Babinet's Principle for Electromagnetic Fields

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1 Problem

In optics, Babinet's principle [1] for complementary screens is that the sum of the wave transmitted through a screen (usually considered to be "black" except for its apertures), plus the wave transmitted through the complementary screen, is the same as if no screen were present. An electromagnetic version of this principle was given by Booker [2], who considered perfectly electrically conducting screens and argued that the electromagnetic fields in the case of the complementary screen (labeled with a $'$) that appear in Babinet's principle should be the dual fields $-\mathbf{B}'$ and $\mathbf{E}'$ rather than the nominal fields $\mathbf{E}'$ and $\mathbf{B}'$. That is, if the fields that would exist in the absence of the screen are labeled with the superscript i (for incident), the incident fields in the complementary case are taken to be $\mathbf{E}'^i = -\mathbf{B}^i$ and $\mathbf{B}'^i = \mathbf{E}^i$.[1] Then, the electromagnetic version of Babinet's principle for the fields on the side of the screen away from the sources is,

$$\mathbf{E}_{\text{away}} + \mathbf{B}'_{\text{away}} = \mathbf{E}^i_{\text{away}}, \qquad \mathbf{B}_{\text{away}} - \mathbf{E}'_{\text{away}} = \mathbf{B}^i_{\text{away}}. \tag{1}$$

Also, the fields on the same side of the screen as the sources are related to the scattered/reflected fields $\mathbf{E}^{s_r}$ and $\mathbf{B}^{s_r}$ if the perfectly conducting screen had no apertures by,

$$\mathbf{E}_{\text{same side}} - \mathbf{B}'_{\text{same side}} = \mathbf{E}^{s_r}_{\text{same side}}, \qquad \mathbf{B}^r_{\text{same side}} + \mathbf{E}'^r_{\text{same side}} = \mathbf{B}^{s_r}_{\text{same side}}. \tag{2}$$

Justify these claims.

It suffices to consider the electromagnetic fields incident on the screen as being a plane electromagnetic wave of angular frequency ω, as a general wave field can by synthesized from such plane waves.[2] Note that the fields dual to a plane electromagnetic wave with linear polarization are those obtained on rotating the direction of the polarization by 90°.

2 Solution

2.1 Dual Fields

Maxwell's equations were extended by Heaviside [4, 5, 6], starting in 1885, to include the possibility of magnetic charges and currents,

$$\boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi\rho, \quad \boldsymbol{\nabla}\cdot\mathbf{B} = 4\pi\rho', \quad \boldsymbol{\nabla}\times\mathbf{E} = -\frac{4\pi}{c}\mathbf{J}' - \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad \boldsymbol{\nabla}\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \tag{3}$$

[1] We can also take the dual incident fields to have the opposite signs, in which case eqs. (1)-(2) hold on reversing the signs of the complementary fields.

[2] See, for example, [3].

where ρ and $\mathbf{J}$ are the densities of electrical charge and current, and ρ' and $\mathbf{J}'$ are the densities of hypothetical magnetic charge and current. The form of these equations is such that if $\mathbf{E}$ and $\mathbf{B}$ are solutions when only electrical charges and currents are present ($\rho' = 0 = \mathbf{J}'$), then if only magnetic charges were present, and with values equal to the electrical charges and currents in the previous case, the electromagnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ obey the duality relations,

$$\mathbf{E}' = -\mathbf{B}, \qquad \mathbf{B}' = \mathbf{E}. \tag{4}$$

2.2 No Screen *vs.* a Conducting Plane

A simple example is a plane electromagnetic wave traveling in empty space. There are no scattered fields in this case, and the incident fields are the total fields,

$$\mathbf{E} = \mathbf{E}^i = E_0\, e^{i(kz-\omega t)}\, \hat{\mathbf{x}}, \qquad \mathbf{B} = \mathbf{B}^i = E_0\, e^{i(kz-\omega t)}\, \hat{\mathbf{y}}, \qquad \mathbf{E}^s = 0 = \mathbf{B}^s. \tag{5}$$

