Conversions of D and H between SI and Gaussian Units

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1 Problem

The electric displacement field **D** has dimensions charge/area and the magnetic field strength **H** has dimensions current/length. In the SI system the units of **D** are C(oulomb)/m^2 and the units of **H** are A/m, while in the Gaussian (cgs) system the units of **D** are sC(statcoulomb)/cm^2 and the units of **H** are Oe (oersteds) where[1,2,3]

$$1 \text{ Oe} = c_{\rm cgs} \text{ sA(statampere)/cm } (= 1 \text{ abampere/cm in the EMU system}), \tag{1}$$

and c is the speed of light in vacuum. The conversions of coulombs to statcoulombs, of amperes to statamperes, are that [1, 10, 11],

$$1 \text{ C} = \frac{c_{\rm cgs}}{10} \text{ sC} \approx 3 \times 10^9 \text{ sC}, \qquad \text{and} \qquad 1 \text{ A} = \frac{c_{\rm cgs}}{10} \text{ sA} \approx 3 \times 10^9 \text{ sA}. \tag{2}$$

The conversion of **D** is that [10, 11],

$$D_{\rm SI} = 1 \text{ C/m}^2 = D_{\rm cgs} = 4\pi c_{\rm cgs} \times 10^{-5} \text{ sC/cm}^2 \approx 12\pi \times 10^5 \text{ sC/cm}^2, \tag{3}$$

and the conversion of **H** is that [1, 10, 11],

$$H_{\rm SI} = 1 \text{ A/m} = H_{\rm cgs} = 4\pi \times 10^{-3} \text{ Oe} = 4\pi c_{\rm cgs} \times 10^{-3} \text{ sA/cm} \approx 12\pi \times 10^7 \text{ sA/cm}. \tag{4}$$

However, from eq. (2) we find the conversions,

$$1 \text{ C/m}^2 = c_{\rm cgs} \times 10^{-5} \text{ sC/cm}^2, \qquad \text{and} \qquad 1 \text{ A/m} = c_{\rm cgs} \times 10^{-3} \text{ sA/cm}. \tag{5}$$

Could it be that the conversions (3)-(4) erroneously include factors of 4π?

This problem was suggested by Neal Carron.

[1]Strictly, the dimensions of $H_{\rm cgs}$ follow from Ampère's law [1], $\boldsymbol{\nabla} \times \mathbf{H}_{\rm cgs} = 4\pi I_{\rm cgs}/c_{\rm cgs}$, as current/(length · velocity), *i.e.*, sA·s/cm^2. Since 1 oersted is not 1 sA·s/cm^2, but c_{cgs} times this, it is convenient to consider that the dimensions, cm/s, of $c_{\rm cgs}$ cancel a factor of s/cm in the dimensions of the oersted, leading to eq. (1). The complication due to the factors of $c_{\rm cgs}$ could be avoided by consideration of the EMU rather Gaussian system of units, but the issue of "extra" factors of 4π remain for this case (and for EMU units) as well.

[2]The oersted is sometimes defined in the EMU system as $1/4\pi$ abamp-turns/cm, meaning that a long solenoid of azimuthal current density $1/4\pi$ abamp-turns/cm produces $H_{\rm EMU} = 1$ Oe according to Ampère's law, $\boldsymbol{\nabla} \times \mathbf{H}_{\rm EMU} = 4\pi \mathbf{J}$ in these units [1]. See, for example, [2], where the preceding is expressed somewhat indirectly. Equivalently, a long wire with current $I = 0.5$ abA produces $H_{\rm EMU} = 2I/r = 1$ Oe $= 1$ abA/cm at radius $r = 1$ cm, and a loop of radius $r = 1$ cm with current $I = 1/2\pi$ abA produces $H_{\rm EMU} = 2\pi I/r = 1$ Oe $= 1$ abA/cm at its center. *One should not misread abamp-turns/cm as abamp/cm to infer that 1 Oe = $(1/4\pi)$ abamp/cm.*

[3]For a classic discussion of dimensions and units, see [3]. For amusing commentary on the oersted, see [4]. Early discussions of dimensions and units in electrodynamics include chap. X of Maxwell's Treatise [5], as well as contributions by Clausius [6], Helmholtz [7], Hertz [8] and Rücker [9].

2 Solution

2.1 Conversions of E and B

We first note that the conversions of the electric field **E** and the magnetic field (or induction or flux density) **B** do not exhibit any "extra" factors of 4π as found above for **D** and **H**.

