Is Particle Decay Exponential in Time?

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The first measurement of the time dependence of radioactive decay was by Rutherford (1900) [1], who found it to be exponential over 4 half lives ($\tau_{1/2} = 0.69/\Gamma$ in $N(t) = N_0\, e^{-\Gamma t}$ where Γ is the decay rate, assumed to be independent of time). Subsequent studies [2, 29, 41, 34, 63, 65, 74, 79] have shown exponential behavior out to 50 half lives, and for times greater than $10^{-10}\tau_{1/2}$ [71, 72].[1] That is, exponential decay is an excellent description for decays of particles at "rest".[2]

While approximate quantum theories are consistent with exponential decay [3, 4, 5, 6, 7], there is an extensive literature to the effect that quantum theory predicts nonexponential decay at very early and very late times [9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 69, 70, 73, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 89, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 104, 105, 107, 108, 109].[3,4] It was estimated in [52] that exponential decay is a good approximation for $10^{-14}\tau_{1/2} < t < 130\,\tau_{1/2}$ in decays of "elementary particles".

It was perhaps first noted in [45] that interactions with the environment can affect the decay of a particle,[5,6] particularly if it is frequently observed. This was called the quantum Zeno effect in [50],[7,8] whose first experimental observation was reported in [68, 92]. A related example is the observation of a nonexponential survival probability at times less than $0.2\tau_{1/2}$ for sodium atoms in an optical trap whose position is accelerated [78]. Much earlier were measurements [23, 24, 26] of nonexponential optical decay of nuclei when filters limited the observed spectral region. Another example, studied in great detail, is the decay

[1]The apparent observation of exponential decay modulated by oscillations of period 7 sec in hydrogen-like ions [90] (the "GSI anomaly") was not confirmed in a later, more precise experiment [106].

[2]There exists one report [88] (apparently unconfirmed) of non-exponential decay in two systems of fluorescent molecules. An issue is that the decay photons in these 1-photon-decay systems can re-excite the molecules, which leads to deviations from simple exponential decay of the system. That is, the behavior of a collection of states whose decay emits only a single particle can be different from the behavior of a single such state.

[3]It was perhaps first noted in [33] that a state which decays must have a spectrum of energies, and if this spectrum has a lower bound, then the decay is not exponential at large times.

[4]It was claimed in [32] (and many subsequent papers) that the survival probability of a decaying particle is a nonmonotonically decreasing function of time (see fig. 2 of [32]), which seems unreasonable to me.

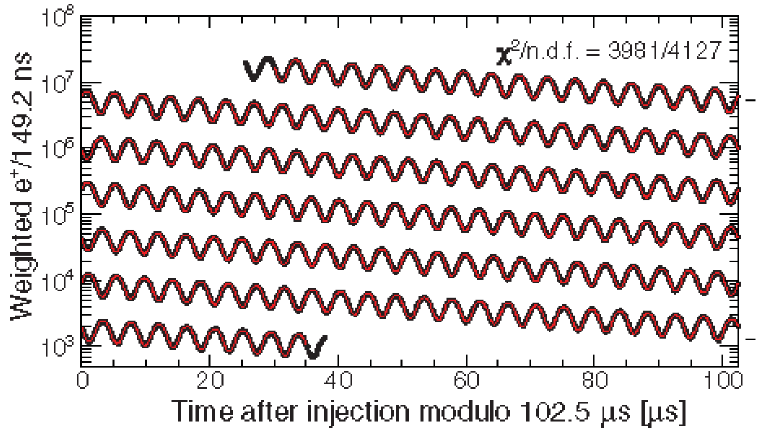
[5]Free neutrons decay (first measured in [8]), but neutrons in stable nuclei and neutron stars do not. The lifetime of neutrons in a magnetic bottle is about 0.1% less than that of neutrons in a beam (see, for example, [103]), which may be due to depletion of the neutrons in the bottle when they form deuterium during collisions (which are more probable in a bottle than in a beam.

[6]Claims that the decay rates of radioactive nuclei on Earth are somehow influenced by the Sun are discussed in [95].

[7]A closely related behavior has been called the quantum watchdog effect [44, 49, 58]. This concept was anticipated by Alan Turing shortly before his death in 1954, as recorded on p. 7 of a letter that year by his student R.O. Gandy to R.H.A. Newman [11].

[8]The case that the short-time decay rate is enhanced by frequent observation was called the anti-Zeno effect in [67].

of transversely spin-polarized muons in a storage ring, where the time distribution of decay electrons observed in a limited solid angle exhibits oscillations about exponential decay over 14 half lives ($\tau_{1/2} = 45$ μs in the experiment), as shown below, Fig. 2 of [110].



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http://kirkmcd.princeton.edu/examples/QM/gandy_june54.pdf
also in R.O. Gandy and C.E.M. Yates (eds.), *The Collected Works of A.M. Turing: Mathematical Logic* (North-Holland, 2001),
http://kirkmcd.princeton.edu/examples/QM/gandy_01.pdf
A slightly more serious contribution to quantum mechanics was "The Turing Paradox"; It is easy to show using standard theory that if a system starts in an eigenstate of some observable, and measurements are made of that observable N times a second, then, even if the state is not a stationary one, the probability that the system will be in the same state after, say, 1 second, tends to one as N tends to infinity; i.e., that continual observation will prevent motion. Alan and I tackled one or two theoretical physicists with this, and they rather pooh-poohed it by saying that continual observation is not possible. But there is nothing in the standard books (e.g., Dirac's) to this effect, so that at least the paradox shows up an inadequacy of Quantum Theory as usually presented.

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