A complementary situation is to have the entire plane $z = 0$ occupied by a perfectly electrically conducting screen. Then, the total fields vanish for $z > 0$, and the scattered fields in this regions are equal and opposite to the incident fields,

$$\mathbf{E}'^i = E_0\, e^{i(kz-\omega t)}\, \hat{\mathbf{x}}, \qquad \mathbf{E}'(z > 0) = 0, \qquad \mathbf{E}'^s(z > 0) = -\mathbf{E}'(z > 0), \tag{6}$$

$$\mathbf{B}'^i = E_0\, e^{i(kz-\omega t)}\, \hat{\mathbf{y}}, \qquad \mathbf{B}'(z > 0) = 0, \qquad \mathbf{B}'^s(z > 0) = -\mathbf{B}'(z > 0). \tag{7}$$

The fields (5)-(7) satisfy what we might naïvely expect Babinet's principle to be for electromagnetism,

$$\mathbf{E}(z > 0) + \mathbf{E}'(z > 0) = \mathbf{E}^i(z > 0), \qquad \mathbf{B}(z > 0) + \mathbf{B}'(z > 0) = \mathbf{B}^i(z > 0). \tag{8}$$

However, if we take the incident fields in the complementary case to be the duals of the original incident fields,

$$\mathbf{E}'^i = -E_0\, e^{i(kz-\omega t)}\, \hat{\mathbf{y}}, \qquad \mathbf{E}'(z > 0) = 0, \qquad \mathbf{E}'^s(z > 0) = -\mathbf{E}'(z > 0), \tag{9}$$

$$\mathbf{B}'^i = E_0\, e^{i(kz-\omega t)}\, \hat{\mathbf{x}}, \qquad \mathbf{B}'(z > 0) = 0, \qquad \mathbf{B}'^s(z > 0) = -\mathbf{B}'(z > 0), \tag{10}$$

the fields (5) and (9)-(10) rather trivially satisfy Booker's version of Babinet's principle for electromagnetism,

$$\mathbf{E}(z > 0) + \mathbf{B}'(z > 0) = \mathbf{E}^i(z > 0), \qquad \mathbf{B}(z > 0) - \mathbf{E}'(z > 0) = \mathbf{B}^i(z > 0). \tag{11}$$

This example is too simple to clarify that eq. (11) rather than eq. (8) is the proper statement of Babinet's principle for electromagnetism.

2.3 Solution via Smythe's Diffraction Integrals

Booker [2] gave only a suggestive argument as to why the relations (1)-(2). An attempt at a more detailed argument for the electromagnetic version of Babinet's principle was perhaps first given by Meixner [7, 8], and variants of this argument appear in [9, 10, 11] and in sec. 2.4 below.

In this section we follow the argument of sec. 10.8 of [11], which utilizes certain diffraction integrals due to Smythe [12, 13].[3] The conducting screen S lies in the plane $z = 0$, the waves are incident from $z < 0$, and we use Gaussian units. Then, the fields (with time-dependence $e^{-i\omega t}$) for $z > 0$ can be computed from the electric fields in the apertures of the screen, according to,[4]

$$\mathbf{E}(z > 0) = \frac{1}{2\pi}\boldsymbol{\nabla} \times \oint_{\text{apertures of } S} \hat{\mathbf{z}} \times \mathbf{E}\frac{e^{ikr}}{r}\, d\text{Area}'', \tag{12}$$

$$\mathbf{B}(z > 0) = \frac{1}{2\pi}\boldsymbol{\nabla} \times \oint_{S} \hat{\mathbf{z}} \times \mathbf{B}\frac{e^{ikr}}{r}\, d\text{Area}'', \tag{13}$$

where $r = \sqrt{(x - x'')^2 + (y - y'')^2 + z^2}$, and $\hat{\mathbf{z}} \times \mathbf{E} = 0$ next to the perfectly conducting screen.

