Magnetic Damping

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(April 14, 2012)

1 Problem

Discuss the most prominent effect on the vertical oscillations of a copper cube of edge a that is suspended from a spring of constant k when the cube is immersed in a uniform, horizontal magnetic field $\mathbf{B}_0$ which is normal to two of the cube's vertical faces. The cube is electrically neutral in the absence of the magnetic field.

2 Solution

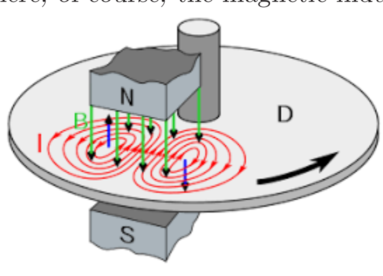
This problem was posed on p. 250 of the April 2012 issue of The Physics Teacher.[1,2]

When a conductor moves through a nonuniform, external magnetic field, the magnetic flux varies through loops fixed inside the conductor, so an electromotive force is induced around the loops, according to Faraday's law (in the rest frame of the conductor), and eddy

[1]Magnetic damping is due to eddy currents, a phenomenon first observed (but not understood) by Arago [1, 2, 5], who reported in 1824: *the results of some experiments that he has conducted on the influence that metals and many other substances exert on a magnetic needle, which has the effect of rapidly reducing the amplitude of the oscillations without altering significantly their duration.* As reported by Babbage and Herschel (1825) [3]: *The curious experiments of M. Arago described by M. Gay Lussac during his visit to London in the spring of the present year (1825), in which plates of copper and other substances set in rapid rotation beneath a magnetized needle, caused it to deviate from its direction, and finally dragged it round with them, naturally excited much attention.*

A step towards an understanding of Arago's phenomenon was made by Christie (1826) [4], who noted that if the copper disk is cut into two or more concentric rings, the effect of a magnet on its rotation is greatly reduced.

In 1831, Art. 131 of [6], Faraday stated: *Future investigations will no doubt ... decide the point whether the retarding or dragging action spoken of* (by Arago) *is always simultaneous with electric currents*, *i.e.*, eddy currents. This statement might have been clearer if Art. 131 had been followed by Art. 123, where it was stated: *These currents are discharged or return in the parts of the plate on each side of and more distant from the place of the pole, where, of course, the magnetic induction is weaker.*



One can reasonably infer that Faraday had a vision of eddy currents as shown above, although Faraday himself never made such a sketch.

The notion of eddy currents is sometimes attributed to Foucault (1855) [7], and are sometimes called Foucault currents, although Foucault attributed their explanation to Faraday.

[2]The pedagogic literature on magnetic damping/braking includes [9]-[81].

currents flow.[3] The Lorentz force on these eddy currents, due to the external magnetic field, opposes the motion, and one speaks of magnetic braking/damping.[4] This effect is (ultra)relativistic, being of order v^2/c^2, where v is the speed of the conductor and c is the speed of light in vacuum. While such relativistic effects are generally small for "ordinary" velocities, the eddy current density obeys $J = \sigma E$, where the conductivity σ for good conductors approaches c^2/v^2 when measured in Gaussian units, such that eddy-current braking is a rare example of an important (ultra)relativistic correction at low velocities.

In the present problem the magnetic field is spatially uniform, so the magnetic flux through a moving loop does not change, and no eddy currents develop. Yet, there exists a very weak magnetic-damping effect, as discussed below.

The cube has mass $m = \rho a^3$, where ρ is the mass density of copper, so in the absence of the magnetic field it oscillates vertically with angular frequency,

$$\omega = \sqrt{\frac{k}{m}}. \tag{1}$$

When the oscillations have amplitude A the vertical velocity has the form,

$$v(t) = A\omega\, e^{i\omega t}, \tag{2}$$

which is surely small in magnitude compared to the speed of light c in vacuum.

In the instantaneous rest frame of the cube there appears to be a horizontal electric field of (time-dependent) magnitude,

$$E_0(t) \approx \frac{v(t)B_0}{c} = \frac{AB_0\,\omega\, e^{i\omega t}}{c} \tag{3}$$

(to order v/c, and in Gaussian units),[5] whose direction is perpendicular to that of $\mathbf{B}_0$, and hence normal to two faces of the cube. Assuming that the frequency ω is low enough that the magnetic field penetrates into the copper cube, the electric field (3) exists throughout its interior, and adds to the electric field $E_1 \approx 4\pi Q/a^2$ associated with surface charge Q (to be determined). The total electric field $E = E_0 + E_1$ gives rise to electrical current density of magnitude $J = \sigma E$ where $\sigma \approx 6 \times 10^{17}$ s is the electrical conductivity of copper.[6] As a consequence, an electrical current,

$$I(t) = a^2 J = a^2 \sigma E, \tag{4}$$

[3]Eddy currents can be induced in conductors at rest by time-dependent magnetic fields. The Lorentz force on the eddy currents can then lead to (dramatic) motion of the conductor, as in Elihu Thomson's jumping ring (1884) [86]-[129]. Here, the magnetic force does the work on the ring.

A small such effect was observed by Ampère and de la Rive in 1821-22 [82, 83], but was largely ignored. A brief mention of this was made by Verdet (1872), sec. 209, p. 357 of [84]. This neglected lore was recounted by Thompson (1894) in [90].

[4]For another example of this phenomenon by the author, see [46].

[5]A lab-frame argument is that the Lorentz force on the conduction electrons can be thought of as due to an effective electric field, given by eq. (3).