The conversion of **E** is that [1, 10, 11],

$$E_{\rm SI} = 1 \text{ V/m} = E_{\rm cgs} = \frac{10^6}{c_{\rm cgs}} \text{ sV/cm}, \tag{6}$$

which is consistent with the conversion from volts to statvolts (sV) [1, 11],

$$1 \text{ V} = \frac{10^8}{c_{\rm cgs}} \text{ sV} \approx \frac{1}{300} \text{ sV}, \tag{7}$$

The conversion of **B** is that [1, 10, 11],

$$B_{\rm SI} = 1 \text{ T(tesla)} = B_{\rm cgs} = 10^4 \text{ G} = \frac{10^4}{c_{\rm cgs}} \text{ sA/cm}, \tag{8}$$

where 1 G(gauss) $= c_{\rm cgs}$ sA/cm = 1 Oe. The check the consistency of eq. (8) we note that a tesla is not a primary SI unit. Using, for example, the Lorentz force law, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}_{\rm SI}$, we see that a magnetic field $B_{\rm SI}$ has dimensions of force/(charge · velocity) = force/(current · distance), *i.e.*, kg/A·s^2. In Gaussian units, the force law includes a factor of $1/c_{\rm cgs}$, so the dimensions of $B_{\rm cgs}$ can be considered as $(1/c_{\rm cgs})$ g/sA·s^2 as well as a gauss, which is also equivalent to $(1/c_{\rm cgs})$ sA/cm. Then, the conversions,

$$1 \text{ T} = 1 \ \frac{\text{kg}}{\text{A} \cdot \text{s}^2} = \frac{10^3 \text{ g}}{c_{\rm cgs}/10 \text{ sA} \cdot \text{s}^2} = \frac{10^4}{c_{\rm cgs}} \frac{\text{g}}{\text{sA} \cdot \text{s}^2} = \frac{10^4}{c_{\rm cgs}} \text{sA/cm} = 10^4 \text{ G}, \tag{9}$$

give a confirmation of eq. (8).

2.2 Conversions of D and H

In secs. 2.2 and 2.3 we consider media of unit relative permittivity and permeability, such that $\mathbf{D}_{\rm SI} = \epsilon_0 \mathbf{E}_{\rm SI}$ and $\mathbf{H}_{\rm S} = \mathbf{H}_{\rm SI}/\mu_0$, where $\mu_0 = 4\pi \times 10^{-7}$ kg·m/A^2·s^2 $= 1/\epsilon_0 c_{\rm SI}^2$, while $\mathbf{D}_{\rm cgs} = \mathbf{E}_{\rm cgs}$ and $\mathbf{B}_{\rm cgs} = \mathbf{H}_{\rm cgs}$.

The conversions of **D** and **H** differ from those for **E** and **B** by the factors of $\epsilon_0 = 1/c_{\rm SI}^2 \mu_0 = 10^4/c_{\rm cgs}^2 \mu_0 = 10^{11}/4\pi c_{\rm cgs}^2$ and $1/\mu_0 = 10^7/4\pi$. It is these factors of 4π that make the conversions of **D** and **H** differ from the conversions of their units.

For example,

$$H_{\rm cgs} = 1 \text{ Oe} = B_{\rm cgs} = 1 \text{ G} = B_{\rm SI} = 10^{-4} \text{ T}, \tag{10}$$

and hence,

$$H_{\rm SI} = \frac{B_{\rm SI}}{\mu_0} = \frac{10^3}{4\pi} \text{ A/m} = H_{\rm cgs} = 1 \text{ Oe}, \tag{11}$$

which confirms the conversion (4).

Similarly,[4]

$$D_{\rm cgs} = 1\ {\rm sC/cm^2} = E_{\rm cgs} = 1\ {\rm sV/cm} = E_{\rm SI} = c_{\rm cgs} \times 10^{-6}\ {\rm V/m}, \tag{12}$$

and hence,

$$D_{\rm SI} = \epsilon_0 E_{\rm SI} = \frac{10^{11}}{4\pi c_{\rm cgs}^2} c_{\rm cgs} \times 10^{-6} = \frac{10^5}{4\pi c_{\rm cgs}}\ {\rm C/m^2} = D_{\rm cgs} = 1\ {\rm sC/cm^2}, \tag{13}$$

which confirms the conversion (3).