We can decompose the total electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ into the sum of "incident" and "scattered" fields,

$$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s, \qquad \mathbf{B} = \mathbf{B}^i + \mathbf{B}^s, \tag{14}$$

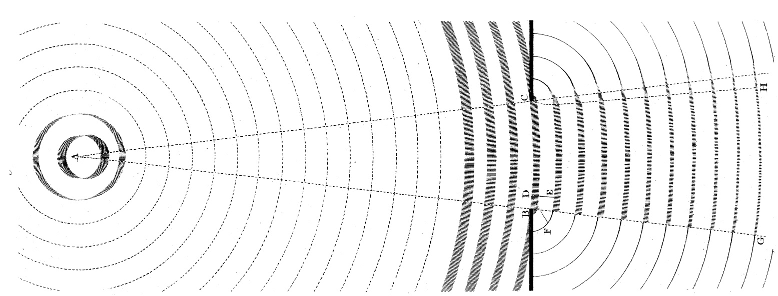
where the incident fields (defined for all z) are those associated with the sources at $z < 0$ in the absence of the screen, and the scattered field are those due only to the charges and currents on the screen. Relations of the forms (12)-(13) hold for both the incident and for the scattered fields.

We now consider the case when the incident fields are the duals of the original incident fields,

$$\mathbf{E}'^i = -\mathbf{B}^i, \qquad \mathbf{B}'^i = \mathbf{E}^i. \tag{15}$$

Only if the screen were made of a hypothetical perfect magnetic conductor in the case of the dual incident fields $\mathbf{E}'^i$ and $\mathbf{B}'^i$ would the scattered fields $\mathbf{E}'^s$ and $\mathbf{B}'^s$ be the duals of

[3]The theory of diffraction has its origin in Huygen's concept of secondary wavelets, as in the left figure below. In 1801, Young [15] supposed that the edges of apertures were sources of Huygen's secondary wavelets, as in the right figure below. In 1815, Fresnel [16] argued that the secondary wavelets have virtual sources over the aperture, rather than on its edges. The mathematical formulation of scalar diffraction theory in terms of diffraction integrals is due to Kirchhoff [17]. The approximate equivalence between Young's and Fresnel's scalar diffraction theory was demonstrated by Rubinowicz [18, 19], who noted that the source of physical scattering is not localized to the mathematical edge of an aperture, but is in the proximity of the edge.



[4]See also Appendix A.3 of [24].

the scattered fields $\mathbf{E}^s$ and $\mathbf{B}^s$. Instead, we suppose the dual fields (15) are incident on the complementary screen S' that is a perfect electrical conductor. The total fields in this case can then be written as,

$$\mathbf{E}' = \mathbf{E}'^i + \mathbf{E}'^s, \qquad \mathbf{B}' = \mathbf{B}'^i + \mathbf{B}'^s. \tag{16}$$

Smythe's integral relations for the dual incident fields and the complementary screen are now,

$$\mathbf{E}'(z > 0) = \frac{1}{2\pi} \boldsymbol{\nabla} \times \oint_{\text{apertures of } S'} \hat{\mathbf{z}} \times \mathbf{E}' \frac{e^{ikr}}{r} \, d\text{Area}'', \tag{17}$$

$$\mathbf{B}'(z > 0) = \frac{1}{2\pi} \boldsymbol{\nabla} \times \oint_{S'} \hat{\mathbf{z}} \times \mathbf{B}' \frac{e^{ikr}}{r} \, d\text{Area}'', \tag{18}$$

since the tangential component of $\mathbf{E}'$ can be nonzero on the plane $z = 0$ only in the apertures of the complementary screen S'.

As mentioned in Appendix A.3 of [24], Smythe's diffraction integrals (12)-(13) and (17)-(18) hold separately for the incident and scattered fields (when the region of integration is the entire plane $z' = 0$). Hence, we can write,

$$\mathbf{E}^s(z > 0) = \frac{1}{2\pi} \boldsymbol{\nabla} \times \oint_{\text{apertures of } S} \hat{\mathbf{z}} \times \mathbf{E}^s \frac{e^{ikr}}{r} \, d\text{Area}'', \tag{19}$$

$$\mathbf{B}^s(z > 0) = \frac{1}{2\pi} \boldsymbol{\nabla} \times \oint_{S} \hat{\mathbf{z}} \times \mathbf{B}^s \frac{e^{ikr}}{r} \, d\text{Area}'', \tag{20}$$

$$\mathbf{E}'^s(z > 0) = \frac{1}{2\pi} \boldsymbol{\nabla} \times \oint_{\text{apertures of } S'} \hat{\mathbf{z}} \times \mathbf{E}'^s \frac{e^{ikr}}{r} \, d\text{Area}'', \tag{21}$$

$$\mathbf{B}'^s(z > 0) = \frac{1}{2\pi} \boldsymbol{\nabla} \times \oint_{S'} \hat{\mathbf{z}} \times \mathbf{B}'^s \frac{e^{ikr}}{r} \, d\text{Area}''. \tag{22}$$

