Force and Torque while Frying an Egg

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As an egg is fried, its thermal energy, and hence its mass, increases.[1,2] When viewed from an inertial frame f in which the center of mass of the egg has position $\mathbf{r}_{\rm cm}$ and constant velocity $\mathbf{v} = \mathbf{v}_{\rm cm} = d\mathbf{r}_{\rm cm}/dt$, the (linear) momentum $\mathbf{P} = m(t)\mathbf{v}_{\rm cm}$ of the egg increases with time, as also does its angular momentum $\mathbf{L} = (\mathbf{r}_{\rm cm} - \mathbf{r}_0) \times \mathbf{P} + I_{\rm cm}\,\boldsymbol{\omega} = (\mathbf{r}_{\rm cm} - \mathbf{r}_0) \times m\mathbf{v}_{\rm cm}$ about any fixed point $\mathbf{r}_0$ (in inertial frame f) not on the trajectory of the center of mass of the egg.

Observers in inertial frame f consider that there exists a nonzero force,

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = \frac{dm}{dt}\mathbf{v}_{\rm cm}, \tag{1}$$

on the egg, exerted by the frying pan. The torque $\boldsymbol{\tau}$ of this force about reference point $\mathbf{r}_0$ is,

$$\boldsymbol{\tau} = (\mathbf{r}_{\rm cm} - \mathbf{r}_0) \times \mathbf{F} = (\mathbf{r}_{\rm cm} - \mathbf{r}_0) \times \frac{dm}{dt}\mathbf{v}_{\rm cm} = \frac{d}{dt}[(\mathbf{r}_{\rm cm} - \mathbf{r}_0) \times m\mathbf{v}_{\rm cm}] = \frac{d\mathbf{L}}{dt}\,. \tag{2}$$

Thus, while the (variable-mass) frying egg moves with constant velocity $\mathbf{v}$ in inertial frame f, without rotation about its center of mass, it is associated with nonzero, time-dependent force, torque, linear and angular momentum.[3,4]

This example was suggested by Daniel Vanzella.

References

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[1] We suppose that the egg and the frying pan are not rotating in their (inertial) rest frame.

[2] As the temperature of the egg increases by ΔT, its mass increases by $\Delta U/c^2 = C\Delta T/c^2$, where C is the heat capacity of the egg and c is the speed of light in vacuum. Thus, we might say that frying an egg is a "relativistic" phenomenon.

[3] The momentum $\mathbf{P}$ of the egg in frame f is "overt", and not "hidden" in the sense discussed in [1].

[4] *Oct. 25, 2020.* Some people have claimed that if an object experiences no torque in its (inertial) rest frame, there is no torque on it in any other inertial frame. A partial history of this controversy can be traced in [2]-[11].

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