2.3 Comments

It remains unusual that the conversions (3)-(4) include a factor of 4π times the factors associated with the conversions of the units. This occurs because **D** and **E**, and also **B** and **H**, have the same dimensions in Gaussian units[5] but different dimensions in SI units (in that electrical current is a separate unit in the SI system but a secondary unit in the Gaussian system[6,7,8]). The relations $\mathbf{D} = \epsilon_0 \mathbf{E} = \mathbf{E}/\mu_0 c_{\rm SI}^2$ and $\mathbf{H} = \mathbf{B}/\mu_0$ in the SI system (in vacuum) involve factors of 4π not present in the corresponding relations ($\mathbf{D} = \mathbf{E}$ and $\mathbf{H} = \mathbf{B}$) in Gaussian units, so that while the conversions (6) and (8) of **E** and **B** match the conversions of their units, this cannot also hold for the conversions (3)-(4) of **D** and **H**.[9,10]

[4] The dimensions in Gaussian units of the electric potential are sV = sC/cm = $\rm g^{1/2}\ cm^{1/2}/s$.

[5] As remarked in footnote 1, the dimensions of **H** (and of **B**) in Gaussian units are sA·s/cm^2. Since the current in sA is charge/second, sC/s, the dimensions of **B** and **H** are also sC/cm^2 = sV/cm, the dimensions of **D** and **E** (in Gaussian units). And, these dimensions are also those of the gauss and the oersted. Of course, in terms of the primary units of the Gaussian system, all these equivalent dimensions are equal to $\rm g^{1/2}/cm^{1/2}{\cdot}s$.

[6] "Extra" factors of 4π also occur in the conversions of **D** and **H** between the SI system and the so-called ESU and EMU systems of units, but not in conversions among the ESU, EMU and Gaussian systems.

[7] This issue is addressed, for example, on p. 431 of [12] where the different forms of Maxwell's equations for **D**, **B** and **H** in Gaussian and SI units (p. 427) are characterized by saying that these fields are "different physical quantities" in the two systems of units, with the implication that the conversion factors are not simply related to the conversions of the various units. Apparently, this view was adopted at an international convention on electrical units in 1930, as reported in [13, 14]. For a dissenting view, see [15].

[8] Conversions between quantities with different dimensions in different systems of units are considered somewhat abstractly in [16]. See also the much earlier paper [17].

[9] The SI system was proposed by Giorgi [18] as a "rationalized" system of units (first advocated by Heaviside [19]) in which factors of 4π would not appear explicitly in Maxwell's equations. See also [2, 14, 20, 21]. However, factors of 4π remain present in these equations for **B** and **E** (contrary to the vision of Heaviside [19]) via the definitions that $\mu_0 = 4\pi \times 10^{-7}$ and $\epsilon_0 = 1/\mu_0 c_{\rm SI}^2$. The resulting peculiar relation between **H** (and **D**) in "rationalized" (SI) and unrationalized (cgs) units has been discussed in [22, 23, 24].

[10] A fourth cgs system (in addition to Gaussian, EMU and ESU) is defined by eqs. (1-153), $\mathbf{B}_4 = \mu_0 \mathbf{H}_4$; (1-172), $\nabla \times \mathbf{B}_4 = \mu_0 \mathbf{J}$; and (1-178), $\nabla \times \mathbf{H}_4 = \mathbf{J}$, of Part 4 (pp. 4–11-14) of [25] in which the statement after eq. (1-153) that $4\pi\mu_0 = 1$ implies that the magnetic fields $\mathbf{B}_4$ and $\mathbf{H}_4$ are related by $\mathbf{B}_4 = \mathbf{B}_{\rm EMU}/(4\pi)^2$ while $\mathbf{H}_4 = \mathbf{H}_{\rm EMU}/4\pi = \mathbf{B}_{\rm EMU}/4\pi = 4\pi \mathbf{B}_4$, which leads to the unusual convention (p. A-12 of [25]) that the oersted is given by 1 Oe$_4$ = $(1/4\pi)$ abA/cm. However, the text on p. A-12 of [25] claims that $\mu_0 = 4\pi$, in which case the equations listed above lead to a fifth version of cgs units where $\mathbf{B}_5 = \mathbf{B}_{\rm EMU} = (4\pi)^2 \mathbf{B}_4$, while $\mathbf{H}_5 = \mathbf{H}_4 = \mathbf{H}_{\rm EMU}/4\pi = \mathbf{B}_{\rm EMU}/4\pi = \mathbf{B}_5/4\pi$.