The forms (19)-(22) are mathematically consistent with the scattered fields in the complementary case being the duals of the scattered fields in the original case, but we cannot expect this to be true as the complementary screen is a perfect electrical conductor, not a perfect magnetic conductor.

To go further, it appears necessary that the scattered fields obey the symmetries,

$$\mathbf{E}_x^s(x, y, -z) = \mathbf{E}_x^s(x, y, z), \qquad \mathbf{B}_x^s(x, y, -z) = -\mathbf{B}_x^s(x, y, z), \tag{23}$$

$$\mathbf{E}_y^s(x, y, -z) = \mathbf{E}_y^s(x, y, z), \qquad \mathbf{B}_y^s(x, y, -z) = -\mathbf{B}_y^s(x, y, z), \tag{24}$$

$$\mathbf{E}_z^s(x, y, -z) = -\mathbf{E}_z^s(x, y, z), \qquad \mathbf{B}_z^s(x, y, -z) = \mathbf{B}_z^s(x, y, z), \tag{25}$$

as assumed in all "proofs" of Babinet's principle in the literature. For the symmetries (23)-(25) to hold there must be no currents the flow from one side of the screen to the other, such that the vector potential due to currents on the screen have no z-component, *i.e.*$A_z^s = 0$. As first noted in [20], and reviewed in [21], there can be no currents on the edges of a plane conducting screen as otherwise the magnetic field energy would be infinite in a finite volume surrounding a portion of the edge, and the relations (23)-(24) do hold in general.

According to the (anti)symmetries (23)-(24), the transverse components of the magnetic field vanish in the apertures of the screen (while equal and opposite nonzero transverse magnetic fields can exist close to the two sides of the conductor). In this case, the region of integration in relations (20) and (22) can be restricted to the conductors of the screens, whose locations correspond to the apertures of the complementary screens,

$$\mathbf{B}^s(z>0) \quad = \quad \frac{1}{2\pi}\boldsymbol{\nabla}\times\oint_{\text{apertures of } S'} \hat{\mathbf{z}}\times\mathbf{B}^s\frac{e^{ikr}}{r}\,d\text{Area}'', \tag{26}$$

$$\mathbf{B}'^s(z>0) \quad = \quad \frac{1}{2\pi}\boldsymbol{\nabla}\times\oint_{\text{apertures of } S} \hat{\mathbf{z}}\times\mathbf{B}'^s\frac{e^{ikr}}{r}\,d\text{Area}''. \tag{27}$$

It is claimed in sec. 10.8 of [11] that the similarity of the forms of eqs. (12) and (27), and that of eqs. (17) and (26) permit us to conclude that,[5]

$$\mathbf{E}(z>0) = -\mathbf{B}'^s(z>0), \qquad \mathbf{E}'(z>0) = \mathbf{B}^s(z>0), \tag{28}$$

and so, recalling eq. (15),

$$\mathbf{B}(z>0) = \mathbf{B}^i(z>0) + \mathbf{B}^s(z>0) = -\mathbf{E}'^i(z>0) + \mathbf{E}'(z>0) = \mathbf{E}'^s(z>0), \tag{29}$$

$$\mathbf{B}'(z>0) = \mathbf{B}'^i(z>0) + \mathbf{B}'^s(z>0) = \mathbf{E}^i(z>0) - \mathbf{E}(z>0) = -\mathbf{E}^s(z>0). \tag{30}$$

The total fields $\mathbf{E}'$ and $\mathbf{B}'$ are not the duals of the total fields $\mathbf{E}$ and $\mathbf{B}$, but of the scattered fields $\mathbf{E}^s$ and $\mathbf{B}^s$. If so, we finally obtain Booker's electromagnetic version (1) of Babinet's principle,

$$\mathbf{E}(z>0) + \mathbf{B}'(z>0) = \mathbf{E}(z>0) - \mathbf{E}^s(z>0) = \mathbf{E}^i(z>0), \tag{31}$$

$$\mathbf{B}(z>0) - \mathbf{E}'(z>0) = \mathbf{B}(z>0) - \mathbf{B}^s(z>0) = \mathbf{B}^i(z>0). \tag{32}$$

2.4 Jones' Argument

A slightly different argument is given in sec. 9.3 of [10], following [8].

