# Ball-Bearing Motor

Kirk T. McDonald
*Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544*
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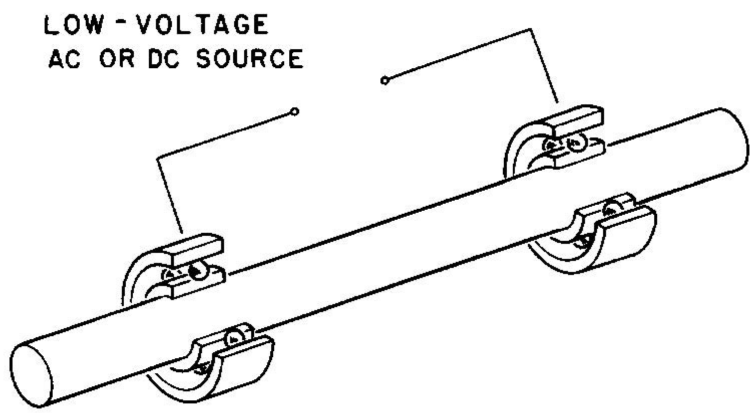
## 1 Problem

Discuss the principle of operation of a so-called ball-bearing motor, a popular form of which is sketched below (from [2]).[1]




The motor can start from rest (although not all observers report this [1]-[24]), and rotates in either sense, if the current (AC or DC!) is large enough.

## 2 Solution

The discovery/invention of the ball-bearing motor is attributed to Milroy [1]. It has been discussed several times, with conflicting explanations [2]-[24].[2,3] This solution is a much simplified version of that given in [10], and is based on the Lorentz force.

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[1]Numerous videos of variants of this device are available on YouTube.

[2]Most authors except Marinov [7] suppose the torque to be due to a Lorentz force, whereas he stated (without supporting argument) that the torque results from thermal expansion of the bearings.

At the end of sec. 2, I note that in some sense Marinov was correct, and the Lorentzian analysis that occupies most of sec. 2 is not the dominant cause of the motion.

It is generally agreed that the first attempt at an electromagnetic explanation, [2], was wrong. Also, the explanation in [11] was later retracted by its author.

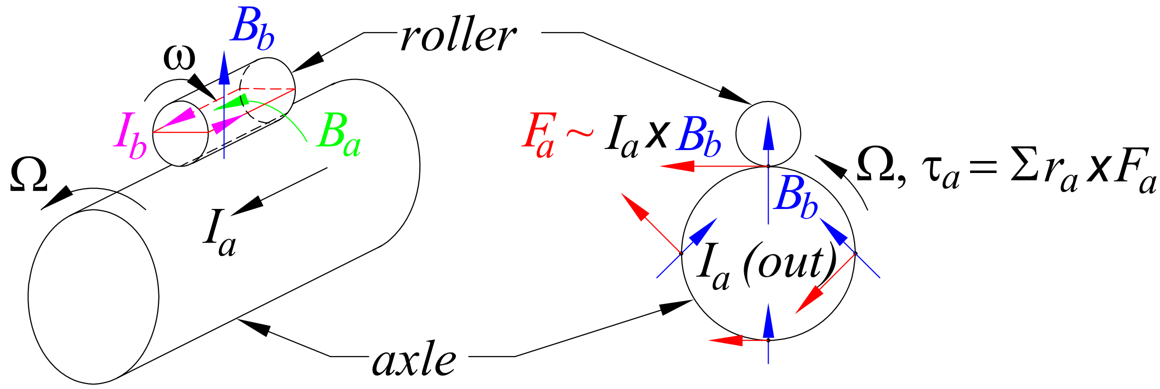
[3]Refs. [6, 19, 22, 23] associate ball-bearing motors with the "Huber effect", after [25]. See also Appendix A below, and [26]-[29]; the last paper reviews the Russian literature on this topic.

Huber considered a kind of railgun in which the crosspiece, a sphere or cylinder, rolled along tracks, supposedly propelled by the Lorentz force on the current in the crosspiece. However, [26] reported sparking between the wheels and the rails on the trailing side of the motion, which could imply that thermal effects, rather than the Lorentz force, are important here.