[6]At high frequencies the conductivity σ has a significant imaginary part, which we neglect in the present low-frequency example.

flows through the copper cube, along the direction of the electric field. This current leads to an accumulation of charge on the faces of the cube normal to the electric field,

$$Q(t) = \int I\, dt = -\frac{ia^2 \sigma E}{\omega}\,, \tag{5}$$

and this surface-charge distribution leads to electric field,

$$E_1(t) \approx \frac{4\pi Q(t)}{a^2} = -\frac{4\pi i \sigma E}{\omega}\,, \tag{6}$$

The total electric field inside the cube is,

$$E = E_0 + E_1 \approx E_0 - \frac{4\pi i \sigma E}{\omega}\,, \tag{7}$$

i.e.,

$$E \approx \frac{E_0}{1 + 4\pi i \sigma/\omega} \approx -\frac{i\omega E_0}{4\pi\sigma} = -\frac{i\omega^2 A B_0\, e^{i\omega t}}{4\pi\sigma c}\,. \tag{8}$$

The electric current is,

$$I(t) = a^2 \sigma E \approx -\frac{i a^2 \omega^2 A B_0\, e^{i\omega t}}{4\pi c}\,, \tag{9}$$

independent of the conductivity σ.[7]

The copper cube presents electrical resistance,

$$R = \frac{1}{a\sigma} \tag{11}$$

to the current flow, and heat is generated at the time-average rate,

$$\langle P \rangle = \frac{|I|^2 R}{2} = \frac{a^3 \omega^4 A^2 B_0^2}{32\pi^2 c^2 \sigma} \equiv C A^2\,, \tag{12}$$

where,

$$C = \frac{a^3 \omega^4 B_0^2}{2c^2 \sigma} = \frac{k^2 B_0^2}{32\pi c^2 \rho^2 \sigma}\,. \tag{13}$$

This is an effect at order v^2/c^2, and will be extremely small due to the additional presence of the large conductivity σ in the denominator.[8]

[7] The result (9) could be gotten more quickly by arguing that the electric field in the interior of a good conductor is essentially zero, so that there must be charge $\pm Q \approx \pm a^2 E_0/4\pi$ on the faces of the cubes normal to E_0 to "short out" this field. Then, the current in the interior of the cube is,

$$I(t) = \frac{dQ}{dt} \approx \frac{a^2}{4\pi}\frac{dE_0}{dt} = \frac{i a^2 \omega^2 A B_0\, e^{i\omega t}}{4\pi c}\,. \tag{10}$$

However, this argument fails when eddy currents are present, so care is required in using it.

[8] When eddy currents are generated the surface charge is negligible, and a factor of σ appears in the numerator rather than the denominator of the expression for $\langle P \rangle$.

The energy deposition (12) results in a gradual decrease of the amplitude A of oscillation, as the energy U of the oscillation system is,[9]

$$U(t) = kA^2(t). \tag{14}$$

Then, the energy-loss equation $dU/dt = -\langle P \rangle$ leads to,

$$\frac{dA^2}{dt} = -\frac{CA^2}{k}, \tag{15}$$

whose solution is,

$$A = A_0 e^{-t/\tau}, \tag{16}$$

with (large!) damping time constant,

$$\tau = 2k/C = \frac{64\pi^2 c^2 \rho^2 \sigma}{k B_0^2}. \tag{17}$$

For parameters typical of laboratory experiments, the time constant τ is larger than the age of the Universe.

2.1 Other Effects at Order v^2/c^2

Even for $\mathbf{B} = 0$ the "relativistic mass" increase with velocity implies that the angular frequency of oscillation is smaller than $\sqrt{k/m}$ by a term of order v^2/c^2.

For nonzero magnetic field the angular frequency is also slightly reduced by the vertical Lorentz force on the horizontal current discussed above.

Whether the tiny frequency shift or the very weak magnetic damping is the more prominent effect is a matter of opinion.

References

Refernces to Magnetic Damping

[1] F. Arago, *L'action que les corps aimantés et ceux qui ne le sont pas exercent les uns sur les autres*, Ann. Chim. Phys. **27**, 363 (1824),
http://kirkmcd.princeton.edu/examples/EM/arago_acp_27_363_24.pdf
Mr. Arago orally communicates the results of some experiments that he has conducted on the influence that metals and many other substances exert on a magnetic needle, which has the effect of rapidly reducing the amplitude of the oscillations without altering significantly their duration. He promises, on this subject, a detailed memoir.
See also p. 445 of [8].

[9] We ignore the tiny change in the magnetic field energy due to the field associated with the current (9).

[2] F. Arago, *L'action que les corps aimantés et ceux qui ne le sont pas exercent les uns sur les autres*, Ann. Chim. Phys. **28**, 325 (1825),
http://kirkmcd.princeton.edu/examples/EM/arago_acp_28_325_25.pdf
http://kirkmcd.princeton.edu/examples/EM/arago_acp_28_325_25_english.pdf

[3] C. Babbage and J.F.W. Herschel, *Account of the repetition of M. Arago's experiments on the magnetism manifested by various substances during the act of rotation*, Phil. Trans. Roy. Soc. London **115**, 467 (1825),
http://kirkmcd.princeton.edu/examples/EM/babbage_ptrsl_115_467_25.pdf

[4] S.H. Christie, *On the mutual action of the particles of magnetic bodies, and on the law of variation of the magnetic forces generated at dfferent distances during rotation*, Phil. Trans. Roy. Soc. London **117**, 71 (1827),
http://kirkmcd.princeton.edu/examples/EM/christie_ptrsl_117_71_27.pdf

