3   Motion in Two Dimensions

Introduction

In this Laboratory you will explore the insight of Descartes that space can be described as having three dimensions. Newton added the understanding that forces and consequent changes in motion in three dimensions can be broken up into three one-dimensional problems that can be analyzed separately.

The classic example is motion under the influence of gravity. On the scale of a person the force of gravity is constant and 'vertical' is defined by the direction of that force. Objects falling through the air have only a vertical force (ignoring air resistance) and so accelerate only vertically.

In this Laboratory you will study motion in only two dimensions – a vertical plane – with the aid of a stroboscopic camera. Calling the horizontal axis x and the vertical axis y the equations of motion of a falling object have the form

$$x = x_0 + v_{0x}t \tag{1}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2, \tag{2}$$

taking the y-axis as positive upwards and g as the acceleration due to gravity.
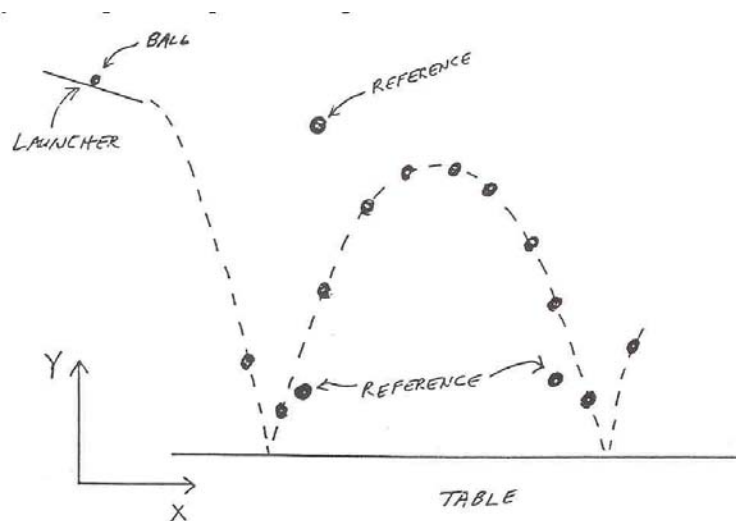

Figure 1: Sketch of the experiment.

The experimental setup is sketched in Fig. 1. You let a ball roll down a short launcher, leading to a motion that should include two bounces on the table. The camera takes a number of short exposures during the motion. A backdrop at the rear of the table is used for alignment and calibration of the camera. By measuring the positions of the various images on the ball's trajectory you should be able to confirm eqs. (1) and (2) and determine the value of g.

3.1   Setup, Alignment and Calibration of the Camera

3.1.1   Camera Setup

Your camera is an Electrim Model EDC-1000 with an CCD (charge-coupled device) that contains 192(H) × 165(V) pixels each 13.75×16 $\mu\mathrm{m}^2$. Each frame has an exposure time between 0.001 and 10 sec and a readout time of about 0.05 sec. Thus the maximum frame rate is about 20/sec; the default is only 4/sec.

The wide-angle lens on your camera has focal length only 3.6 mm. It has both a focusing ring and an iris ring. Only a rather limited region on the iris ring will result in a good exposure. On the other hand, the focus ring is not very sensitive.

The camera is controlled by a Windows *program in your PC: select the* Physics 101 *program group, and then* Camera*. There are several ways to control the camera: basic functions can be operated from the bars along the right side of the screen; along the top are pull-down menus as well as a row of function buttons; some parameters can also be set via bars along the bottom.*

Click on the bar next to Live Mode *until it reads* On *to let the camera acquire images continuously.* Anti-Blooming *should be* On*; if not, click on the corresponding bar. 'Blooming' is the tendency for an overexposed pixel to leak charge into neighboring pixels.*

One way to take a picture is to click on the bar next to Single Frame*. (You may have to click more than once before the computer responds. When you wish to save a captured image to disk use* Save Image *under the* File *menu; use a .tif extension and store the image in the directory* c:\Phys101*.*

A captured image can be altered in a number of ways using options from the Image Utilities *menu. Try it.*

Images can be Print*ed from the* File *menu. If your image is predominatly black, please use* Print Negative *instead to conserve toner. A* Paintbrush *window pops up with your image converted to .bmp format. Click on* Print *from the* Paintbrush *file menu to bring up the* Print *window. You can choose the* Number of Copies *as desired, and set the* Scaling *to 400% for the image to fill the page.* Windows *experts can save the .bmp file to disk for use in the screen saver.*

Exit *the* Paintbrush *window after printing to keep from loading the PC with too many windows.*

3.1.2   Alignment

In preparation for data taking the camera should be positioned so the the three reference circles on the backdrop appear at corners of the image. Adjust the lighting and iris ring so that the thumbtacks in the center of to white circles can be seen. Capture an image of the reference marks and Save it to disk.