The "Huber effect" was described already in 1858 by Gore [30]-[32]. He reported [30] that after the electric current started, the roller first vibrated, and eventually rolled either toward of away from the current source. Sparking on the trailing side of the roller was reported in [31]. See also p. 195 of [33]. For some general comments about railguns, see [34].

Because the motor is weak, it is helpful to reduce friction on the axle by connecting the high-current lead to the outer races of the ball bearings, as shown above. However, I believe that this is not required in principle.

It is simpler to analyze the interaction of a rotating, current-carrying axle[4] with a single roller bearing, both of whose axes are fixed,[5] as shown below.




The axle has angular velocity $\mathbf{\Omega}$, and the angular velocity $\boldsymbol{\omega} = -(a/b)\mathbf{\Omega}$ of the roller has the opposite sign, where $a$ and $b$ are the radii of the axle and bearing, respectively. The axial current $\mathbf{I}_a$ in the axle generates azimuthal magnetic field on the roller, $\mathbf{B}_a \approx 2\mathbf{I}_a/c\,a$ in Gaussian units. The Lorentz force $\mathbf{F} = e\mathbf{v}_b/c \times \mathbf{B}_a = e(\boldsymbol{\omega} \times \mathbf{r}_b)/c \times \mathbf{B}_a = e\mathbf{E}_{\rm eff}$ on the conduction electrons of charge $e$ in the roller leads to a current $\mathbf{I}_b \propto \sigma_b E_{\rm eff} \approx 2\sigma_b\,\omega\,bI_a/c^2\,a$ that circulates around the roller.[6] This current $\mathbf{I}_b$ (associated with magnetic moment $\mu_b = \pi I_b\,b^2/c$) generates a (dipole) magnetic field $\mathbf{B}_b \approx [3(\boldsymbol{\mu}_b \cdot \hat{\mathbf{r}})\,\hat{\mathbf{r}} - \boldsymbol{\mu}_b]/r^3$ that is generally perpendicular to the axle inside the latter. The consequent magnetic forces $\mathbf{F}_a \propto \mathbf{I}_a/c \times \mathbf{B}_b$ on current filaments in the axle vary over the axle,[7] but the strongest force is near the line of contact of the axle and roller, $F_{a,\max} \approx I_a\mu_b/c\,b^3 \approx \pi I_a I_b/c^2\,b \approx 2\pi\sigma_b\,\omega I_a^2/c^4\,a$, where the force exerts a torque on the axle, $\tau_a \approx aF_{a,\max} \approx 2\pi\sigma_b\,\omega I_a^2/c^4 = -2\pi a\sigma_b\,\Omega I_a^2/c^4\,b$, that has the same sense as the angular velocity $\mathbf{\Omega}$, thereby increasing (or at least maintaining against friction) the angular velocity of the axle.[8,9]

This argument holds for either sign of angular velocity $\mathbf{\Omega}$ (whose sign is opposite to that of angular velocity $\boldsymbol{\omega}$), but does not explain the sign of $\mathbf{\Omega}$.

The magnitude of the torque scales as $a\sigma_b\Omega I_a^2/c^4\,b \propto I_a^2$ (in Gaussian units), where $c$

[4]The axle could be either a hollow or a solid conductor.

[5]In practice, the roller (or ball) bearings are encased in a "race" that can rotate with respect to both the axle and the outer sleeve. Then, the axes of the roller bearings move azimuthally, which complicates the motion, but which does not change the essence of the analysis given below.

[6]When there is no rolling, there is no current $I_b$, and no Lorentz force $\mathbf{I}_a/c \times \mathbf{B}_b$ on the axle. Hence, the motor cannot start from rest due to Lorentz-force effects (unlike a railgun). Since a ball-bearing motor is observed to be able to start from rest, the magnetic analysis given here cannot be the full story.

[7]The current filaments have helical form due to the rotation of the axle, but their azimuthal component does not lead to an azimuthal torque.