[5] F. Arago, *Note concernant les Phénomènes magnétiques auxquels le mouvement donne naissance*, Ann. Chim. Phys. **32**, 213 (1826),
http://kirkmcd.princeton.edu/examples/EM/arago_acp_32_213_26.pdf

[6] M. Faraday, *Experimental Researches in Electricity*, Phil. Trans. Roy. Soc. London **122**, 125 (1832), kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_122_163_32.pdf

[7] L. Foucault, *De la chaleur produite par l'influence de l'aimant sur les corps en mouvement*, Comptes Rendus Acad. Sci. **41**, 450 (1855),
http://kirkmcd.princeton.edu/examples/EM/foucault_cras_41_450_55.pdf

[8] W.F. Magie, *A Source Book in Physics* (Harvard U. Press, 1963),
http://kirkmcd.princeton.edu/examples/EM/magie_63.pdf

[9] H.R. Hertz, *Uber der Verteilung der Electtricität auf der Oberfläche bewegter Leiter*, Ann. d. Phys. **13**, 266 (1881), http://kirkmcd.princeton.edu/examples/EM/hertz_ap_13_266_81.pdf
http://kirkmcd.princeton.edu/examples/EM/hertz_ap_13_266_81_english.pdf

[10] H. Lamb, *On Electrical Motions in a Spherical Conductor*, Phil. Trans. Roy. Soc. London A **174**, 519 (1883), http://kirkmcd.princeton.edu/examples/EM/lamb_ptrsla_174_519_83.pdf

[11] L.V. King *Electromagnetic shielding at radio frequencies*, Phil. Mag. **15**, 201 (1933),
http://kirkmcd.princeton.edu/examples/EM/king_pm_15_201_33.pdf

[12] W.R. Smythe, *On Eddy Currents in a Rotating Disk*, Trans. Am. Inst. Elec. Eng. **61**, 42 (1942), http://kirkmcd.princeton.edu/examples/EM/smythe_taiee_61_681_42.pdf

[13] J.R. Wait, *A conducting sphere in a time varying magnetic field*, Geophysics **16**, 666 (1951), http://kirkmcd.princeton.edu/examples/EM/wait_geophysics_16_666_51.pdf

[14] J.R. Reitz, *Forces on Moving Magnets due to Eddy Currents*, J. Appl. Phys. **41**, 2067 (1970), http://kirkmcd.princeton.edu/examples/EM/reitz_jap_41_2067_70.pdf

[15] J.W. Luquire, W.E. Deed and C.V. Dodd, *Axially Symmetric Eddy Currents in a Spherical Conductor*, J. Appl. Phys. **41**, 3976 (1970),
http://kirkmcd.princeton.edu/examples/EM/luquire_jap_41_3976_70.pdf

[16] L.C. Davis and J.R. Reitz, *Eddy Currents in Finite Conducting Sheets*, J. Appl. Phys. **42**, 4119 (1971), http://kirkmcd.princeton.edu/examples/EM/davis_jap_42_4119_71.pdf

[17] J.R. Reitz and L.C. Davis, *Force on a Rectangular Coil Moving above a Conducting Slab*, J. Appl. Phys. **43**, 1547 (1972),
http://kirkmcd.princeton.edu/examples/EM/reitz_jap_43_1547_72.pdf

[18] P.L. Richards and M. Tinkham, *Magnetic Suspension and Propulsion Systems for High-Speed Transportation*, J. Appl. Phys. **43**,2680 (1972),
http://kirkmcd.princeton.edu/examples/EM/richards_jap_43_2680_72.pdf

[19] L.C. Davis, *Drag force on a magnet moving near a thin conductor*, J. Appl. Phys. **43**, 4256 (1972), http://kirkmcd.princeton.edu/examples/EM/davis_jap_43_4256_72.pdf

[20] G. Wouch and A.E. Lord Jr, *Eddy currents: Levitation, metal detectors, and induction heating*, Am. J. Phys. **46**, 464 (1978),
http://kirkmcd.princeton.edu/examples/EM/wouch_ajp_46_464_78.pdf

[21] M.P. Perry and T.B. Jones, *Eddy current induction in a solid conducting cylinder with a transverse magnetic field*, IEEE Trans. Magn. **14**, 227 (1978),
http://kirkmcd.princeton.edu/examples/EM/perry_ieeetm_14_227_78.pdf

[22] W.T. Doyle and R. Gibson, *Demonstration of eddy current forces*, Am. J. Phys. **47**, 470 (1979), http://kirkmcd.princeton.edu/examples/EM/doyle_ajp_47_470_79.pdf

[23] J. Walker and W.H. Wells, *Drag Force on a Conducting Spherical Drop in a Nonuniform Magnetic Field*, ORNL/TN-6976 (Sept. 1979),
http://physucs.princeton.edu/~mcdonald/examples/fluids/walker_ornl_tm6976_79.pdf

[24] J.R. Gosselin, P. Rochon and N. Gauthier, *Study of eddy currents in a cylindrical wire: an undergraduate laboratory experiment*, Am. J. Phys. **50**, 440 (1982),
http://kirkmcd.princeton.edu/examples/EM/gosselin_ajp_50_440_82.pdf

[25] C. Emson, J. Simkin and C. Trowbridge, *Further developments in three dimensional Eddy current analysis*, IEEE Trans. Mag. **21**, 2231 (1987),
http://kirkmcd.princeton.edu/examples/EM/emson_ieee_tm_21_2231_85.pdf

