Face in a Mirror

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1 Problem

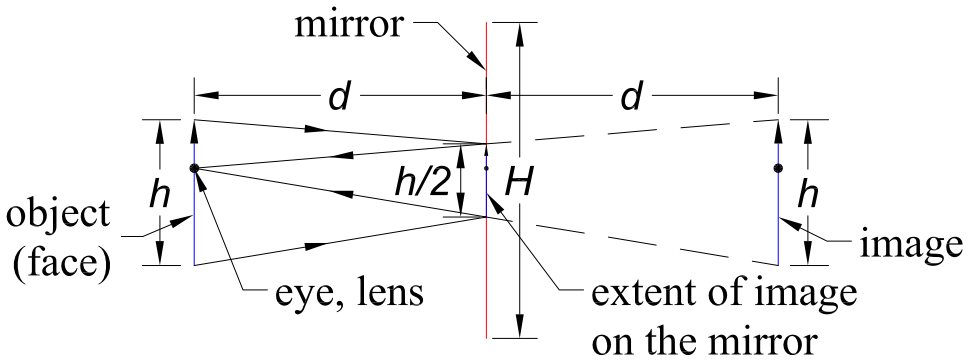
Show that the linear extent (*i.e.*, height) on a mirror occupied by the image of your face, of height h, is $h/2$ independent of the distance d between your face and the mirror.

Of course, the image appears to be smaller (subtends less angle) the farther you are from the mirror.

This problem was suggested by Derek Abbott.

2 Solution

The sketch below illustrates the paths of light rays from the bottom and top of an object (face, or camera) when reflected off a mirror at distance d from the object, to arrive at your eye (or a the lens of a camera). The dashed lines show virtual light rays associated with the (virtual) image of your face at distance $2d$ from your face.



The similar triangles, whose bases are the virtual image of height h, and the extent of the image in the plane of the mirror, have horizontal lengths $2d$ and d respectively. Hence, the extent of the image on the mirror is $h/2$.

In the plane of the mirror (of height H), the image occupies fraction $h/2H$ of the extent of the mirror independent of distance d. This could be verified by placing pieces of tape on the mirror at the bottom and top of the extent of the image there for some distance d, and then varying d while noting that the image on the plane of the mirror always fits exactly between the two pieces of tape.

Of course, the virtual image of your face (and the mirror itself) appears to you to be smaller for larger distances d between you and the mirror.