[8]Another argument notes that parallel currents attract, such that a current filament along the top of the axle is attracted to the left current around the bearing, and repelled from the right current. The net force on the current filament in the axle is to the left, and the reaction force on the bearing is to the right.

[9]*June 1, 2020.* Derek Abbott notes that the Lorentz force due to $\mathbf{B}_b$ on conduction electrons near the top of the roller opposes the electric force that drives current $I_a$, and so slightly reduces that current compared to the case of no rotation. This is somewhat counterintuitive, but does not violate conservation of energy.

is the speed of light. This behavior was observed in the experiments of [23].[10] Since the conductivity $\sigma_b$ is of order $c$ in Gaussian units, the torque scales as $I_a^2/c^3$. This confirms that a ball-bearing motor is a very weak device.

*An alternative configuration is for the axle to be held fixed while the bearings rotate about it. If the bearing race were fixed to the outer sleeve of the bearing, then a torque (clockwise in the above figure) on the bearings could be transmitted to the latter, providing another type of motor. This configuration would result in friction between the bearings and the bearing race, which might limit the utility of the motor.*

*In yet another variant (thanks to Alexis Bacot), the axle is fixed and nonconducting, and the current enters and leaves the system through lead wires attached to the inner races of the bearings; the outer races of the two bearings are attached to a conducting cylinder that rotates along with them. The currents in the lead wires generate the magnetic field* $\mathbf{B}_a$*, which has sufficient radial field component on the bearings that the above analysis still applies.*

*March 2016*. Experiments by the author with a ball-bearing motor (thanks to Omelan Stryzk), and with the configuration of Gore/Huber, strongly suggest that thermal effects are responsible for the startup of the rolling motion of these systems, which was observed to be in both possible directions in different trials. Furthermore, no motion was observed in the Gore/Huber configuration when the rods and roller were made of Invar (with "zero" thermal expansion).

It remains that once the a ball-bearing motor is started, Lorentz-force effects can contribute to its steady state. The ball-bearing motor is another example of a system that in which magnetic forces/torques do work [36].[11]

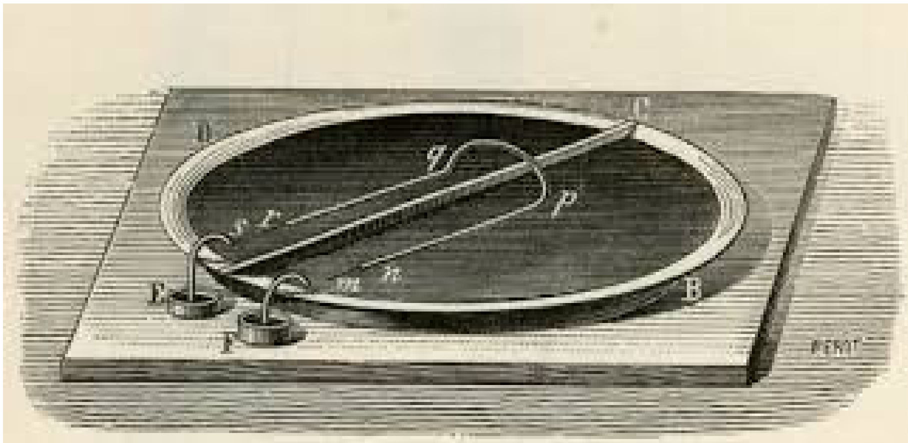
[10]The view that the ball-bearing motor is driven by thermal effects also predicts that the torque would scale as $I_a^2 R_a$, where $R_a$ is the resistance of the axle.

[11]For another recent, amusing example of a system in which magnetic forces do work, see [37].