[26] H.D. Wiederick *et al.*, *Magnetic braking: Simple theory and experiment*, Am. J. Phys. **55**, 500 (1987), http://kirkmcd.princeton.edu/examples/EM/wiederick_ajp_55_500_87.pdf

[27] J.R. Bowler, *Eddy current calculations using half-space Green's functions*, J. Appl. Phys. **61**, 833 (1987), http://kirkmcd.princeton.edu/examples/EM/bowler_jap_61_833_87.pdf

[28] W.M. Saslow, *Electromechanical implications of Faraday's law: A problem collection*, Am. J. Phys. **55**, 986 (1987), http://kirkmcd.princeton.edu/examples/EM/saslow_ajp_55_986_87.pdf

[29] M.A. Heald, *Magnetic braking: Improved theory*, Am. J. Phys. **56**, 521 (1988), http://kirkmcd.princeton.edu/examples/EM/heald_ajp_56_521_88.pdf

[30] S. Fahy, C. Kittel and S.G. Louie, *Electromagnetic screening by metals*, Am. J. Phys. **56**, 989 (1988), http://kirkmcd.princeton.edu/examples/EM/fahy_ajp_56_989_88.pdf

[31] K. Weigelt, *Expressions for solution of the Laplace and Helmholtz equations in three-dimensional eddy-current problems*, Ark. Elek. **72**, 195 (1989), http://kirkmcd.princeton.edu/examples/EM/weigelt_ae_72_195_89.pdf

[32] J.H. Wouterse, *Critical torque and speed of eddy current brake with widely separated soft iron poles*, IEE Proc. B **138**, 153 (1991), http://kirkmcd.princeton.edu/examples/EM/wouterse_ieeepb_138_153_91.pdf

[33] W.M. Saslow, *How a superconductor supports a magnet, how magnetically "soft iron attracts a magnet, and eddy currents for the uninitiated*, Am. J. Phys. **59**, 16 (1991), http://kirkmcd.princeton.edu/examples/EM/saslow_ajp_59_16_91.pdf

[34] J.A.M. Clack and T.P. Toepker, *Magnetic Induction Experiment*, Phys. Teach. **28**, 236 (1990), http://kirkmcd.princeton.edu/examples/EM/clack_pt_28_236_90.pdf

[35] M. Marcuso *et al.*, *Magnetic drag in the quasistatic limit: A computational method*, Am. J. Phys. **59**, 1118 (1991), http://kirkmcd.princeton.edu/examples/EM/marcuso_ajp_59_1118_91.pdf

[36] M. Marcuso *et al.*, *Magnetic drag in the quasistatic limit: Experimental data and analysis*, Am. J. Phys. **59**, 1123 (1991), http://kirkmcd.princeton.edu/examples/EM/marcuso_ajp_59_1123_91.pdf

[37] W.M. Saslow, *Maxwells theory of eddy currents in thin conducting sheets, and applications to electromagnetic shielding and MAGLEV*, Am. J. Phys. **60**, 693 (1992), http://kirkmcd.princeton.edu/examples/EM/saslow_ajp_60_693_92.pdf

[38] K.R. Davey, *Equivalent Surface Sources Using the Second Order Vector Potential Formulation*, IEEE Trans. Mag. **29**, 1337 (1993), http://kirkmcd.princeton.edu/examples/EM/davey_ieeetm_29_1337_93.pdf

[39] S.J. Norton and J.L. Bowler, *Theory of eddy current inversion*, J. Appl. Phys. **73**, 501 (1993), http://kirkmcd.princeton.edu/examples/EM/norton_jap_73_501_93.pdf

[40] C.S. MacLatchy, P. Backman and L. Bogan, *A quantitative magnetic braking experiment*, Am. J. Phys. **61**, 1096 (1993), http://kirkmcd.princeton.edu/examples/EM/maclatchy_ajp_61_1096_93.pdf

[41] J.A. Manzanares *et al.*, *An experiment on magnetic induction pulses*, Am. J. Phys. **62**, 702 (1994), http://kirkmcd.princeton.edu/examples/EM/manzanares_ajp_62_702_94.pdf

[42] L.H. Cadwell, *Magnetic damping: Analysis of an eddy current brake using an airtrack*, Am. J. Phys. **64**, 917 (1996), http://kirkmcd.princeton.edu/examples/EM/cadwell_ajp_64_917_96.pdf

[43] T.P. Theodoulidis *et al.*, *Analytical and Numerical Solution of the Eddy-Current Problem in Spherical Coordinates Based on the Second-Order Vector Potential Formulation*, IEEE Trans. Mag. **33**, 2461 (1997),
http://kirkmcd.princeton.edu/examples/EM/theodoulidis_ieeetm_33_2461_97.pdf

[44] J. M. Aguirregabiria, A. Hernández and M. Rivas, *Magnetic braking revisited*, Am. J. Phys. **65**, 851 (1997), http://kirkmcd.princeton.edu/examples/EM/aguirregabiria_ajp_65_851_97.pdf

[45] K.D. Hahn *et al.*, *Eddy current damping of a magnet moving through a pipe*, Am. J. Phys. **66**, 1055 (1998), http://kirkmcd.princeton.edu/examples/EM/hahn_ajp_66_1066_98.pdf

[46] K.T. McDonald, *Pitching Pennies into a Magnet* (Jan. 15, 1997),
http://kirkmcd.princeton.edu/examples/pennies.pdf

