

The Falling Cat and All That: A Short Story of the “Geometric Phase”

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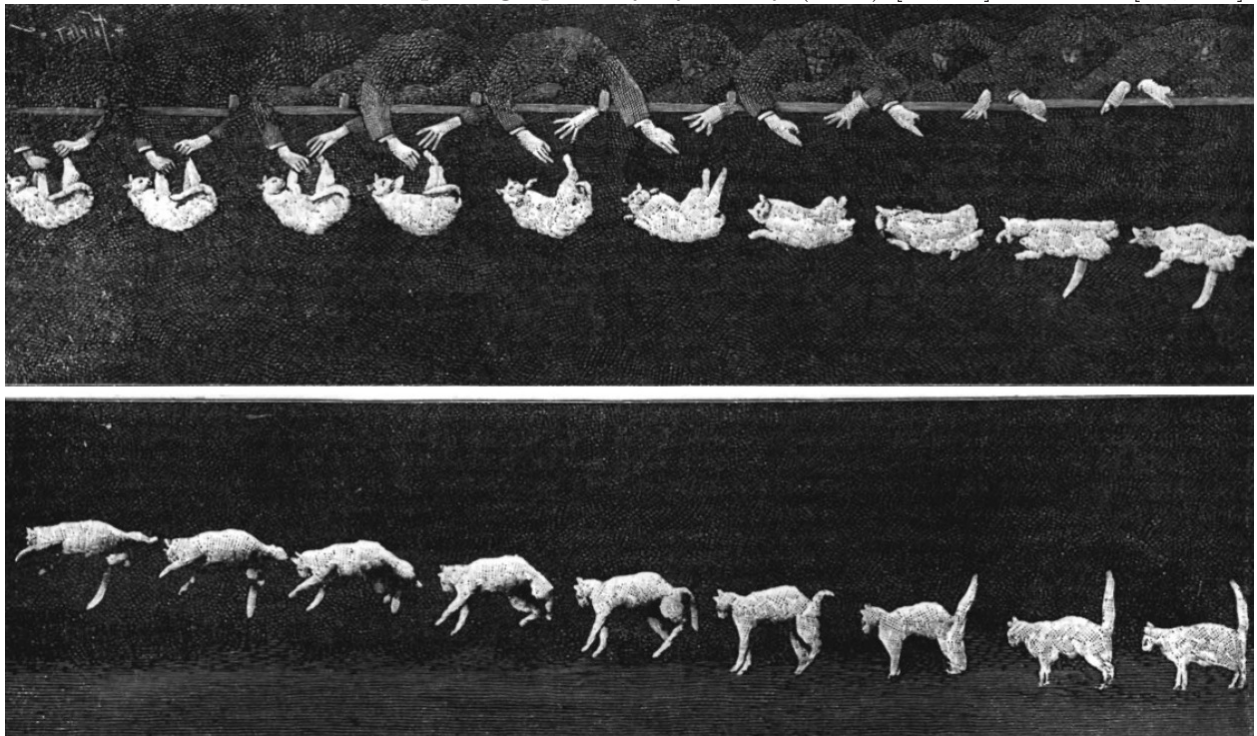
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As an undergraduate at Cambridge U., Maxwell performed “research” with cats [9] that we can now say was an exploration of effects of a “geometric phase”:

There is a tradition in Trinity that when I was here I discovered a method of throwing a cat so as not to light on its feet, and that I used to throw cats out of windows. I had to explain that the proper object of research was to find how quick a cat could turn around, and that the proper method was to let the cat drop on a table or bed from about two inches (*probably two feet*), and even then the cat lands on her feet.

The cat is dropped, upside down, with zero angular momentum, and yet manages to rotate itself by 180° about a horizontal axis and land feet-first.

This was first documented photographically by Marey (1894) [10, 15]. See also [28, 148].



OK. We might well say that the cat somehow changed its phase angle about a horizontal axis by 180° , but how is this “phase” related to a larger context, now often referred to as the concept of the “geometric phase”?¹

¹Subsequent studies of the cat problem without mention of the “geometric phase” include [11]-[15, 17]-[24, 27, 28, 29, 33, 36, 42, 43, 45, 51]-[54, 65, 66, 72, 73, 80, 82, 86, 92, 97, 148, 199, 208, 243, 246, 252, 257, 261, 280]. Discussions that mention the “geometric phase” include [163, 185, 204, 216, 221, 236, 241, 244, 275, 283, 304].

