Physics 101 Learning Guide

Department of Physics
Princeton University

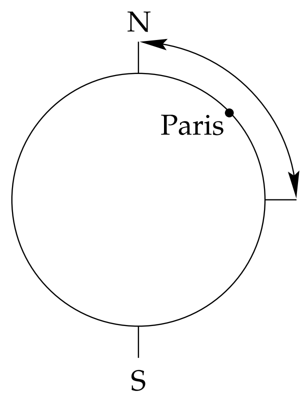
The "Learning Guide" provides a programmed guide for working through physics problems. In addition to breaking tough problems up into sub-components, there are hints and the answers are arranged to limit the possibility that you might see the solution to a problem before you have had a chance to think about it.

Many faculty members have contributed to the current compilation. The first learning guide appeared in 1973 and was last updated by D. Nice and T. Shutt in 2003.

The Department dedicates this work to the memory of Professor Tomas R. Carver, who did so much to improve physics instruction at Princeton.

Learning Guide 1

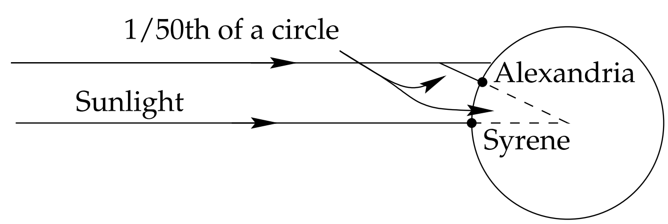
Problem I: The Meter



The International Prototype Meter was originally intended to have a length equal to exactly 1.0×10^{-7} times the distance from the equator to the North Pole on the meridian passing through Paris. The Earth is now known to be closely approximated by a sphere of radius 6.3676×10^6 meters.

1. What is the circumference of the earth in kilometers? Key 21
2. By what fraction of its length does the meter fall short of its intended length? Key 42

Problem II: The Circumference of the Earth



During the third century BC, Eratosthenes "measured" the circumference of the Earth as follows. At a time when the sun was directly overhead in Syrene (modern Aswan, Egypt), he observed that at Alexandria, the angle between a vertical pole and a line drawn from the pole to the sun was "one-fiftieth of a circle." The distance between Syrene and Alexandria is 830 km.

1. Using Eratostenes' data, calculate the Earth's circumference. Key 20
2. What assumption about the distance from the Earth to the Sun has been made in making the calculation? Key 41

(If you are having difficulty, see Helping Question 1.)

Problem III: The size of the Sun and the Moon

1. To an observer on earth, the moon, which is 3.8×10^5 km away, subtends an angle of $0.52°$. What is the moon's diameter? Key 40

2. It happens that the sun subtends almost the same angle at earth as the moon. The distance to the sun is 1.5×10^8 km. What is the diameter of the sun? Key 18

Problem IV: Atomic Scale of Length

Diamond, which is made of carbon atoms, has a density of $3.2\,\mathrm{gm/cm^3}$. The mass of a carbon atom is 2.0×10^{-26} kg.

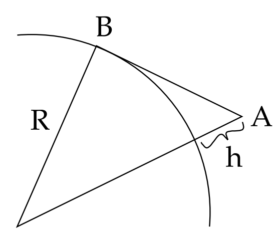
1. What is the volume occupied by a single carbon atom in diamond? Key 39

2. If this volume is a cube, what is the length of a side of the cube? Key 17

3. How many carbon atoms are there in a meter cubed of diamond? Key 38

(If you cannot get this, see Helping Question 2.)

Problem V: The Lookout

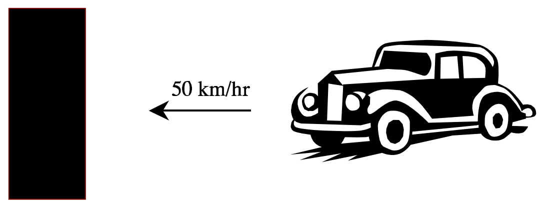


1. Show that the distance from the horizon to the eye of an observer, A a height h above the surface of a spherical earth radius R, is $L = (h^2 + 2Rh)^{1/2}$.
(See Helping Questions 3 and 4 if necessary.)