[47] Y. Kraftmakher, *Eddy currents: Contactless measurement of electrical resistivity*, Am. J. Phys. **68**, 375 (2000), http://kirkmcd.princeton.edu/examples/EM/kraftmakher_ajp_68_375_00.pdf

[48] P. J. Salzman, J.R. Burke and S.M. Lea, *The effect of electric fields in a classic introductory physics treatment of eddy current forces*, Am. J. Phys. **69**, 586 (2001),
http://kirkmcd.princeton.edu/examples/EM/salzman_ajp_69_586_01.pdf

[49] A. Singh, Y.N. Mohapatra and S. Kumar, *Electromagnetic induction and damping: Quantitative experiments using a PC interface*, Am. J. Phys. **70**, 424 (2002),
http://kirkmcd.princeton.edu/examples/EM/singh_ajp_70_424_02.pdf

[50] K. Lee and K. Park, *Modeling Eddy Currents With Boundary Conditions by Using Coulomb's Law and the Method of Images*, IEEE Trans. Mag. **38**, 1333 (2002),
http://kirkmcd.princeton.edu/examples/EM/lee_ieeetm_38_1333_02.pdf

[51] K. Lee and K. Park, *Analysis of an eddy-current brake considering finite radius and induced magnetic flux*, J. Appl. Phys. **92**, 5532 (2002),
http://kirkmcd.princeton.edu/examples/EM/lee_jap_92_5532_02.pdf

[52] M.I. González, *Experiments with eddy currents: the eddy current brake*, Eur. J. Phys. **25**, 463 (2004), http://kirkmcd.princeton.edu/examples/EM/gonzalez_ejp_25_463_04.pdf

[53] J. Iñiguez *et al.*, *Study of the conductivity of a metallic tube by analysing the damped fall of a magnet*, Eur. J. Phys. **25**, 593 (2004),
http://kirkmcd.princeton.edu/examples/EM/iniguez_ejp_25_593_04.pdf

[54] M.H. Partovi and E.J. Morris, *Electrodynamics of a magnet moving through a conducting pipe*, Can. J. Phys. **84**, 253 (2006),
http://kirkmcd.princeton.edu/examples/EM/partovi_cjp_84_253_06.pdf

[55] J. Pellicer-Porres *et al.*, *Force characterization of eddy currents*, Am. J. Phys. **74**, 267 (2006), http://kirkmcd.princeton.edu/examples/EM/pellicer-porres_ajp_74_267_06.pdf

[56] X. Xie *et al.*, *Investigation of magnetic damping on an air track*, Am. J. Phys. **74**, 974 (2006), http://kirkmcd.princeton.edu/examples/EM/xie_ajp_74_974_06.pdf

[57] Y. Levin, F.L. da Silveira and F. B. Rizzato, *Electromagnetic braking: A simple quantitative model*, Am. J. Phys. **74**, 815 (2006),
http://kirkmcd.princeton.edu/examples/EM/levin_ajp_74_815_06.pdf

[58] M.K. Roy, M.K. Harbola and H.C. Verma, *Demonstration of Lenzs law: Analysis of a magnet falling through a conducting tube*, Am. J. Phys. **75**, 728 (2007),
http://kirkmcd.princeton.edu/examples/EM/roy_ajp_75_728_07.pdf

[59] A. Garg, *Conductors in quasistatic electric fields*, Am. J. Phys. **76**, 615 (2008),
http://kirkmcd.princeton.edu/examples/EM/garg_ajp_76_615_08.pdf

[60] C.P. Bidinosti, E.M. Chappie and M.E. Hayden, *The sphere in a uniform rf fieldrevisited*, Con. Mag. Reson. **31B**, 191 (2007),
http://kirkmcd.princeton.edu/examples/EM/bidinosti_cmr_31b_191_07.pdf

[61] G. Ireson and J. Twidle, *Magnetic braking revisited: activities for the undergraduate laboratory*, Eur. J. Phys. **30**, 855 (2009),
http://kirkmcd.princeton.edu/examples/EM/ireson_ejp_29_745_08.pdf

[62] J. Iñiguez *et al.*, *The electromagnetic field in conductive slabs and cylinders submitted to a harmonic longitudinal magnetic field*, Am. J. Phys. **77**, 1974 (2009),
http://kirkmcd.princeton.edu/examples/EM/iniguez_ajp_77_1074_09.pdf

[63] G. Donoso, C.L. Ladera and P. Martin, *Magnet fall inside a conductive pipe: motion and the role of the pipe wall thickness*, Eur. J. Phys. **30**, 855 (2009),
http://kirkmcd.princeton.edu/examples/EM/donoso_ejp_30_855_09.pdf

[64] N. Derby and S. Olbert, *Cylindrical magnets and ideal solenoids*, Am. J. Phys. **78**, 229 (2010), http://kirkmcd.princeton.edu/examples/EM/derby_ajp_78_229_10.pdf

[65] G. Donoso, C.L. Ladera and P. Martin, *Damped fall of magnets inside a conducting pipe*, Am. J. Phys. **79**, 193 (2011),
http://kirkmcd.princeton.edu/examples/EM/donoso_ajp_79_193_11.pdf

[66] J.A. Molina-Bolivar and A.J. Abella-Palacios, *A laboratory activity on the eddy current brake*, Eur. J. Phys. **33**, 697 (2012),
http://kirkmcd.princeton.edu/examples/EM/molina_ejp_33_697_12.pdf