In retrospect, we identify a demonstration by Arago and Fresnel (1819) [2, 3, 313] of changes in the interference pattern in a double-slit experiment on rotations of the plane of polarization of light along its path as an effect of a “geometric phase”. These studies helped confirm Young’s arguments that light is a wave phenomenon, and led Fresnel to the understanding that light is transversely polarized. But Fresnel’s extra phase (*une demi-ondulation*, p. 102 of [3]) was largely ignored,² even after being rediscovered in 1956 by Pancharatnam (1956) [35] (who made no mention of Fresnel). Another example from the 1800’s involving a “geometric phase” is Foucault’s pendulum (1852) [7, 8], but like the falling-cat problem, explanations via classical physics seemed sufficient, and these examples did not directly lead to new insights.

Rather, a key step was taken by Yang only in 1954 [32] (see also [34]) when he inverted arguments by Fock (1926) [25], London (1927) [26] and Pauli (1941) [30] that Schrödinger’s equation for an electric charge e of mass m in electromagnetic fields described by potentials $A_\mu = (V, \mathbf{A})$ can be written, in Gaussian units with c as the speed of light,

$$\frac{(-i\hbar\mathbf{D})^2}{2m}\psi = i\hbar D_0\psi, \quad \text{using the “altered” (covariant) derivative } D_\mu = \partial_\mu - \frac{iq}{\hbar c}A_\mu. \quad (1)$$

This behavior is gauge invariant only if the gauge transformation of the potentials, $A_\mu(x_\nu) \rightarrow A_\mu + \partial_\mu\chi(x_\nu)$, is accompanied by a phase change of the wavefunction, $\psi(x_\nu) \rightarrow e^{-iq\chi(x_\nu)/\hbar c}\psi$. Following Yang, Fock’s argument can be inverted such that the additional requirement of local phase invariance of the form $\psi(x_\nu) \rightarrow e^{-iq\chi(x_\nu)/\hbar c}\psi$ implies the existence of an interaction described by a potential A_μ (and charge q) which satisfies gauge invariance and modifies Schrödinger’s equation via the altered derivative D_μ . This insight has led to the understanding that quantum field theories could/should be “gauge theories”, based on potentials that are not “observable”, but which, together with local phase invariance of the quantum wave function, highly constrain the possible theories.

Soon thereafter (but prior to the coining of the term “geometric phase” in 1983 [79]), further examples that we now associate with a “geometric phase” were published, including [35, 37, 38, 50, 57, 59, 61, 62, 64, 69, 70, 71].

In 1974, Yang noted [55] that gauge theories are associated with a path-dependent “gauge phase factor” that is now often called the “geometric phase”. In a follow-up paper [58], Wu and Yang noted that the Aharonov-Bohm effect [38]³ contains such a “gauge phase factor”, $e^{iq\oint A_\mu dx^\mu/\hbar c}$, and made a connection of this with the mathematical concept of fiber bundles (perhaps the first such in physics).

Simultaneous with Yang’s effort in 1974, Wilson [56] discussed a “weight factor” that involves an exponential of an integral of a gauge potential around a closed loop, which is essentially the same as Yang’s “gauge phase factor”. Such factors are now sometimes called “Wilson loops”.

The term “geometric phase” was introduced by Berry (1983) in an influential paper [79] titled “Quantal phase factors accompanying adiabatic changes”. This has led to an

²Fresnel’s work [1] on conical refraction was discussed by Lloyd [4, 5] and by Hamilton [6], which bears an indirect relation to the issue of “geometric phases”.

³Papers related to the Aharonov-Bohm effect include [31, 39, 40, 41, 44, 46, 47, 48, 49, 50, 63, 67, 68, 69, 70, 74, 76, 77, 78, 83, 87, 94, 96, 100, 101, 142, 152, 184, 186, 207, 231, 239, 245, 250, 255, 259, 260, 263, 264, 281, 284, 295].

impression that the “geometric phase” is a quantum effect, and is only associated with “adiabatic” processes, neither of which are actually requirements for the existence of the “gauge phase factors” anticipated by Wu and Yang (who were briefly mentioned by Berry). Examples of this “phase” were given by Berry, including the Aharonov-Bohm effect, in which it was related to effects along a geometric path. Berry ended the paper with a comment that the concept of “geometric phase” should apply to classical electromagnetic waves (without giving any examples).⁴

