEOUS AXIS OF ROTATION JUST AFTER THE IMPULSE?

③ c) A BODY WITH A SYMMETRY AXIS $(I_1 \neq I_2 = I_3)$ IS ROTATING SUBJECT TO AN EXTERNAL DRAG TORQUE

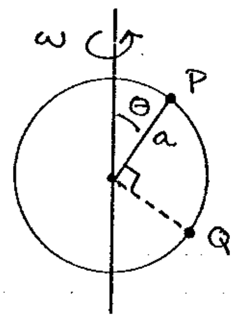
$$\bar{N} = -C\,\bar{\omega} \qquad \text{DUE TO AIR RESISTANCE}$$

USE EULER'S EQUATIONS TO SHOW THAT ω_1 DECREASES EXPONENTIALLY WITH TIME. COMBINE THE EQUATIONS FOR ω_2 & ω_2 TO SHOW THAT THE ANGLE BETWEEN $\bar{\omega}$ AND ↑ VARIES LIKE

$$\tan\Theta = \frac{\sqrt{\omega_2^2+\omega_3^2}}{\omega_1} = \tan\Theta_0\, e^{-\frac{C(I_1-I_2)}{I_1 I_2}t}$$

SO $\Theta \to 0$ OR $90°$ AS $t \to \infty$.

④ ω HOOPS! A SPHERE OF MASS M, RADIUS a IS SPINNING ABOUT A DIAMETER WITH ANGULAR VELOCITY ω.



IGNORE GRAVITY. THE C.M. IS INITIALLY AT REST.

SUDDENLY A POINT P ON THE SPHERE (AT ANGLE Θ TO THE DIAMETER OF ROTATION) IS FIXED.

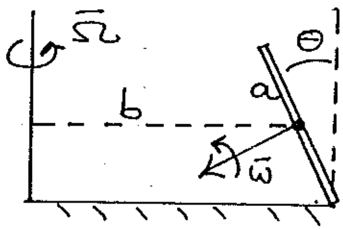
a) WHAT IS THE INSTANTANEOUS AXIS OF ROTATION JUST AFTER THE MOMENT WHEN P IS FIXED? IT MAY BE USEFUL TO EXPRESS YOUR ANSWER WITH RESPECT TO AXES WHICH EMPHASIZE POINT P. (WHAT IS $\bar{\omega}$ JUST AFTER...?)

b) WHAT IS THE VELOCITY OF POINT Q, LOCATED 90° AROUND THE SPHERE FROM P, JUST AFTER P IS FIXED? ANS: $|\bar{V}_Q| = a\omega\left|\frac{2}{7}\sin\theta - \cos\theta\right|$
P AND Q AND THE CENTER OF THE SPHERE ALL LIE IN A PLANE.

c) WHAT IS THE IMPULSE REQUIRED TO FIX P? GIVE BOTH MAGNITUDE AND DIRECTION.

d) WHAT IS THE MAGNITUDE AND DIRECTION OF THE FORCE REQUIRED TO MAINTAIN STEADY MOTION AFTER P IS FIXED? $|\bar{F}| = \frac{4}{49} M a \omega^2 \sin^2\Theta$.

⑤ STEADY MOTION OF A ROLLING WHEEL



A WHEEL IN THE FORM OF A HOOP OF MASS M AND RADIUS a IS ROLLING WITHOUT SLIPPING, AND MAKES ANGLE θ TO THE VERTICAL. ITS C.M. MOVES IN A CIRCLE OF RADIUS b WITH ANGULAR VELOCITY $\bar{\Omega}$. IT IS ROTATING ABOUT ITS AXLE WITH ANGULAR VELOCITY $\bar{\omega}$.

CONSIDER STEADY MOTION $\Rightarrow |\bar{\omega}|$, $\bar{\Omega}$ AND θ ARE CONSTANTS.

a) WHAT IS THE RELATION BETWEEN ω AND Ω FOR ROLLING WITHOUT SLIPPING? YOU MUST GET THIS RIGHT BEFORE PROCEEDING.

b) WHAT IS THE VECTOR TORQUE $\bar{N}$ ABOUT THE C.M.?

WE NOW WANT TO USE PARTS a) & b) TO FIND A RELATION BETWEEN Ω, a AND b.

c) USE THE METHOD OF EULER'S EQUATIONS.

THAT IS, CHOOSE AS AXES A SET OF PRINCIPAL AXES ROTATING WITH THE WHEEL. FIND THE PRINCIPAL MOMENTS OF INERTIA.