[67] A. Bonanno *et al.*, *Foucault dissipation in a rolling cylinder: a webcam quantitative study*, Eur. J. Phys. **32**, 419 (2011),
http://kirkmcd.princeton.edu/examples/EM/bonanno_ejp_32_419_11.pdf

[68] J. Lévêque *et al.*, *Eddy current in a rotating cylinder in a static field by a stochastic method*, Eur. Phys. J. Appl. Phys. **57**, 30901 (2011),
http://kirkmcd.princeton.edu/examples/EM/leveque_epjap_57_30901_12.pdf

[69] P. Onorato and A. De Ambrosis, *Magnetic damping: Integrating experimental and theoretical analysis*, Am. J. Phys. **80**, 27 (2012),
http://kirkmcd.princeton.edu/examples/EM/onorato_ajp_80_27_12.pdf

[70] R.P. Uhlig *et al.*, *Lorentz Force Eddy Current Testing: a Prototype Model*, J. Nond. Eval. **31**, 357 (2012), http://kirkmcd.princeton.edu/examples/EM/uhlig_jne_31_357_12.pdf

[71] A. Muiznieks and I. Dudareva, *Quantitative analysis of the damping of magnet oscillations by eddy currents in aluminum foil*, Am. J. Phys. **80**, 804 (2012), http://kirkmcd.princeton.edu/examples/EM/muiznieks_ajp_80_804_12.pdf

[72] F.G. Tomasel and M.C. Marconi, *Rolling magnets down a conductive hill: Revisiting a classic demonstration of the effects of eddy currents*, Am. J. Phys. **80**, 100 (2012), http://kirkmcd.princeton.edu/examples/EM/tomasel_ajp_80_800_12.pdf

[73] C.L. Ladera and G. Donoso, *Oscillations of a springmagnet system damped by a conductive plate*, Eur. J. Phys. **34**, 1187 (2013), http://kirkmcd.princeton.edu/examples/EM/ladera_ejp_34_1187_13.pdf

[74] M.A. Nurge *et al.*, *A slowly rotating hollow sphere in a magnetic field: First steps to de-spin a space object*, Am. J. Phys. **84**, 181 (2016), http://kirkmcd.princeton.edu/examples/EM/nurge_ajp_84_181_16.pdf

[75] M.A. Nurge *et al.*, *A thick-walled sphere rotating in a uniform magnetic field: The next step to de-spin a space object*, Am. J. Phys. **85**, 596 (2017), http://kirkmcd.princeton.edu/examples/EM/nurge_ajp_85_596_17.pdf

[76] A. Bonanno, G. Bozzo and P. Sapia, *An innovative experimental sequence on electromagnetic induction and eddy currents based on video analysis and cheap data acquisition*, Eur. J. Phys. **33**, 697 (2012), http://kirkmcd.princeton.edu/examples/EM/bonanno_ejp_38_065203_17.pdf

[77] M.L. Honke and C.P. Bidinosti, *The metallic sphere in a uniform ac magnetic field: A simple and precise experiment for exploring eddy currents and non-destructive testing*, Am. J. Phys. **86**, 430 (2018), http://kirkmcd.princeton.edu/examples/EM/honke_ajp_86_430_18.pdf

[78] M.A. Nurge, R.C. Youngquist and S.O. Starr, *Drag and lift forces between a rotating conductive sphere and a cylindrical magnet*, Am. J. Phys. **86**, 443 (2018), http://kirkmcd.princeton.edu/examples/EM/nurge_ajp_86_443_2018.pdf

[79] H. Chebbi and D. Prémel, *The fast computation of eddy current distribution and probe response in homogenized composite material based on semi-analytical approach*, Eur. Phys. J. Appl. Phys. **89**, 10901 (2020), http://kirkmcd.princeton.edu/examples/EM/chebbi_epjap_89_10901_20.pdf

[80] I.N. Gorbatyy and I.P. Zhura, *Eddy currents in multilayer coils*, Am. J. Phys. **89**, 284 (2021), http://kirkmcd.princeton.edu/examples/EM/gorbatyy_ajp_89_284_21.pdf

[81] F. Behroozi, *Rotation of Spherical Magnet Falling Through a Slit Copper Pipe*, Phys. Teach. **59**, 243 (2021), http://kirkmcd.princeton.edu/examples/EM/behroozi_pt_59_243_21.pdf

[82] A.M. Ampère, *Sur une mouvelle Expérience lectro-magnétique*, J. Phys. Chem. **93**, 447 (1821), p. 448, http://kirkmcd.princeton.edu/examples/EM/ampere_jpc_93_447_21.pdf

[83] A. de la Rive *fils*, *Sur l' Action qu'exerce le globe terrestre sur une portion mobile du circuit voltaïque*, Ann. Chem. Phys. **21**, 24 (1822), p. 47, http://kirkmcd.princeton.edu/examples/EM/delarive_acp_21_24_22.pdf

[84] É. Verdet, *Conferences du Physique*, Part 1 (Paris, 1872), Sec. 209, p. 357, http://kirkmcd.princeton.edu/examples/EM/verdet_72_pt1.pdf

[85] C.V. Boys, *A Magneto-electric Phenomenon*, Proc. Phys. Soc. London **6**, 218 (1884), http://kirkmcd.princeton.edu/examples/EM/boys_ppsl_6_218_84.pdf
Phil. Mag. **18**, 216 (1884), http://kirkmcd.princeton.edu/examples/EM/boys_pm_18_216_84.pdf

