Hering's Flux-Linkage Paradox

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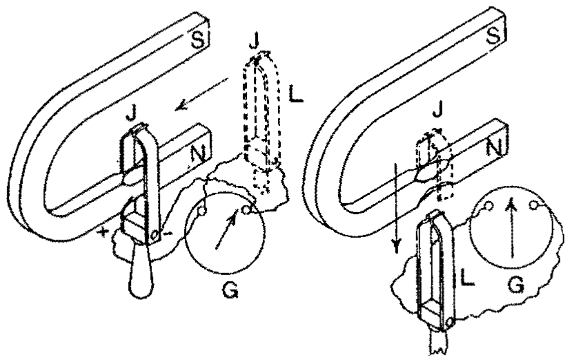
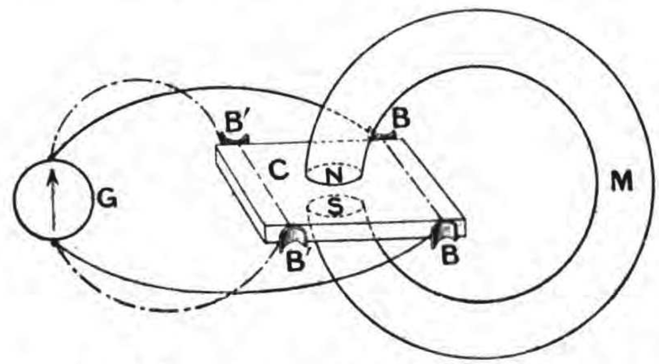
1 Problem

In 1908, Hering [1] performed an experiment, sketched on the left below, in which a pair of spring clips was moved (vertically in the figure) with respect to a permanent horseshoe magnet and a galvanometer was used to detect any current induced in the loop consisting of the galvanometer, lead wires, sliding contacts. Initially, the circuit surrounded the N pole of the magnet such that the circuit linked magnetic flux, but after the clips passed by that pole are were below it, no flux was linked.





In 1915, p. 559 of [2], Hering described a variant of the experiment sketched on the right above, in which the field of the permanent magnet was perpendicular to a fixed copper plate in the magnet gap, connected to a galvanometer by sliding contacts that were moved from inside of the magnet to outside.[1]

Was the reading of the galvanometer nonzero?

2 Solution

The experimental result was that the galvanometer never registered any current, although the magnetic flux through the circuit changed with time.[2] Hering considered this to be a violation of Faraday's "flux rule", and concluded that the whole edifice of the electrodynamics of Faraday and Maxwell was therefore doubtful.[3,4]

[1] This experiment was described by Faraday (1831) in Art. 101 of [3], but with the connections to the copper plate fixed rather than sliding. In that case, the galvanometer reading was nonzero.

[2] When the copper plate was removed from the second experiment, and the sliding contacts touched one another to form a closed loop, the galvanometer did detect a current when the loop was deformed so that it "cut" magnetic field lines.

[3] Hering also considered [4]-[9] that the Biot-Savart law could not explain Ampère's "hairpin" experiment [10], as reviewed in [11].

[4] For a review of "flux rule exceptions" by the author, see [12].

The issue is that in circuits with moving conductors, a simple application of the "flux rule" is not necessarily sufficient, as anticipated by Faraday in secs. 114-119 of his first paper on magnetic induction [13]. In addition to $\mathcal{EMF}_\Phi = -d\Phi/dt$ associated with a changing magnetic flux Φ through the circuit, one should also consider a possible "motional $\mathcal{EMF}$" associated with a conductor moving through a magnetic field.[5]

In the configuration shown on the right on p. 1 above, we suppose the copper plate (which remains at rest) has width l, and the sliding contacts are pulled with velocity v perpendicular to the magnetic field B, which is approximated as uniform. Then, the $\mathcal{EMF}$ associated with the changing magnetic flux is,

$$\mathcal{EMF}_\Phi = -\frac{d\Phi}{dt} = Blv, \tag{1}$$

acting counterclockwise in the circuit as viewed from above. Meanwhile, in the rest frame of the sliding contacts, there is an electric field $\mathbf{E}' = \mathbf{v} \times \mathbf{B}$ over the width l of the plate, and so the motional $\mathcal{EMF}$ is,[6]

$$\mathcal{EMF}_{\text{motional}} = vBl, \tag{2}$$

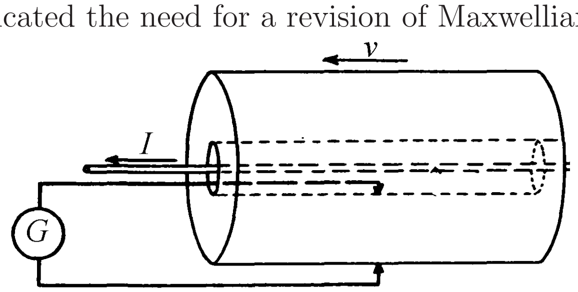
acting clockwise. This $\mathcal{EMF}$ also acts in the lab frame.