You now should verify that the camera is aligned so that the reference marks (the thumbtacks) define horizontal and vertical axes. If they don't, rotate the camera until the alignment is better. The following procedure permits you to be quantitative.

If Live Mode *is* On*, turn it off to free up the computer. Minimize the* EDC-1000 Imaging software *window to an icon by clicking on the button with a single downward arrow in the upper right corner of the screen. Click on the* Point-n-Click *icon in the* Physics 101 *window*

to activate the image digitizing window. Use Open Image *of the* File *menu to bring your captured and saved image into this window.*

When the mouse cursor is moved onto the image the x-y coordinates are displayed in CCD-pixel units. If you are not in Cartesian *mode of the* Point/Vector Mode *menu, go to this mode. Center the cursor on each of the three reference marks and click to record their positions. When you have a good alignment there should be only 0 or 1 unit difference between the x-coordinates of the two left reference marks, and between the y-coordinates of the two bottom reference marks. If the alignment is poor, minimize the* Point-n-Click *window and restore the* EDC-1000 *window. Turn* Live Mode *on and rotate the camera as needed. If a simple rotation of the camera is insufficient you may need a translation as well.*

You may wish to print out your table of the digitized reference marks. However, this cannot be done directly from Point-n-Click *(whose* Print *command prints the camera image). Instead, use* Save Data *to save your file to disk with a* .txt *extension. Minimize the window and click on* Notepad. Open *your data file and* Print *it from the* Notepad *window.* Point-n-Click *data files are in directory* c:\phys101. *(It's probably simpler just to write the numbers into your book!)*

Print Negative *the image of final alignment for your lab book.*

3.1.3   Calibration

Calibration consists of converting CCD-pixel units to centimeters. A slight complication is that we desire calibration constants relevant to the images of the bouncing ball to be acquired later, rather than those for the reference marks on the backdrop. That is, because the camera is closer to the plane of the ball than to the backdrop, an image spanning a given number of pixels corresponds to a smaller distance at the plane of the ball than on the backdrop.

From the digitization of the reference image performed above calculate Δx(pixels) *between the two bottom reference marks, and* Δy(pixels) *between the two left marks. Use a meter stick to determine the corresponding* Δx(cm) *and* Δy(cm). *Also measure the distance* D *from the front face of the camera to the backdrop, and distance* d *from the camera to the plane of the bouncing ball = plane of the launcher. Your calibration constants are then*

$$C_x = \frac{d}{D}\frac{\Delta x(\text{cm})}{\Delta x(\text{pixels})} \qquad \text{and} \qquad C_y = \frac{d}{D}\frac{\Delta y(\text{cm})}{\Delta y(\text{pixels})}. \tag{3}$$

3.2   Trajectory of a Bouncing Ball

The main part of the lab consists of comparison of the path of a bouncing ball with eqs. (1) and (2). Use the Multiple Frame Acquisition mode of the camera to capture a stroboscopic image of the ball's flight, then digitize this image with Point-n-Click and analyze the data with StatMost.

Each lab group can analyze a common image of the bouncing ball.

Practice the technique of bouncing the ball before recording images. The ball should bounce first near one of the lower white circles on the backdrop, and then follow a high arc before bouncing again near the other lower white circle. The plane of the ball's flight should

be parallel to the backdrop. If the ball hits the backdrop during it's flight, motion according to eqs. (1) and (2) will not occur.

3.2.1   Multiple Frame Acquisition

To take the stroboscopic image activate the EDC-1000 *window. Use* Set Exposure Time *of the* Image Capture *menu to reduce the exposure time to, say, 10 ms = 0.01 sec. (The* delay time *can be left at 0.)*

You can, of course, check the state of the camera by setting Live Mode *on. Because of the short exposure time you will likely need to open the iris of the lens. Even so it may not be easy to see the image of the ball after capturing a multiple frame exposure. Also, with a short exposure time the image may contain streaks. Some image processing will be necessary before you can judge whether the camera is set up properly.*

Click on the bar next to MFA frame number *and to choose a multiple exposure of 12 or 16 frames. To take a multiple exposure, first turn* Live Mode *off. Click on the bar next* MFA Start *to bring up the window* Multiple Frame Acquisition *that actually initiates the capture. When you hit the* spacebar *the computer takes a multiple exposure and averages the frames. The person who operates the computer and the person who launches the ball should coordinate their timing so a full arc of the bouncing ball is captured by the camera.*

Save *the image to disk if you intend to process it further. The average time between exposures is displayed in the box to the right of* MFA Frame Time. *The units are milliseconds (ms = 0.001 s).*

To begin the image processing first click on the bar to the right of Background Image *at a time when the ball is not in front of the backdrop but the lighting is the same as during your multiple frame exposure. Then use* Subtract background frame *of the* Image Utilities *menu to produce a new image that contains almost nothing but the multiple images of the ball.*