2.4 Conversions of ϵ_r, $\mathbf{P}$, $\chi_{\rm E}$, μ_r, $\mathbf{M}$ and $\chi_{\rm M}$ (Aug. 4, 2020)

For linear media, we write,

$$\mathbf{D}_{\rm SI} = \epsilon_r \epsilon_0 \mathbf{E}_{\rm SI}, \qquad \mathbf{D}_{\rm cgs} = \epsilon_r \mathbf{E}_{\rm cgs}, \qquad \mathbf{B}_{\rm SI} = \mu_r \mu_0 \mathbf{H}_{\rm SI}, \qquad \mathbf{B}_{\rm cgs} = \mu_r \mathbf{H}_{\rm cgs}, \tag{14}$$

where the relative permittivity ϵ_r and the relative permeability μ_r have the same (dimensionless) values in both SI and cgs units.

We also have that the fields $\mathbf{D}$ and $\mathbf{B}$ are related to $\mathbf{E}$ and $\mathbf{H}$ via the densities $\mathbf{P}$ of electric polarization and $\mathbf{M}$ of magnetization by,

$$\mathbf{D}_{\rm SI} = \epsilon_0 \mathbf{E}_{\rm SI} + \mathbf{P}_{\rm SI}, \qquad \mathbf{D}_{\rm cgs} = \mathbf{E}_{\rm cgs} + 4\pi \mathbf{P}_{\rm cgs}, \tag{15}$$

$$\mathbf{B}_{\rm SI} = \mu_0 (\mathbf{H}_{\rm SI} + \mathbf{M}_{\rm SI}), \qquad \mathbf{B}_{\rm cgs} = \mathbf{H}_{\rm cgs} + 4\pi \mathbf{M}_{\rm cgs}, \tag{16}$$

and also that, by definition of the electric and magnetic susceptibilities, $\chi_{\rm E}$ and $\chi_{\rm M}$,

$$\mathbf{P}_{\rm SI} = \chi_{\rm E,SI} \epsilon_0 \mathbf{E}_{\rm SI}, \qquad \mathbf{P}_{\rm cgs} = \chi_{\rm E,cgs} \mathbf{E}_{\rm cgs}, \tag{17}$$

while the form,

$$\mathbf{M} = \chi_{\rm M} \mathbf{H}, \tag{18}$$

holds in both SI and cgs units.

Combining eqs. (14), (15) and (17), we see that,

$$\mathbf{D}_{\rm SI} = (1 + \chi_{\rm E,SI}) \epsilon_0 \mathbf{E}_{\rm SI}, \qquad \text{while} \qquad \mathbf{D}_{\rm cgs} = (1 + 4\pi \chi_{\rm E,cgs}) \mathbf{E}_{\rm cgs}, \tag{19}$$

which implies that,

$$\chi_{\rm E,SI} = \epsilon_r - 1, \qquad \chi_{\rm E,cgs} = \frac{\epsilon_r - 1}{4\pi}, \qquad \chi_{\rm E,SI} = 4\pi \chi_{\rm E,cgs}. \tag{20}$$

Similarly, combining eqs. (14), (16) amd (18), we see that,

$$\mathbf{B}_{\rm SI} = (1 + \chi_{\rm M,SI}) \mu_0 \mathbf{H}_{\rm SI}, \qquad \text{while} \qquad \mathbf{B}_{\rm cgs} = (1 + 4\pi \chi_{\rm M,cgs}) \mathbf{H}_{\rm cgs}, \tag{21}$$

which implies that,

$$\chi_{\rm M,SI} = \mu_r - 1, \qquad \chi_{\rm M,cgs} = \frac{\mu_r - 1}{4\pi}, \qquad \chi_{\rm M,SI} = 4\pi \chi_{\rm M,cgs}. \tag{22}$$

Finally, inspection of eq. (15) indicates that the conversion of $\mathbf{P}$ from SI to cgs units is like that of $\mathbf{D}$, but divided by 4π,

$$P_{\rm SI} = 1\ {\rm C/m^2} = P_{\rm cgs} = c_{\rm cgs} \times 10^{-5}\ {\rm sC/cm^2} \approx 3 \times 10^5\ {\rm sC/cm^2}, \tag{23}$$

recalling eq. (3). Similarly, inspection of eq. (16) indicates that the conversion of $\mathbf{M}$ from SI to cgs units is,

$$M_{\rm SI} = 1\ {\rm A/m} = M_{\rm cgs} = \frac{\mu_0}{4\pi} \frac{B_{\rm SI}}{B_{\rm cgs}} = 10^{-3}\ {\rm sA/cm}. \tag{24}$$

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