Meson Theory of Hyperdeuterons

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1 Problem

Estimate the relative binding energies of the 64 possible pairs of baryons in the basic octet: $n,\ p,\ \Lambda,\ \Sigma^-,\ \Sigma^0,\ \Sigma^+,\ \Xi^-,\ \Xi^0$.

For this, use a simplified one-pion-exchange model that the nuclear force is entirely due to exchanges of a single π meson, and that the operator $g^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ characterizes the charge independence of this interaction.[1] Here g is a coupling constant, and $\boldsymbol{\tau}$ is the isospin-1 operator (because pions form an $I = 1$ multiplet). That is, ignore electromagnetic effects and spin-dependent effects.

(A harder version of the problem would be to deduce that $g^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ is the appropriate operator.)

A hint is that the Hamiltonian relevant to binding of the dibaryons is $H \propto g^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$. Hence, you should consider the matrix elements $\langle B_1B_2|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2|B_1B_2\rangle$, where B is any member of the baryon octet. As for electricity, we infer that a negative matrix element implies an attractive force, and bound states, while a positive matrix element implies repulsion.

Note that,

$$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 = \frac{1}{2}\left(\boldsymbol{\tau}^2 - \boldsymbol{\tau}_1^2 - \boldsymbol{\tau}_2^2\right). \tag{1}$$

Also, charge independence means you don't have to look at each of the 64 dibaryon pairs separately, but you can more simply consider pairs of isospin multiplets, each of which leads to one or more multiplets of total isospin exactly as for combinations of ordinary spin. For this, note that the nucleons, N, and the cascade particles, Ξ, each form an isodoublet, the Λ is an isosinglet, and the Σ's form an isotriplet.

Give the isospin wavefunctions of the candidate bound states.

I found that 11 of the 64 pairs should have bounds states, and that none of these would be more weakly bound than the deuteron.

No dibaryon bound state other than the deuteron has ever been observed, although searches continue.[2] *The lightest known hypernucleus is* ${}^3_\Lambda H$,[3] *and even its antiparticle has*

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[1]If this interaction is represented by a Feynman diagram with single pion exchange, then each of the $BB\pi$ vertices has strength $g\boldsymbol{\tau}$.

[2]See, for example, B.H. Kim *et al.*, *Search for an H-Dibaryon with a Mass near* $2m_\Lambda$ *in* $\Upsilon(1S)$ *and* $\Upsilon(2S)$ *Decays*, Phys. Rev. Lett. **110**, 222002 (2013),
http://kirkmcd.princeton.edu/examples/EP/kim_prl_110_222002_13.pdf.
For a review of ongoing modeling of such possible states, see S.R. Beane *et al.*, *Light nuclei and hypernuclei from quantum chromodynamics in the limit of SU(3) flavor symmetry*, Phys. Rev. D **87**, 034506 (2013),
http://kirkmcd.princeton.edu/examples/EP/beane_prd_87_034506_13.pdf.

[3]R.J. Prem and P.H. Steinberg, *Lifetimes of Hypernuclei,* ${}_\Lambda H^3$, ${}_\Lambda H^4$, ${}_\Lambda He^4$, Phys. Rev. **136**, B1803 (1964), http://kirkmcd.princeton.edu/examples/EP/prem_pr_136_B1803_64.pdf.

been observed.[4] *A Σ-hypernucleus is ${}^4_\Sigma$He.*[5] *A handful of examples of hyper-He nuclei containing two Λ's have been reported.*[6]

2 Solution

To deal with all 64 pairs of dibaryons from the basic spin-1/2 octet, n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^-, Ξ^0, we need a compact analysis. For this, we note that for $\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_1$,

$$(\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)^2 = \boldsymbol{\tau}^2 = \boldsymbol{\tau}_1^2 + \boldsymbol{\tau}_2^2 + 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \tag{2}$$

$$\langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left(\langle \boldsymbol{\tau}^2 \rangle - \langle \boldsymbol{\tau}_1^2 \rangle - \langle \boldsymbol{\tau}_2^2 \rangle\right) = \frac{1}{2}[(\tau(\tau+1) - (\tau_1(\tau_1+1) - (\tau_2(\tau_2+1)], \tag{3}$$

noting that the expectation value of the (iso)spin operator $\boldsymbol{\tau}^2$ is $\tau(\tau+1)$. Thus, the strength of the $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ interaction is the same for all members of a multiplet of total isospin τ, and in the simplest model is the same for any dibaryon isospin multiplet of the same τ.

So, we consider the possible dibaryon isospin multiplets.