Jones begins with a justification of the symmetries (23)-(25), and then proceeds assuming that these symmetries hold. He invokes the spirit of the image method for the case of a perfectly electrically conducting plane with no apertures. In the case the fields for $z<0$ are the same as if there were no conducting plane but image charge and current densities existed for $z>0$ according to $\rho^{\text{image}}(x,y,z>0) = -\rho(x,y,z<0)$, $J^{\text{image}}_{x,y}(x,y,z>0) = -J_{x,y}(x,y,z<0)$ and $J^{\text{image}}_z(x,y,z>0) = J_z(x,y,z<0)$. Jones supposes these relations also hold if the conducting plane has apertures.

As noted in sec. 2.3, the symmetries (23)-(25) imply that certain components of the scattered fields vanish in the apertures of the screen at $z=0$, namely,

$$E^s_z = B^s_{x,y} = 0, \qquad E_z = E^i_z, \quad B_{x,y} = B^i_{x,y}, \qquad \text{in the apertures at } z=0. \tag{33}$$

[5]In [11] the complementary incident fields are taken to be $\mathbf{E}'^i = \mathbf{B}^i$ and $\mathbf{B}'^i = -\mathbf{E}^i$ rather than those of eq. (15), which leads to various reversals of signs compared to those in this note.

Likewise, next to the material of the screen we can write,

$$E_z(0^+) + E_z(0^-) = 2E^i_z(0), \quad B_{x,y}(0+) + B_{x,y}(0^-) = 2B^i_{x,y}(0), \quad \text{next to conductor.} \tag{34}$$

The claim is that in the case of the complementary, electrically conducting screen the following fields are solutions to Maxwell's equations,

$$\mathbf{E}'(z) = \begin{cases} -\mathbf{B}(z) + \mathbf{B}^i_\parallel(-z) - B^i_z(-z)\,\hat{\mathbf{z}} & (z \le 0), \\ \mathbf{B}(z) - \mathbf{B}^i(z) = \mathbf{B}^s(z) & (z \ge 0), \end{cases} \tag{35}$$

$$\mathbf{B}'(z) = \begin{cases} \mathbf{E}(z) + \mathbf{E}^i_\parallel(-z) - E^i_z(-z)\,\hat{\mathbf{z}} & (z \le 0), \\ -\mathbf{E}(z) + \mathbf{E}^i(z) = -\mathbf{E}^s(z) & (z \ge 0). \end{cases} \tag{36}$$

By considering the case that the complementary screen is vacuum, for which the total fields are the same as the incident fields, we see that the incident fields are the dual fields given by eq. (15).[6]

Next to the complementary screen the fields (35)-(36) are, noting that the incident fields are continuous at $z = 0$,

$$\mathbf{E}'(z = 0^\pm) = \begin{cases} -\mathbf{B}^s_\parallel(0^-) - (2B^i_z(0) + B^s_z(0^+))\,\hat{\mathbf{z}} & (z = 0^-), \\ \mathbf{B}(0^+) - \mathbf{B}^i(0) = \mathbf{B}^s(0^+) & (z = 0^+), \end{cases} \tag{40}$$

$$\mathbf{B}'(z = 0^\pm) = \begin{cases} 2\mathbf{E}^i_\parallel(0) + \mathbf{E}^s_\parallel(0^-) + E^s_z(0^-)\,\hat{\mathbf{z}} & (z = 0^-), \\ -\mathbf{E}(0^+) + \mathbf{E}^i(0) = -\mathbf{E}^s(0^+) & (z = 0^+). \end{cases} \tag{41}$$

Since the conductor of the complementary screen corresponds to the apertures in the original screen, eq. (33) tells us that the tangential electric field $\mathbf{E}'_\parallel$ and the normal magnetic field B'_z vanish next to the conductor of the complementary screen. Thus, the fields (35)-(36) satisfy the boundary conditions at the screen in the complementary case. It is therefore plausible that these fields are indeed solutions to Maxwell's equations for the dual incident fields (15) and the complementary electrically conducting screen.[7] Then, the representation of Babinet's principle by eqs. (31)-(32) follows at once.