A paper by Simon (1983) [81], which cited prepublication results of Berry, introduced the notion that the “geometric phase” is an “(an)holonomy” associated with motion of a system around a closed path.⁵

In 1984, Wilczek became interested in the concept of the “geometric phase”, first in a quantum context [85], but then in 1987, together with Shapere, he discussed its application to the phenomenon of “swimming” at low Reynolds number [118], which is in the realm of classical dynamics. In 1989, they noted [137] that in the falling-cat problem the angle (180°) through which the cat rotates (with zero angular momentum at all times) is a “geometric phase”.⁶

In the 1980’s, Witten [75, 136], Atiyah [121, 135] and others (inspired in part by Schwarz [60, 71, 315]) developed “topological quantum field theory”, also called “Chern-Simons theory”,⁷ which emphasize certain “topological invariants”, some of which are versions of Yang’s “gauge phase factor”, “Wilson loops”, and Berry’s “geometric phase”.⁸ “Topological field theory” has led to extensive, ongoing experimental studies that provide laboratory realizations of various exotic features of the theory.⁹

In 2002, Wisdom noted [218] that in curved space (and curved spacetime), but not in Euclidean/flat space, a deformable object can have net motion while its “linear” momentum remains zero at all times. In examples where this is accomplished by cyclic motion, a “geometric phase” is involved. Discussions of this include [232, 236, 238, 248, 251], which point out that in curved space the appealing notion of a center of mass is not well defined.

⁴Berry has since written extensively on the “geometric phase”, including [88, 112, 120, 122, 140, 162, 168, 180, 181, 195, 200, 230, 258, 265, 267, 306, 307].

⁵Later papers by others on “geometric phases” in quantum systems include:

General quantum systems: [89, 91, 93, 95, 106, 114, 116, 117, 123, 127, 130, 131, 132, 133, 147, 153, 156, 158, 159, 160, 166, 171, 173, 174, 176, 179, 190, 191, 197, 202, 206, 210, 224, 228, 229, 240, 266, 273, 285, 291, 312, 315, 320].

Optical systems: [84, 102, 103, 104, 105, 113, 125, 128, 134, 143, 145, 154, 164, 170, 172, 175, 189, 196, 209, 211, 212, 213, 214, 219, 223, 226, 227, 234, 235, 242, 247, 249, 253, 256, 262, 268, 269, 271, 272, 274, 277, 282, 286, 290, 293, 294, 296, 298, 300, 301, 302, 303, 305, 308, 313, 316, 317, 321, 322].

Systems with spin-1/2: [108, 110, 114, 115, 215, 217, 220, 222, 224, 270, 278, 292, 311].

Chemical systems: [119, 167, 169, 177, 182, 192, 201, 203, 289, 314].

Other gauge systems: [98, 107, 187, 299].

⁶Other works on these themes by Shapere and Wilczek include [138, 139, 144, 150, 319].

Works by others on “geometric phases” in classical physics include [90, 108, 110, 124, 141, 146, 149, 151, 155, 161, 163, 165, 176, 178, 225, 197, 198, 237, 251, 254, 297],

with those related to locomotion including [157, 163, 194, 205, 279, 323, 324].

⁷J.H. Simons is not related to B. Simon.

⁸The term “topological” has the implication of independence of shape/path, while the “geometric phase” is related to path-dependent effects.

⁹See, for example, a review by J.E. Moore, http://kirkmcd.princeton.edu/examples/CM/moore_15.pdf

Recently, an experimental demonstration has been made [318] that a “free”, deformable object on the surface of a sphere can have motion which results in a change of its position while returning to its original shape, with zero linear momentum at all times (with respect to the surface of the sphere).¹⁰

Longer reviews of this topic include [150, 156, 179, 309].

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¹⁰A person swimming on the surface of a lake or ocean is moving on a “sphere”, *i.e.*, a curved space, but this motion is not “free”, due to the viscosity of the water, and hence ordinary swimming does not directly illustrate the significance of a “geometric phase”. Zeb Rocklin (private communication) estimates that part of the net displacement per stroke with relative motions of order 1 meter which is associated with a “geometric phase” (and zero linear momentum at all times with respect to the 2-d curved space defined by the surface of the water) could be about 10^{-12} m, the size of a large atomic nucleus.

- http://kirkmcd.princeton.edu/examples/mechanics/foucault_cras_32_135_51.pdf
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