[86] E. Thomson, *Novel Phenomena of Alternating Currents*, Electrician **19**, 104 (1887), http://kirkmcd.princeton.edu/examples/EM/thomson_electrician_19_104_87.pdf

[87] J.A. Fleming, *Electro-Magnetic Repulsion*, Electrician **26**, 567, 601 (1891), http://kirkmcd.princeton.edu/examples/EM/fleming_electrician_26_567_91.pdf

[88] J. Borgman, *Sur les actions mécaniques des courants variables*, Compes Rendus Acad. Sci. **110**, 233, 849 (1890), http://kirkmcd.princeton.edu/examples/EM/borgman_cras_110_233_90.pdf

[89] G.T. Walker, *Repulsion and Rotation produced by Alternating Currents*, Phil. Trans. Roy. Soc. A **183**, 279 (1892), http://kirkmcd.princeton.edu/examples/EM/walker_ptrsla_183_279_92.pdf

[90] S.P. Thompson, *Note on a Neglected Experiment of Ampère*, Proc. Phys. Soc. London **13**, 493 (1894), http://kirkmcd.princeton.edu/examples/EM/thompson_ppsl_13_493_94.pdf
Phil. Mag. **39**, 534 (1895), http://kirkmcd.princeton.edu/examples/EM/thompson_pm_39_534_95.pdf

[91] J.A. Fleming, *The Alternate Current Transformer in Theory and Practice*, Vol. 1 (Electrician, 1896), pp. 307-329, http://kirkmcd.princeton.edu/examples/EM/fleming_96_v1.pdf

[92] F.T. Holmes, *Axial Magnetic Suspensions*, Rev. Sci. Instr. **8**, 444 (1937), http://kirkmcd.princeton.edu/examples/EM/holmes_rsi_8_444_37.pdf

[93] W. Kemper, *Schwebende Aufhängung durch elektromagnetische Kräfte: eine Möglichkeit für eine grundsätzlich neue Fortbewegungsart*, Elek. Z. **59**, 391 (1938), http://kirkmcd.princeton.edu/examples/EM/kemper_ez_59_391_38.pdf

[94] B.D. Bedford, L.H.B. Peer and L. Tonks, *The Electromagnetic Levitator*, Gen. Elec. Rev. **42**, 246 (1939), http://kirkmcd.princeton.edu/examples/EM/bedford_ger_42_246_39.pdf

[95] E.C. Okress *et al.*, *Electromagnetic Levitation of Solid and Molten Metals*, J. Appl. Phys. **23**, 545 (1952), http://kirkmcd.princeton.edu/examples/EM/okress_jap_23_545_52.pdf

[96] A.H. Boerdijk, *Technical Aspects of Levitation*, Phillips Res. Rep. **11**, 45 (1955), http://kirkmcd.princeton.edu/examples/EM/boerdijk_prr_11_45_56.pdf

[97] W. Brisley and and B.S. Thornton, *Electromagnetic levitation calculations for axially symmetric systems*, Brit. J. Appl. Phys. **14**, 682 (1963),
http://kirkmcd.princeton.edu/examples/EM/brisley_bjap_14_682_63.pdf

[98] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics*, Vol. 2 (Addison-Wesley, 1964), sec. 16-2, https://www.feynmanlectures.caltech.edu/II_16.html

[99] H.E. White and H. Weltin, *Experimental Levitator*, Am. J. Phys. **31**, 925 (1963),
http://kirkmcd.princeton.edu/examples/EM/white_ajp_31_925_63.pdf

[100] E.R. Laithwaite, *Electromagnetic levitation*, Proc. IEE **112**, 2361 (1965),
http://kirkmcd.princeton.edu/examples/EM/laithwaite_piee_112_2361_65.pdf

[101] E.J. Churchill and J.D. Noble, *A demonstration of Lenz's law?* Am. J. Phys. **39**, 285 (1971), http://kirkmcd.princeton.edu/examples/EM/churchill_ajp_39_285_71.pdf

[102] D.J. Sumner and A.K. Thakkrar, *Experiments with a 'jumping ring' apparatus*, Phys. Educ. **7**, 238 (1972), http://kirkmcd.princeton.edu/examples/EM/sumner_pe_7_238_72.pdf

[103] W.R. Towler and J.W. Beams, *Magnetic suspension for lecture and classroom demonstrations*, Am. J. Phys. **39**, 285 (1971),
http://kirkmcd.princeton.edu/examples/EM/towler_ajp_44_478_76.pdf

[104] A.R. Quinton, *The ac repulsion demonstration of Elihu Thomson*. Phys. Teach. **17**, 42 (1979), http://kirkmcd.princeton.edu/examples/EM/quinton_pt_17_40_79.pdf

[105] S.Y. Mak and K. Young, *Floating metal ring in an alternating magnetic field*, Am. J. Phys. **54**, 808 (1986), http://kirkmcd.princeton.edu/examples/EM/mak_ajp_54_808_86.pdf

[106] P.J. Ford and R.A.L. Sullivan, *The jumping ring revisited*, Phys. Educ. **26**, 380 (1991),
http://kirkmcd.princeton.edu/examples/EM/ford_pe_26_380_91.pdf

[107] T.D. Rossing and J.R. Hull, *Magnetic Levitation*, Phys. Teach. **29**, 552 (1991),
http://kirkmcd.princeton.edu/examples/EM/rossing_pt_29_552_97.pdf

