Macroscopic Magnetic Forces Can Do Work

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A recent article [1] compounded all-too-prevalent misunderstandings about the phrase "magnetic forces do no work", making the absurd claim that the force $I\,d\mathbf{l} \times \mathbf{B}$ on a current element $I\,d\mathbf{l}$ in a magnetic field $\mathbf{B}$ is a static force, and is an electric force, not a magnetic force. If the current element moves (in the direction of $d\mathbf{l} \times \mathbf{B}$), this force does work, and is not a static force even if the $\mathbf{B}$ field is static.

Static forces do no work. Work is only done in dynamic situations. When work is done by electromagnetic fields, there is an associated flow of electromagnetic energy, described by the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$. Hence, there must always be both (nonstatic) electric and magnetic fields present when electromagnetic work is done.

It was reviewed in [1] that in a microscopic view of conduction electrons associated with a macroscopic electric current I, these electrons are held inside the conductor by a so-called Hall electric field, due to surface charges on the conductor. The Hall field exerts an electric force on the conduction electrons which is equal and opposite to the magnetic Lorentz force on them, if the conductor is not accelerating. However, it was claimed in [1] that we should therefore consider the magnetic Lorentz force to be the electric Hall force, which is akin to claiming that a force is the same as mass times acceleration because they are equal according to Newton's 2$^{\text{nd}}$ law, $\mathbf{F} = m\mathbf{a}$.

A more sophisticated view, given, for example, in sec. 6.7 of [2] and sec. 4.3 (and 5.1) of [3], is that the rate of transfer of energy between an electromagnetic field and a (macroscopic) current density $\mathbf{J}$ is $\mathbf{J} \cdot \mathbf{E}$, which does not involve the magnetic field $\mathbf{B}$. It is true that in a microscopic view of the current element, the magnetic part, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, of the Lorentz force on an electric charge q with velocity $\mathbf{v}$ does no work, in the sense that $\mathbf{F} \cdot \mathbf{v} = 0$. However, this does not mean that the magnetic force, $\mathbf{J} \times \mathbf{B}$, on a macroscopic electric-current density $\mathbf{J}$ does no work. Too many authors try to impose a result of microscopic electrodynamics on macroscopic electrodynamics, including refs. 6-10 of [1].

Magnetic field energy can be transformed into electric field energy, which can then transfer kinetic energy to conduction electrons. This microscopic kinetic energy is then quickly transformed into heat as well as macroscopic kinetic energy. This occurs, for example, when a superconducting magnet goes "normal", possibly breaking into pieces that fly apart. Saying that magnetic field energy cannot be transformed into heat or kinetic energy, as implied in [2, 3], because intermediate steps are involved, is a poor use of language.

Another issue is that when considering permanent magnets, whose magnetism is not associated with classical motion of electric charges, and to which the Lorentz force law does not apply, the magnetic forces do work. Permanent-magnetic field energy is converted into kinetic energy, for example, in the attraction of a refrigerator magnet to the permeable metal of the fridge. Another example of this conversion is discussed in [4].

A side issue is that the macroscopic, magnetic Lorentz force, $I d\mathbf{l} \times \mathbf{B}$, is called the Laplace force in [1], which usage will be unfamiliar to most American readers, although this is the custom in France.

A lengthy discussion of these topics is given in [5].

References

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