The fields (35)-(36) are defined for $z < 0$, and we find that Babinet's principle for this region is that stated in eq. (2),

$$\mathbf{E}(z < 0) - \mathbf{B}'(z < 0) = -\mathbf{E}^i_\parallel(z > 0) + \mathbf{E}^i_z(z > 0)\,\hat{\mathbf{z}} = \mathbf{E}^{s_r}(z < 0), \tag{42}$$

$$\mathbf{B}(z > 0) + \mathbf{E}'(z < 0) = \mathbf{B}^i_\parallel(z > 0) - \mathbf{B}^i_z(z > 0)\,\hat{\mathbf{z}} = \mathbf{B}^{s_r}(z < 0), \tag{43}$$

[6]In this case the original screen fills the entire plane $z = 0$ and reflects the incident wave. Here, the original fields obey the relations (where z is a positive quantity, and the superscript s_r indicates that the scattered fields are those for total reflection),

$$\mathbf{E}^{s_r}_x(x, y, -z) = -\mathbf{E}^i_x(x, y, z), \qquad \mathbf{B}^{s_r}_x(x, y, -z) = \mathbf{B}^i_x(x, y, z), \tag{37}$$

$$\mathbf{E}^{s_r}_y(x, y, -z) = -\mathbf{E}^i_y(x, y, z), \qquad \mathbf{B}^{s_r}_y(x, y, -z) = \mathbf{B}^i_y(x, y, z), \tag{38}$$

$$\mathbf{E}^{s_r}_z(x, y, -z) = \mathbf{E}^i_z(x, y, z), \qquad \mathbf{B}^{s_r}_z(x, y, -z) = -\mathbf{B}^i_z(x, y, z). \tag{39}$$

[7]Although Jones' argument is not completely compelling, it seems more convincing than that of sec. 2.3.

recalling eqs. (37)-(39) for the scattered fields in the case of total reflection by the electrically conducting plane $z = 0$.[8]

2.5 Solution via an Integral Equation for a Scalar Field Component

The arguments presented in secs. 2.3-4 appear to the authors to have logical gaps, so it may be worthwhile to consider other arguments. Already in 1897 Rayleigh gave an argument involving an integral equation for a scalar component of the electromagnetic fields [27, 28], and came very close to enunciating the electromagnetic version of Babinet's principle. Some 50 years later, Fox [29, 30] (who considered only sound waves) and Copson [31, 32, 33, 34, 35] revived this theme. For the electromagnetic fields to be characterized by a single scalar field component, the problem must be restricted to the (interesting) case of incident fields normal to the screen, and with the apertures in the screen having edges all along one axis, say, the y-axis, and finally with the incident fields having either $\mathbf{E}^i$ or $\mathbf{B}^i$ along the y-axis. Then, following lengthy preliminary arguments, one finds the electromagnetic versions (1)-(2) of Babinet's principle to hold.

2.6 Solution via Analysis of Edge Scattering

A different approach is used in [36], where following the spirit of Young [15] rather than Fresnel [16], the scattered waves are considered to have their source along the edges of the apertures. As remarked on p. 161 of [19], in the approximation of edge currents (which conflicts with the comments on the bottom of p. 4 above), there exist phase changes between the diffracted wave on the two sides of screen, from which the electromagnetic version of Babinet's principle can be deduced [36].

2.7 Examples of Babinet's Principle

The only two cases for which "exact" results have been obtained which obey the electromagnetic form of Babinet's principle are Sommerfeld's famous solution [37, 38, 39] for the diffraction of electromagnetic waves by a conducting half plane, and the transmission of waves through a conducting planar grating [40, 41], although this principle was not noticed in the original studies.

Two laboratory demonstrations of the electromagnetic Babinet's principle are reported in [42, 43].

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