The Hooper-Monstein Paradox

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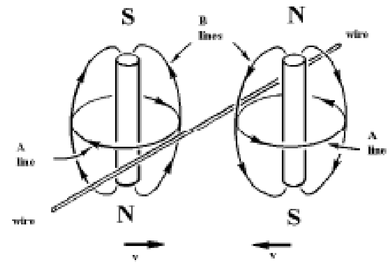
1 Problem

In 1969, Hooper, pp. 53-54 of [1] (see also [2]), considered two antiparallel magnets equidistant from a conducting wire, as shown below.



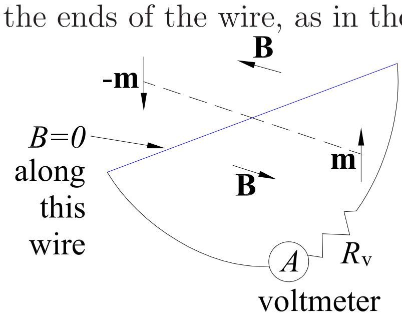
In any quasistatic configuration, the conducting wire would be an equipotential, with zero potential difference between its two ends. This includes the case that the two magnets move towards, or away from, the wire, always keeping the same distance from it, during which motion the magnetic field along the wire is always zero. However, if one measures the voltage difference between the ends of the wire during the above motion of the two magnets, a nonzero result is observed.

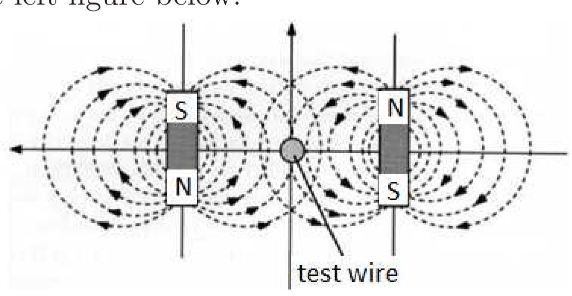
How can this be so?

2 Solution

This example illustrates that even in "classical" examples, the act of measurement can influence the observed result.

To measure the voltage difference between the ends of the wire, they must be connected to a "voltmeter",[1] which includes a resistor $R_{\rm v}$ and an ammeter A, together with two "leads" attached the ends of the wire, as in the left figure below.[2]





The loop consisting of the (blue) wire and the voltmeter has nonzero magnetic flux $\Phi = \int \mathbf{B}\cdot d\mathbf{Area}$ through it, which increases when the two magnetic dipoles $\mathbf{m}$ and $-\mathbf{m}$

[1] General discussions by the author on "voltmeters" and "voltage" are given in [3, 4].

[2] The right figure is from p. 198 of [5].

move towards one another. An $\mathcal{EMF}$ of magnitude $\mathcal{E} = d\Phi/dt$ is induced in the loop during this motion, and the observed voltage drop across the wire is,

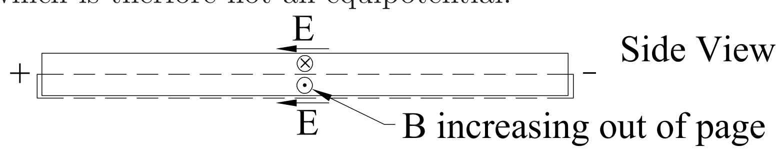
$$V \approx \mathcal{E}\frac{R}{R_{\rm v}}\,, \tag{1}$$

where R is the electrical resistance of the wire and $R_{\rm v}$ is the (much larger) internal series resistance of the voltmeter. In practice, this effect is small, and a signal of only 3 μV was reported in the experiment described in [2].

2.1 Isolated Wire

This section added April 2, 2022, based on a comment by Dennis Allen Jr.

If the wire between the two magnets (which are approaching one another) is isolated, the (weak) horizontal magnetic field inside the wires changes with time, such that a small electric field is induced in/on the wire according to Faraday's law, $\oint \mathbf{E}\cdot d\mathbf{l} = -(d/dt)\int \mathbf{B}\cdot d{\rm Area}$, which leads to an accumulation of positive charge on the "left" end of the wire, and negative charge on its "right" end. When the magnets are moving, a small current flows inside the (resistive) wire, which is therfore not an equipotential.



In particular, consider the loop shown by the dashed box in the sketch above, which is in a "vertical" plane that includes the axis of the wire. This loop is "below" the axis of the wire, and the magnetic field is out of the page there. As the two permanent magnets approach one another as in the figure on p. 1, the magnetic flux through the loop increases, which leads to the electric field $\mathbf{E}$ to the "left" as shown at the "bottom" of the wire.

References

[1] W.J. Hooper, *New Horizons In Electric, Magnetic & Gravitational Field Theory* (1969), http://kirkmcd.princeton.edu/examples/EM/hooper_69.pdf

[2] C. Monstein, *Electromagnetic Induction Without Magnetic Field*, Deutsche Physik **1**(4) (1992), http://kirkmcd.princeton.edu/examples/EM/monstein_dp_1-4-92.pdf
Elektromagnetische Induktion ohne Magnetfeld, NET-J. + SAFE-News, **2**(6/7), 22 (1997), http://kirkmcd.princeton.edu/examples/EM/monstein_hooper.pdf

[3] K.T. McDonald, *What Does an AC Voltmeter Measure?* (March 16, 2008), http://kirkmcd.princeton.edu/examples/voltage.pdf

[4] K.T. McDonald, *Voltage Drop, Potential Difference and $\mathcal{EMF}$* (Sept. 18, 2012), http://kirkmcd.princeton.edu/examples/volt.pdf

[5] O. Crane, J.-M. Lehner and C. Monstein, *Central Oscillator and Space-Quanta-Medium* (Universal Export Publishers, 2000), http://kirkmcd.princeton.edu/examples/EM/crane_00.pdf