Dineutrons

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1 Problem

1. The dineutron, nn, is a partner with pp and np states in a triplet of isospin,[1] while there also exists an np state (deuteron) that is an isosinglet. The dinucleon isotriplet states do not appear to be bound, while the isosinglet deuteron is (with binding energy 2.23 MeV).[2,3] Use a simplified one-pion-exchange model that the nuclear force is entirely due to exchanges of a single π meson, and that the operator $g^2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ characterizes the charge independence of this interaction.[4] Here g is a coupling constant, and $\boldsymbol{\tau}$ is the isospin-1 operator (because pions form an $I = 1$ multiplet). That is, ignore electromagnetic effects and spin-dependent effects. Note that,

$$\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = \frac{1}{2}\left(\boldsymbol{\tau}^2 - \boldsymbol{\tau}_1^2 - \boldsymbol{\tau}_2^2\right). \tag{1}$$

   Show that the isospin structure of the interaction matrix element implies that the isosinglet deuteron is bound while the isotriplet, (pp, np, nn), is not.[5]

2. Discuss the possibility of a weakly bound dineutron state due to the interaction of the magnetic moments of the neutrons.

2 Solution

2.1 Isopin Relations in Meson Theory of the Nuclear Force

Following the hint in eq. (1),

$$(\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)^2 = \boldsymbol{\tau}^2 = \boldsymbol{\tau}_1^2 + \boldsymbol{\tau}_2^2 + 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \tag{2}$$

$$\langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left(\left\langle \boldsymbol{\tau}^2 \right\rangle - \left\langle \boldsymbol{\tau}_1^2 \right\rangle - \left\langle \boldsymbol{\tau}_2^2 \right\rangle\right) = \frac{1}{2}[(\tau(\tau+1) - (\tau_1(\tau_1+1) - (\tau_2(\tau_2+1)], \tag{3}$$

noting that the expectation value of the (iso)spin operator $\boldsymbol{\tau}^2$ is $\tau(\tau+1)$. Thus, the strength of the $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ interaction is the same for all members of a multiplet of total isospin τ.

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[1] Following Chadwick's discovery of the neutron in 1932 [1], Heisenberg introduced the concept of isospin, wherein he postulated that the proton and neutron are members of an isodoublet $N = (p, n)$ where $p = |I = \frac{1}{2}, I_3 = \frac{1}{2}\rangle$ and $n = |I = \frac{1}{2}, I_3 = -\frac{1}{2}\rangle$, with mathematical properties as for a spin-1/2 doublet.

[2] The deuteron bound state was discovered in 1931 by Urey [3], six months before Chadwick discovered the neutron [1].

[3] Evidence for an unbound dineutron state has been reported in [4].

[4] If this interaction is represented by a Feynman diagram with single pion exchange, then each of the $NN\pi$ vertices has strength $g\boldsymbol{\tau}$.

[5] See [5] for an extension of this problem to all 64 possible pairs of baryons in the basic octet: n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^-, Ξ^0.

In the present problem both 1 and 2 represent members of the $I = 1/2$ nucleon isodoublet N. Two such nucleons (with $\tau_1 = \tau_2 = 1/2$) couple to $\tau = 0$ and $\tau = 1$ dinucleon multiplets,

$$\tau = 0: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[0(0+1) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\right] = -\frac{3}{4}, \tag{4}$$

$$\tau = 1: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[1(1+1) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\right] = \frac{1}{4}. \tag{5}$$

This model suggests that the (antisymmetric) isosinglet state $(pn - np)/\sqrt{2}$ (deuteron) would be bound (with relative binding energy 3/4), while (symmetric) isotriplet states pp (diproton), $(pn + np)/\sqrt{2}$ (excited deuteron) and nn (dineutron) would be unbound (by 1/4 relative energy units). As the binding energy of the deuteron is 2.23 MeV, the isotriplet states are predicted to be unbound by about 700 keV (with the diproton being the least bound, taking electromagnetic energy into account).[6]