[108] E.R. Laithwaite, *Induction coil guns for hypervelocities*, IEE Proc. Elec. Power Appl. **142**, 215 (1995), http://kirkmcd.princeton.edu/examples/EM/laithwaite_ieepepa_142_215_95.pdf

[109] J. Hall, *Forces on the Jumping Ring*, Phys. Teach. **35**, 80 (1997),
http://kirkmcd.princeton.edu/examples/EM/hall_pt_35_80_97.pdf

[110] C.S. Schneider and J.P. Ertel, *A classroom jumping ring*, Am. J. Phys. **66**, 686 (1998),
http://kirkmcd.princeton.edu/examples/EM/schneider_ajp_66_686_98.pdf

[111] N. Barry, *Dynamics of Jumping Rings and Collars*, Int. J. Elec. Eng. Ed. **35**, 357 (1998), http://kirkmcd.princeton.edu/examples/EM/barry_ijeee_35_357_98.pdf

[112] N. Barry, *Elihu Thomson's Jumping Ring in a Levitated Closed-Loop Control Experiment*, IEEE Trans. Ed. **42**, 72 (1999),
http://kirkmcd.princeton.edu/examples/EM/barry_ieeete_42_72_99.pdf

[113] P.J.H. Tjossem and V. Cornejo, *Measurements and mechanisms of Thomsons jumping ring*, Am. J. Phys. **68**, 238 (2000),
http://kirkmcd.princeton.edu/examples/EM/tjossem_ajp_68_238_00.pdf

[114] P. Tanner *et al.*, *A pulsed jumping ring apparatus for demonstration of Lenz's law*, Am. J. Phys. **69**, 911 (2001), http://kirkmcd.princeton.edu/examples/EM/tanner_ajp_69_911_01.pdf

[115] A. McAlexander, *PSSC Turbo Ring Flinger*, Phys. Teach. **43**, 613 (2005),
http://kirkmcd.princeton.edu/examples/EM/mcalexander_pt_43_613_05.pdf

[116] H.M. Aguilar, *Magnetic Levitation and Newton's Third Law*, Phys. Teach. **45**, 278 (2007), http://kirkmcd.princeton.edu/examples/EM/aguilar_pt_45_278_07.pdf

[117] J.M. Bostock-Smith, *The jumping ring and Lenz's law–an analysis*, Phys. Educ. **43**, 265 (2008), http://kirkmcd.princeton.edu/examples/EM/bostock_pe_43_265_08.pdf

[118] R.N. Jeffery and F. Amiri, *The Phase Shift in the Jumping Ring*, Phys. Teach. **46**, 350 (2008), http://kirkmcd.princeton.edu/examples/EM/jeffery_pt_46_350_08.pdf

[119] M. Baylie *et al.*, *The jumping ring experiment*, Phys. Ed. **44**, 27 (2009),
http://kirkmcd.princeton.edu/examples/EM/baylie_pe_44_27_09.pdf

[120] P.J.H. Tjossem and E.C. Brost, *Optimizing Thomsons jumping ring*, Am. J. Phys. **79**, 353 (2011), http://kirkmcd.princeton.edu/examples/EM/tjossem_ajp_79_353_11.pdf

[121] J. Taweepong, K. Thamaphat and S. Limsuwan, *Jumping ring experiment: effect of temperature, non-magnetic material and applied current on the jump height*, Proc. Eng. **32**, 982 (2012), http://kirkmcd.princeton.edu/examples/EM/taweepong_pe_32_982_12.pdf

[122] F. Waschke, A. Strunz and J.-P. Meyn, *A safe and effective modification of Thomson's jumping ring experiment*, Eur. J. Phys. **33**, 1625 (2012),
http://kirkmcd.princeton.edu/examples/EM/waschke_ejp_33_1625_12.pdf

[123] A. Zangwill, *Modern Electrodynamics* (Cambridge U. Press, 2012), Appl. 14.7, p. 492,
http://kirkmcd.princeton.edu/examples/EM/zangwill_em_12.pdf

[124] G. Donoso and C.L. Ladera, *The naked toy model of a jumping ring*, Eur. J. Phys. **35**, 015002 (2014), http://kirkmcd.princeton.edu/examples/EM/donoso_ejp_35_015002_14.pdf

[125] C.L. Ladera and G. Donoso, *Unveiling the physics of the Thomson jumping ring*, Am. J. Phys. **83**, 341 (2015), http://kirkmcd.princeton.edu/examples/EM/ladera_ajp_83_341_15.pdf

[126] R.N. Jeffery and F. Amiri, *DC-Powered Jumping Ring*, Phys. Teach. **54**, 112 (2016),
http://kirkmcd.princeton.edu/examples/EM/jeffery_pt_54_112_16.pdf

[127] T. Foster *et al.*, *A New Version of an Old Demonstration Experiment Using the Elihu Thomson Jumping Ring Apparatus*, Phys. Teach. **54**, 488 (2016),
http://kirkmcd.princeton.edu/examples/EM/foster_pt_54_488_16.pdf

[128] D.J. Griffiths, *Introduction to Electrodynamics*, 4$^{\rm th}$ ed. (Cambridge U. Press, 2017), Ex. 7.6, p. 316, http://kirkmcd.princeton.edu/examples/EM/griffiths_em4_p316.pdf

[129] R.N. Jeffery and F. Amiri, *Thomson's Jumping Ring Over a Long Coil*, Phys. Teach. **56**, 176 (2018), http://kirkmcd.princeton.edu/examples/EM/jeffery_pt_56_176_18.pdf