Electric and Magnetic Hoses

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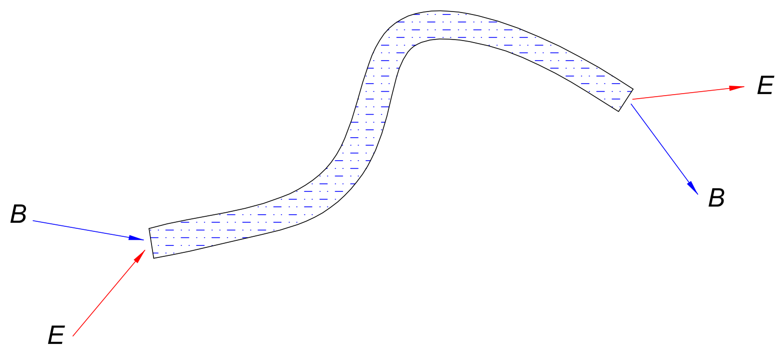
1 Problem

An electric or magnetic "hose" is a hypothetical tubular medium such that the static electric and magnetic fields at the input face are the same as those at the output face, as if these fields were somehow transported from one end of the "hose" to the other.



Deduce the properties of this medium for a "hose" whose faces are the planes $z = 0$ and $z = a$.

2 Solution

This problem is discussed in [1], where a lengthy solution is given in the language of optics. As noted there, waveguides and fiberoptic cables can be considered as "electromagnetic hoses", but these devices do not function as such in the static limit. Here, we consider a "hose" as a boundary-value problem in electro- and magnetostatics.

2.1 Electrical Properties

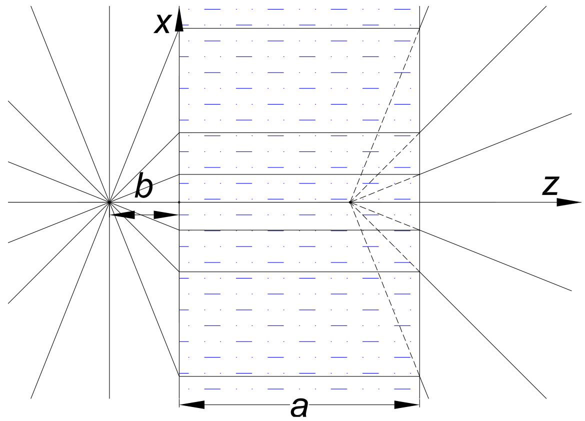
It suffices to consider the electric field due to a single electric charge at $(x, y, z) = (0, 0, -b)$. Then, the desired pattern of electric field lines is sketched on the next page.

Inside the "hose" medium the $\mathbf{E}$-field lines should be only in the z-direction. However, the tangential component of the electric field is continuous across the surfaces $z = 0$ and $z = a$, so it must be that E_z is very large inside the medium.

If we suppose that there are no free charges inside or on the surface of the medium, then D_z is continuous across the surfaces $z = 0$ and $z = a$. Assuming that the relative permittivity is zero outside the medium and ϵ inside it, we have that,

$$E_z(z = 0^-) = \epsilon E_z(z = 0^+), \tag{1}$$

so that the medium must have $\epsilon \approx 0$.



2.2 Magnetic Properties

It suffices to consider the magnetic field due to a single (hypothetical) magnetic charge at $(x, y, z) = (0, 0, -b)$. Then, the desired pattern of magnetic field lines is the same as that for the electric field, sketched above.

Inside the medium the **B**-field lines should be only in the z-direction. If we suppose that there are no magnetic charges inside or on the surface of the medium, then B_z is continuous across the surfaces $z = 0$ and $z = a$, so the transverse component of the magnetic field must be much smaller inside the medium than outside it.

The tangential component of the magnetic field **H** is continuous across the surfaces $z = 0$ and $z = a$, so to have small tranverse $B_\perp = \mu H_\perp$ inside the medium requires the relative permeability μ to be very small, assuming that the relative permeability is unity outside the medium.

In sum, the "hose" medium should have $\epsilon \approx 0 \approx \mu$. More particularly, the medium could be anisotropic with $\epsilon_\perp$ and $\mu_\parallel$ of any value so long as $\epsilon_\parallel \approx 0 \approx \mu_\perp$, where $\parallel$ is parallel to the axis of the "hose".

No ordinary medium has these properties, but so-called (isotropic) metamaterials[1] can have zero or negative ϵ and μ, for nonzero wave frequencies. However, such materials do not have these desirable properties in the static limit. A static magnetic (but not electric) hose has been demonstrated in [1], via an anisotropic material with $\mu_\perp \approx 0$ and large $\mu_\parallel$.

References

[1] C. Navau *et al.*, *Long-Distance Transfer and Routing of Static Magnetic Fields*, Phys. Rev. Lett. **112**, 253901 (2014),
http://kirkmcd.princeton.edu/examples/EM/navau_prl_112_253901_14.pdf

[2] K.T. McDonald, *Doubly Negative Metamaterials* (Apr. 14, 2010),
http://kirkmcd.princeton.edu/examples/metamaterials.pdf

[1]For some comments by the author on metamaterials, see [2].