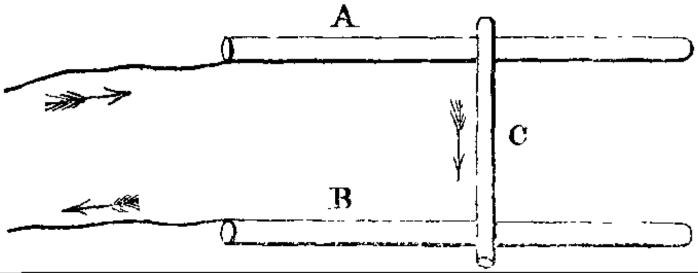
# A Appendix: The Gore/Huber Effect

*This Appendix written May 20, 2020.*

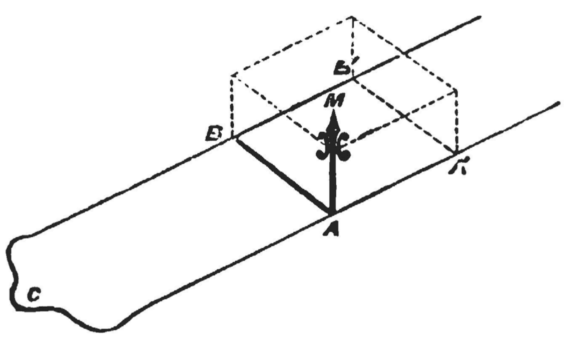
Gore [30]-[32] (1858), and Huber [25, 27] (1959), considered variants of the experiment of Ampère and de la Rive [38] (1822), sketched below, in which the crosspiece $pq$ was replaced by a roller, and the conductors $qr$ and $np$ were fixed rails (rather than part of the "hairpin" $npqr$ floating on mercury).




One of Gore's configurations, from [30], is sketched below.



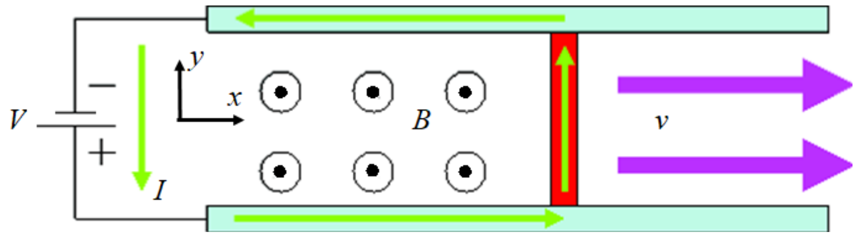

This configuration, but with a sliding crosspiece (now called a railgun [34]) rather than a roller, was discussed by Maxwell in Arts. 594-596 of [39], as sketched in the figure below.[12]

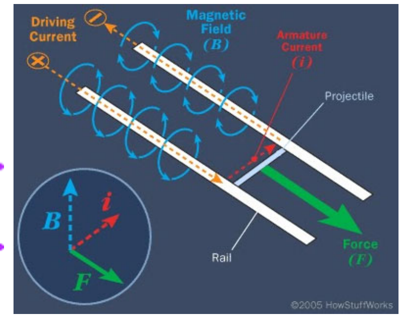



A Lorentz-force explanation of the railgun experiment (and of Ampère's) is that when a battery is connected to the "rails", a magnetic field is generated with a component in the

[12]In Arts.594-596 consider this example as a dynamo, if the crosspiece is slid by a mechanical force while a magnetic flux $\Phi(t)$ is linked by the circuit, then the induced $\mathcal{EMF}$ is $-d\Phi/dt$, eq. (14), Art. 595. In Art. 603, eq. (11), Maxwell discussed the $\mathbf{I} \times \mathbf{B}$ force, but did not apply this to the railgun.

$z$-direction (out of the page in the left figure below) when the current in the crosspiece is in the $+y$ direction, such that the $\mathbf{I} \times \mathbf{B}$ force on the moving crosspiece is in the $+x$ direction, away from the battery (for either polarity of the latter). [13]