However, the ball may still not be visible. Try clicking on Streak correction, *and then on (most significantly)* Optimized contrast. *If the images of the ball are not visible now the iris may have been misadjusted, or you may have taken the exposure before or after the ball was in front of the backdrop.*

When you have a processed image that shows at least 8 exposures of the ball in flight, Save *it to disk.*

Print *a copy of your processed image, following the procedure in sec. 3.1.1.* **Please use the** Print Negative **option to conserve toner.** *Figure 2 shows a processed image of a ball in flight.*

3.2.2   Digitization

Use the Point-n-Click *window to digitize the center of the ball at each of the multiple exposures. However, include only frames taken after the first bounce and before the second. It is helpful to digitize from left to right.* Save *the digitized data to disk using a* .txt *extension.*

3.2.3   Analysis

Use StatMost *to create a* New Sheet Document *and use* Import Data *on your* .txt *file in directory* c:\phys101. *The data file should be* Delimited *by* Commas.
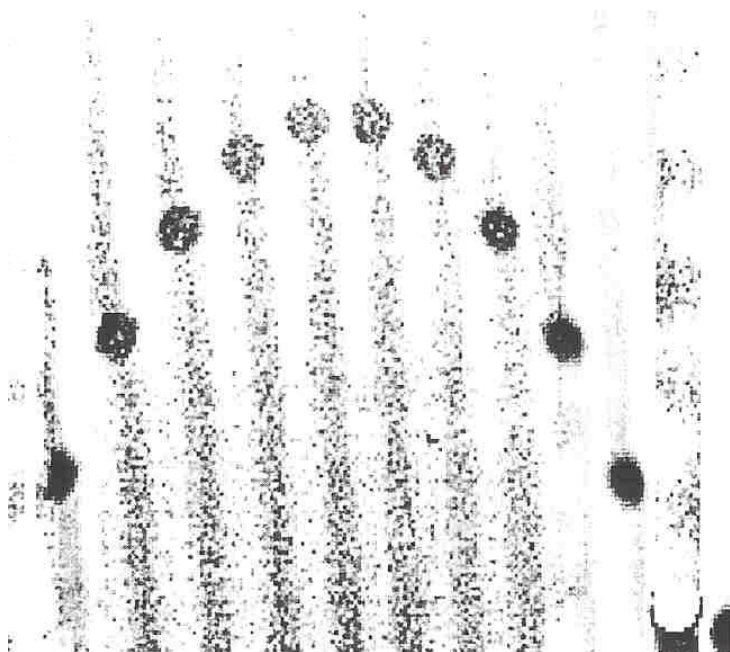
Figure 2: Processed multiple frame exposure of a ball in flight.

The two columns of your new spreadsheet contain the x- and y-coordinates of the ball's trajectory in pixel units. Convert these to cm using the calibration constants you calculated in sec. 3.1.3. Select Data, *then* Transform, *then* Simple Math. *Enter an equation such as* X=0.5*A *if your calibration constant is $C_x = 0.5$ and the x pixel values are in column* A, *etc.*

Also enter the exposure times into the spreadsheet. This might be done with a trick such as filling a new column with integers 0, 1, ... n and then using Simple Math *to multiply this column by the time (in seconds) between exposures found in sec. 3.2.1. Label the resulting new column as* Time.

Perform an analysis of y vs. time using Polynomial Regression *with the* Graph *option. Use a polynomial of one order higher than expected. That is, since y should vary quadratically with time, fit it to a third-order polynomial to see if the $n = 3$ coefficient is 'small'.*

Extract a value of the acceleration of gravity from the $n = 2$ coefficient of the y vs. t analysis. If this value is not within 1 or 2 standard deviations of the nominal value, or if the order-3 coefficient is not statistically close to zero check your calibration constants, time between exposures, etc., and repeat the analysis or even take a new picture.

Make similar analyses for your data on x vs. time and y vs. x. What functional form do you expect for the curve of y vs. x given eqs. (1) and (2)?

Print your spreadsheets, graphs and statistical-analysis summaries only once you have obtained good results.

Do your results confirm that Descartes and Newton gave valid descriptions of the motion of simple systems?

3.2.4   Simplified Error Analysis

As check on whether the results of your analysis are reasonable a simplified error analysis is useful. As in sec. 2.2.2 of Lab 2, the present experiment can be thought of as a measurement of the relation

$$y = \frac{1}{2}gt^2, \qquad \text{so that} \qquad g = \frac{2y}{t^2}, \tag{4}$$

and hence according to eq. (16) of Lab 1

$$\frac{\sigma_g}{g} \approx \sqrt{\left(\frac{\sigma_y}{y}\right)^2 + 4\left(\frac{\sigma_t}{t}\right)^2}. \tag{5}$$

Estimate σ_y and σ_t based on your understanding of the apparatus to arrive at an estimate for σ_g. Compare with your results from sec. 3.2.3.