NOTE THAT IN THE LAB FRAME THE TOTAL ANGULAR VELOCITY IS $\bar{\omega}_{TOT} = \bar{\omega} + \bar{\Omega}$. CALCULATE $\dot{\bar{\omega}}_{TOT}$ IN EITHER THE LAB FRAME OR THE ROTATING FRAME. CAN YOU INTERPRET THE RESULT IN BOTH FRAMES?

TO PLUG INTO EULER'S EQUATIONS, IT IS CONVENIENT TO CHOOSE A TIME WHEN ONE PRINCIPAL AXIS IS HORIZONTAL, AND THE OTHER TWO ARE IN THE VERTICAL PLANE. THEN ONLY ONE COMPONENT OF EULER'S EQUATIONS SHOULD SURVIVE, LEADING TO

$$\Omega^2 = \frac{2g\tan\theta}{4b + a\sin\theta}.$$

d) INSTEAD OF USING EULER'S EQUATIONS, WE MIGHT HAVE CHOOSEN A SET OF PRINCIPAL AXES WHICH DON'T ROLL WITH $\bar{\omega}$, BUT MERELY ROTATE ABOUT THE VERTICAL WITH $\bar{\Omega}$. THIS IS POSSIBLE BECAUSE A WHEEL HAS A SYMMETRY AXIS. (AGAIN CHOOSE ONE AXIS HORIZONTAL FOR SIMPLICITY.

NOW WE CAN GO BACK TO BASICS. OUR FRAME ROTATES WITH $\bar{\Omega}$,

SO $\bar{N} = \frac{d\bar{L}}{dt} = \frac{\delta \bar{L}}{\delta t} + \bar{\Omega} \times \bar{L}$

BUT NOW $\frac{\delta \bar{L}}{\delta t} = 0$ BY OUR CHOICE OF ROTATING FRAME.

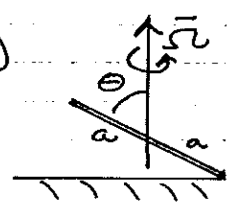
USE $\bar{L} = \bar{\bar{I}} \cdot \bar{\omega}_{TOT}$ TO PLUG INTO THE TORQUE EQUATION, WHICH SHOULD LEAD TO THE SAME RESULT AS IN PART c).

6 SPINNING COIN

A SPECIAL CASE OF THE ROLLING WHEEL IS WHEN THE C.M. DOES NOT MOVE

$\Rightarrow b = 0$ IN PROB 5.

THIS IS THE CASE OF A COIN SPINNING ON A TABLE, JUST BEFORE IT COMES TO REST. TRY IT! NOTICE THAT THE COIN WOBBLES RAPIDLY, BUT THE FIGURE ON THE FACE PRECESSES SLOWLY.

a) 

DERIVE AN EXPRESSION FOR $\Omega^2(\Theta)$ BY AN ANALYSIS IN THE LAB FRAME. NOTE THAT THE TOTAL ANGULAR MOMENTUM IS ABOUT THE INSTANTANEOUS AXIS IN THIS CASE. WHAT IS $\bar{L}(\Theta, \Omega)$ FOR A DISC OF MASS M, RADIUS a.

FINALLY SHOW $\Omega^2 = 4g/a\cos\Theta$.

b) AS THE POINT OF CONTACT MOVES, THE COIN ROLLS WITHOUT SLIPPING. BY WHAT ANGLE DOES THE ORIENTATION OF THE FIGURE ON THE COIN'S FACE IN ONE REVOLUTION OF THE POINT OF CONTACT IN A CIRCLE ON THE TABLE?

SHOW THAT THE FIGURE ORIENTATION PRECESSES SLOWLY, IN THE SAME SENSE AS $\bar{\Omega}$, AND AT AN ANGULAR VELOCITY

$$\Omega(1 - \sin\Theta).$$

AS $\Theta \to 90°$ THE PRECESSION FREQUENCY $\to 0$, EVEN THOUGH $\Omega \to \infty$.

Ph 205 Set 9

6

(3) a)

13. A coin in a horizontal plane is tossed into the air with angular velocity components ω_1 about a diameter through the coin and ω_3 about the principal axis perpendicular to the coin. If ω_3 were equal to zero, the coin would simply spin around its diameter. For ω_3 nonzero, the coin will precess. What is the minimum value of ω_3/ω_1 for which the wobble is such that the same face of the coin is always exposed to an observer looking from above? This is a clever way to cheat at flipping coins!