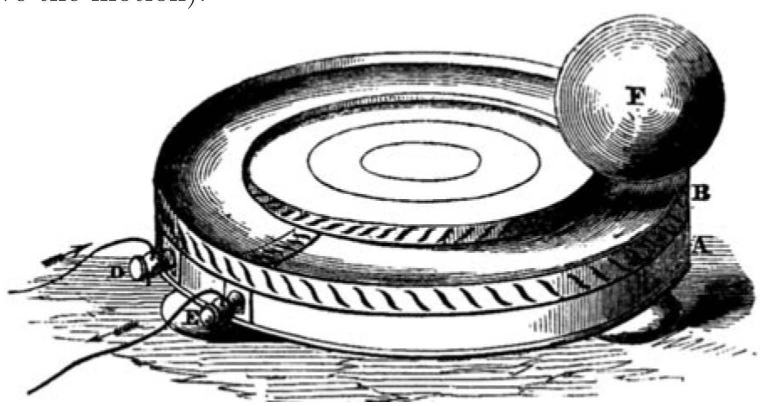



In these experiments with sliding contacts, the current density (for a given total current) at the sliding contact is not as large as that in experiments with a rolling contact, and the Lorentz-force explanation is largely satisfactory.[14]

In contrast, in the experiment of Gore [30] sketched on the previous page, with roller C as the crosspiece, the roller can move in either direction, after some initial vibration. In addition, sparking is observed at the points of contact between the roller and the rails, where the current density is maximal.[15]

Furthermore, when Gore studied the motion of a sphere on circular tracks, as sketched below, the sphere would roll steadily in complete circles, in either direction, after the vibratory startup. Whereas, the Lorentz-force on the sphere is away from the connection of the battery to the rails, and goes to zero at the diametrically opposite point, such that the motion of the sphere would not be steady, and would not complete a full circle (if only the Lorentz force drove the motion).




It seems clear that in the experiments of Gore (and also in those of Huber), the effect was primarily thermal (as inferred by Gore, but not by Huber).

[13] *June 16, 2024.* An alternative configuration has the upper (negative) terminal of the battery connected to the far right end of the upper rail. This again leads to a magnetic field out of the paper near the intersection of the upper rail with the crosspiece, and again implies a Lorentz force on the rail the right.

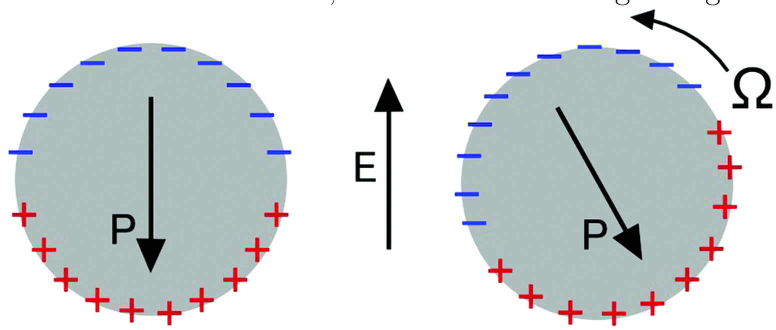
[14] The Lorentz force in railgun experiments can start the motion from rest, unlike the Lorentz forces and torques in the ball-bearing motor.

[15] *June 16, 2024.* The motion occurs even when the apparatus is in vacuum (Derek Abbott, private communcation), which suggests that pressure effects associated with the hot plasma of sparks is not the dominant driving force. It remains that localized thermal expansion of the roller and rail near the point of contact can be important (as argued by Marinov [7]).

## B Appendix: The Quincke Effect

*This Appendix written May 27, 2020.*

Another surprising motor is that based on the Quincke effect [40]-[57],[16] in which a dielectric, but slightly conducting, sphere placed in a dielectric, but slightly conducting, liquid in a uniform, external electric field, as sketched below (from [56]), will rotate about an axis perpendicular to the electric field, if the latter is strong enough.[17]




The external electric field **E** polarizes the dielectric sphere, with nominal polarization **P** opposite to the electric field. However, under certain conditions, there occurs a (nominally classical) spontaneous symmetry breaking such that the polarization vector **P** takes on an angle with respect to the electric field, as in the right figure above. The resulting torque, $\mathbf{P} \times \mathbf{E}$ rotates the sphere, while the polarization vector **P** maintains a constant direction relative to **E**.

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[16] The early papers on the Quincke effect all credit Hertz' dissertation (1880) [58] for inspiration.

[17] The related phenomenon of ionic currents in a liquid surrounding a fixed sphere in a rotating electric field has been studied since 1892 [59], and is sometimes called the Born-Lertes effect [60].

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