Uncertainties in the Measurement of the Momentum and Position of an Electron

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1 Problem

An electron is emitted from a source at the origin at time $t = 0$, and later is observed at time t by a detector at position x. We can say that the velocity of the electron when it was detected was $v = x/t$, and therefore its momentum is $P = mv = mx/t$. By making t arbitrarily large, perhaps we can make the uncertainty in momentum P arbitrarily small.

Consider the detector to be a single atom that "signals" the detection of the electron by emission of a photon upon de-excitation of the atom after its interaction with the electron. Show that the resulting uncertainties in the measurement of position and momentum obey,

$$\Delta x \Delta P \geq \hbar. \tag{1}$$

2 Solution

This problem was suggested by a colloquium by Freeman Dyson at Princeton University on Sept. 20, 2001, in which he appeared to claim that the procedure stated above could evade the Heisenberg uncertainty principle (1).[1]

The uncertainty in the electron's position after detection by the atom is at least the wavelength $\lambda\!\!\!^{-}$ of the photon emitted by the excitation of the atom,

$$\Delta x \geq \lambda\!\!\!^{-} = \frac{\hbar c}{\Delta E}, \tag{2}$$

where ΔE is the energy of that photon.

For that atom to have emitted the photon as a result of its interaction with the electron, the electron must have imparted at least energy ΔE to the atom, and in general it imparted a larger energy than this to the atom. Therefore, the energy of the electron is uncertain by an amount of order ΔE. This implies that the momentum of the electron is uncertain by the amount,

$$\Delta P = \frac{\Delta E}{v} \geq \frac{\Delta E}{c}\,. \tag{3}$$

The product of the uncertainties (2) and (3) in position and momentum of the electron after its detection by the atom is therefore,

$$\Delta x \Delta P \geq \frac{\hbar c}{\Delta E}\frac{\Delta E}{c} = \hbar. \tag{4}$$

[1]A similar claim is discussed/refuted in sec. 2-2, p. 2-3, of Vol. III of R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics* (Addison Wesley, (1964), https://www.feynmanlectures.caltech.edu/III_02.html

The size of the detector atom does not enter directly into the above argument. Hence, the limit (4) cannot be evaded by imagining larger or smaller atoms. Rather, the limit arises simply because the mechanism of detection is an electromagnetic interaction that involves the emission of a photon as the signal.