2.2 Bound State via Interaction of Neutron Magnetic Moments

The neutron has a magnetic moment of magnitude,

$$\mu = \Gamma \frac{e\hbar}{mc}, \tag{6}$$

where $\Gamma = -1.79$, $-e$ is the charge of the electron and m is the mass of the neutron. If two neutrons are separated by distance r larger than their diameters ($\approx 2 \times 10^{-15}$ cm), their magnetic interaction energy is,

$$U = -\boldsymbol{\mu}_1 \cdot \mathbf{B}_2 = -\boldsymbol{\mu}_2 \cdot \mathbf{B}_1 = \frac{\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2 - 3(\boldsymbol{\mu}_1 \cdot \hat{\mathbf{r}})(\boldsymbol{\mu}_2 \cdot \hat{\mathbf{r}})}{r^3}, \tag{7}$$

in Gaussian units. The interaction energy is minimal for antiparallel magnetic moments, both perpendicular to the line of centers of the neutrons, $\boldsymbol{\mu}_1 = -\boldsymbol{\mu}_2 \perp \hat{\mathbf{r}}$, with,

$$U_{\rm min} = -\frac{\Gamma^2}{r^3}\left(\frac{e\hbar}{mc}\right)^2. \tag{8}$$

The (attractive) force between the neutrons in this case is,

$$\mathbf{F} = -\boldsymbol{\nabla} U = -\frac{3\Gamma^2}{r^4}\left(\frac{e\hbar}{mc}\right)^2 \hat{\mathbf{r}}. \tag{9}$$

If the two neutrons are in a circular orbit of radius $R = r/2$, their velocity v obeys,

$$\frac{mv^2}{R} = F = \frac{3\Gamma^2}{16R^4}\left(\frac{e\hbar}{mc}\right)^2, \qquad {\rm KE}_{\rm total} = mv^2 = \frac{3\Gamma^2}{16R^3}\left(\frac{e\hbar}{mc}\right)^2, \tag{10}$$

[6] In 1971 by Dyson [6] commented that if nuclear force were strong enough that the diproton were bound, the Universe would be very different and would not support life as we know it. Such comments are incorporated in the notion of "fine tuning" of the fundamental constants of Nature. Of course, in the simple one-pion-exchange model considered here, the diproton is unbound for any value of the nuclear coupling constant g.

so the total energy of the system is positive, and no magnetically bound state (with a circular orbit) exists,

$$E = \mathrm{KE} + U_{\min} = \frac{\Gamma^2}{8R^3}\left(\frac{e\hbar}{mc}\right)^2 > 0. \tag{11}$$

Indeed, this is an example of a more general result that for central potentials of the form $U(r) = -\alpha/r^n$, stable circular orbits (bound states) exist only if $n < 2$ (as is satisfied by the Coulomb potential).

A curiosity is that there does exist a bounded motion for the central potential (8), but eventually the two neutrons would collide, such that it is not appropriate to call this motion a "bound state" in the usual sense.

To characterize this possibility, we note that for a central potential $U(R)$ the orbit lies in a plane and the component L of angular momentum perpendicular to that plane is constant. The equation of motion can be expressed in terms of the radial coordinate only via the effective potential,[7]

$$U_{\rm eff} = U + \frac{L^2}{mR^2} = -\frac{\alpha}{R^3} + \frac{L^2}{mR^2} \qquad \text{with} \qquad \alpha = \frac{\Gamma^2}{8}\left(\frac{e\hbar}{mc}\right)^2, \tag{12}$$

for the case of two equal-mass neutrons. This effective potential peaks at $R_0 = 3\alpha m/2L^2$ with value $U_{\rm eff,max} = 4L^6/27\alpha^2 m^3$. For motion with total energy $E < U_{\rm eff,max}$ and initial radius $R < R_1 < R_0$, where R_1 is the smaller positive root of the equation $E = -\alpha/R^3 + L^2/mR^2$, the subsequent motion covers the interval $0 < R < R_1$. This motion is bounded, but not stable as it eventually includes $R = 0$.

References

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[7] See, for example, sec. 3.3 of [7].

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