1. The $1/2 \times 1/2$ multiplets are NN, $\Xi\Xi$ and $N\Xi$, which lead to $\tau = 0$ and $\tau = 1$ multiplets with $\tau_1 = \tau_2 = 1/2$.

$$\tau = 0: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[0(0+1) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\right] = -\frac{3}{4}, \tag{4}$$

$$\tau = 1: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[1(1+1) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\right] = \frac{1}{4}. \tag{5}$$

   This model suggests that there would be bound isosinglet states $(pn - np)/\sqrt{2}$ (deuteron), $(\Xi^0\Xi^- - \Xi^-\Xi^0)/\sqrt{2}$ and $(p\Xi^- - n\Xi^0)/\sqrt{2}$.

2. The $1/2 \times 0$ multiplets are the $N\Lambda$ and $\Xi\Lambda$ states, with $\tau = 1/2$, $\tau_1 = 1/2$ and $\tau_2 = 0$.

$$\tau = \frac{1}{2}: \qquad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - (0)(0+1)\right] = 0. \tag{6}$$

   These states are not bound in this model.

3. The 0×0 multiplet is the state $\Lambda\Lambda$, with $\tau = 0 = \tau_1 = \tau_2 = 0$.

$$\tau = 0: \qquad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}[(0)(0+1) - (0)(0+1) - (0)(0+1)] = 0. \tag{7}$$

   This state is not bound in this model.

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[4] STAR Collaboration, *Observation of an Antimatter Hypernucleus*, Science **328**, 58 (2010), http://kirkmcd.princeton.edu/examples/EP/star_science_328_58_10.pdf.

[5] T. Nagae *et al.*, *Observation of a ${}^4_\Sigma$He Bound State in the ^{4}He(K^-, π^-) Reaction at 600 MeV/c*, Phys. Rev. Lett. **80**, 1605 (1998), http://kirkmcd.princeton.edu/examples/EP/nagae_prl_80_1605_98.pdf.

[6] See, for example, K. Nakazawa *et al.*, *Double-Λ Hypernuclei via the Ξ^- Hyperon Capture at Rest Reaction in a Hybrid Emulsion*, Nucl. Phys. A **835**, 207 (2010), http://kirkmcd.princeton.edu/examples/EP/nakazawa_npa_835_207_10.pdf.

4. The 1×0 multiplet is the states $\Sigma\Lambda$, with $\tau = 1 = \tau_1$ and $\tau_2 = 0$.

$$\tau = 1: \qquad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[(1)(0+1) - (1)(1+1) - (0)(0+1)\right] = 0. \tag{8}$$

   These states are not bound in this model.

5. The $1/2 \times 1$ multiplets are the $N\Sigma$ and $\Xi\Sigma$ states, with $\tau = 1/2$ or $3/2$, $\tau_1 = 1/2$ and $\tau_2 = 1$.

$$\tau = \frac{1}{2}: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - 1\,(1+1)\right] = -1, \tag{9}$$

$$\tau = \frac{3}{2}: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[\left(\frac{3}{2}\right)\left(\frac{3}{2}+1\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right) - 1\,(1+1)\right] = \frac{1}{2}. \tag{10}$$

6. The 1×1 multiplets are the $\Sigma\Sigma$ states, with $\tau = 0$, 1 and $\tau_1 = \tau_2 = 1$.

$$\tau = 0: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[0\,(0+1) - 1\,(1+1) - 1\,(1+1)\right] = -2, \tag{11}$$

$$\tau = 1: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[1\,(1+1) - 1\,(1+1) - 1\,(1+1)\right] = -1, \tag{12}$$

$$\tau = 2: \quad \langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \rangle = \frac{1}{2}\left[2\,(2+1) - 1\,(1+1) - 1\,(1+1)\right] = 1. \tag{13}$$

   This model suggests that the most tightly bound state would be isosinglet $(\Sigma^+\Sigma^- - \Sigma^0\Sigma^0 + \Sigma^-\Sigma^+)/\sqrt{3}$, and the isotriplet $\Sigma^+\Sigma^0$, $\Sigma^+\Sigma^-$, $\Sigma^0\Sigma^-$ is also bound.[7]

Taking Coulomb effects into account (which don't conserve isospin), the $\Sigma^+\Sigma^-$ part of the isosinglet would be the most tightly bound dibaryon in this model.

Unfortunately, the data do not support this model.

[7]Note that the state $\Sigma^0\Sigma^0$ does not contribute to the $\Sigma\Sigma$ isotriplet. (Similarly, the isovector meson ρ^0 does not decay to $\pi^0\pi^0$.)