As the two $\mathcal{EMF}$s, (1) and (2), have equal and opposite senses, the total $\mathcal{EMF}$ in the lab frame is zero, and the galvanometer reads nothing, as found by Hering in his experiments.

3 Cullwick's Flux-Linkage Paradox

Another early flux-linkage paradox was reported by Blondel (1914) [14, 15] (and reviewed by the author in [16], which includes references to several other discussions of "flux-rule exceptions").

We now briefly discuss the flux-linkage "paradox", illustrated below, due to Cullwick [17], who gave an explanation similar to that given above for Hering's paradox, but who also felt that these examples indicated the need for a revision of Maxwellian electrodynamics.



[5]This was pointed out by Steinmetz in a letter appended to Hering's 1908 paper, p. 1352 of [1].

[6]In 1908, an argument based on the rest frame of the sliding contacts would not have occured to most people (although Faraday often considered possible effects in moving parts of circuits), so an argument in the lab-frame would have to suppose that if $\mathcal{EMF}_\Phi$ created a current in the circuit with the sliding contacts, then the moving charges in that current would have a component of their velocity equal to the velocity $\mathbf{v}$ of the sliding contacts, and so would experience a Lorentz force $e\mathbf{v} \times \mathbf{B}$ in the lab frame. This chain of logic, involving a "virtual current", apparently was unconvincing to those inclined to doubt Maxwell's electrodynamics.

A steady current I flows in a fixed wire along the axis of an annular metal cylinder that moves with velocity v parallel to its axis. A galvanometer is connected to contacts on the inner and outer radii of the cylinder, which contacts remain fixed in the lab as the cylinder slides past them.

Does the reading of the galvanometer depend on the permeability of the cylinder, which could be made of copper or of iron?

The (azimuthal) magnetic field $B_{\rm in}$ (at a given distance from the axis) inside a cylinder of (relative) permeability μ is μ times the field $B_{\rm out}$ outside the cylinder (at the same radial distance); $\mathbf{B}_{\rm in} = \mu \mathbf{B}_{\rm out}$.

If only the $\mathcal{EMF}_\Phi$ associated with the changing magnetic flux were relevant, then the galvanometer reading would be larger with an iron cylinder than with a copper one, as this $\mathcal{EMF}$ is proportional to $(B_{\rm in} - B_{\rm out})v$. That is, $\mathcal{EMF}_\Phi$ is zero for a moving copper cylinder.

However, the circuit includes the moving metal cylinder, in whose rest frame there exists a radial electrical field of strength $\mathbf{v} \times \mathbf{B}_{\rm in}$, and hence there is a motional $\mathcal{EMF}$ in the galvanometer circuit as well, whose sign is opposite to that of $\mathcal{EMF}_\Phi$.[7]

The total $\mathcal{EMF}$ in the galvanometer circuit is proportional to $(B_{\rm in} - B_{\rm out})v - vB_{\rm in} = -vB_{\rm out}$, and is independent of the permeability μ, as observed by Cullwick in lab experiments.

References

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[2] C. Hering, *The Laws of Induction*, Electrician **73**, 344 (1915),
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[3] M. Faraday, *Experimental Researches in Electricity*, Phil. Trans. Roy. Soc. London **122**, 125 (1832), kirkmcd.princeton.edu/examples/EM/faraday_ptrsl_122_163_32.pdf

[4] C. Hering, *The Stretching of a Conductor by Its Current*, J. Franklin Inst. **171**, 73 (1911), kirkmcd.princeton.edu/examples/EM/hering_jfi_171_73_11.pdf

[5] C. Hering, *Revision of Some of the Electromagnetic Laws*, J. Franklin Inst. **192**, 599 (1921), kirkmcd.princeton.edu/examples/EM/hering_jfi_192_599_21.pdf

[6] C. Hering, *Electrodynamic Forces in Electric Furnaces*, Trans. Am. Electrochem. Soc. **39**, 313 (1921), kirkmcd.princeton.edu/examples/EM/hering_taec_39_313_21.pdf

[7] C. Hering, *Electrodynamic Motions in Electric Furnaces*, Trans. Am. Electrochem. Soc. **41**, 303 (1922), kirkmcd.princeton.edu/examples/EM/hering_taec_41_303_22.pdf

[7]Despite advocating the theory of relativity in his book [18], Cullwick seemed uncomfortable using it for his circuit, discussed on pp. 134-135.

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[12] K.T. McDonald, *Is Faraday's Disk Dynamo a Flux-Rule Exception?* (July 27, 2019), http://kirkmcd.princeton.edu/examples/faradaydisk.pdf

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