2. Show that $L \simeq (2Rh)^{1/2}$ if $h \ll R$
(If you have trouble, Helping Question 5 can be used.)

3. In the same approximation, what is the furthest distance at which an island of height $h = 300\,\mathrm{m}$ can be seen by the lookout in the crow's nest of a ship $100\,\mathrm{m}$ above the surface of the water? Key 14
(Use Helping Questions 6 and 7 if you have to.)

Problem VI: Fasten Your Seat Belts!



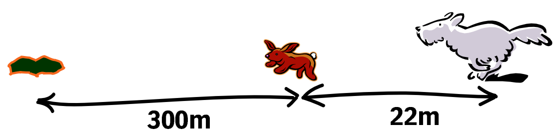
1. A car travelling at 50 km/hr hits a stone wall and is brought to rest. A passenger wearing a seatbelt comes to rest in 1 m. What is the acceleration experienced by this passenger? Key 34

2. A passenger in the same car and not wearing a seatbelt strikes the windshield and comes to rest in 0.01 m. What is the acceleration experienced by this passenger? Key 12

3. An airplane achieves a speed of 200 km/hr from rest along a 1 km long runway. What is the acceleration of the airplane, and how long is it on the runway? Key 33

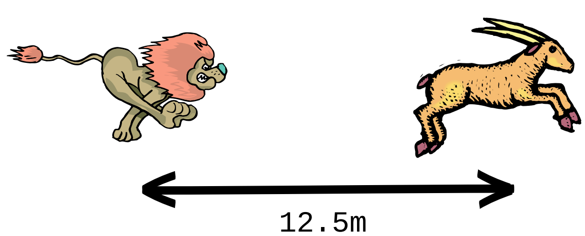
(If you have trouble, review Helping Question 8.)

Problem VII: The Hounded Hare



A dog is chasing a rabbit which has 300 m to go before it reaches its burrow. The rabbit is running at 15 m/s and the dog at 16 m/s. The dog is 22 m behind the rabbit. Does the dog catch the rabbit? Key 32
(See Helping Questions 9 and 10.)

Problem VIII: The Unfortunate Antelope

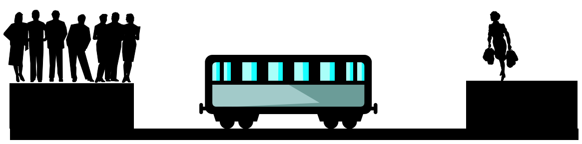


A grazing antelope notices a lion attacking it when the lion is 12.5 m away and moving towards the antelope at 5 m/s. The antelope begins to accelerate away from the lion at 3 m/s^2 and the lion simultaneously begins to accelerate at 2 m/s^2.

1. How long does the antelope's flight last? Key 9
2. How far has the antelope travelled when the lion catches up with it? Key 30

(Go to Helping Question 11 if necessary.)

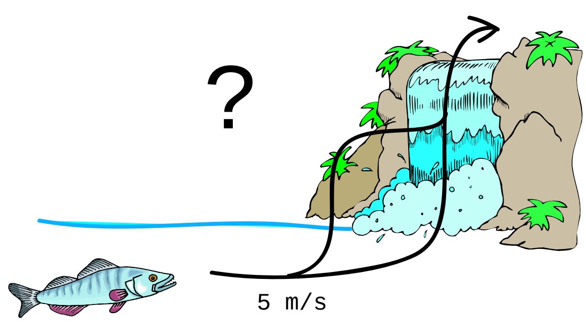
Problem IX: The Subway



Suppose that the maximum possible acceleration for passengers standing on a subway is 0.1 m/s^2 and that the subway stations are located 0.7 km apart.

1. What is the minimum possible time taken between stations? Key 29
2. What is the maximum speed in meters per second and in kilometers per hour that the train can attain between stations, assuming that it accelerates uniformly for half of the journey and then decelerates uniformly for the remaining half? Key 7
3. What is the average speed on such a journey, in meters per second and kilometers per hour? Key 28

Problem X: The Salmon versus the Waterfall



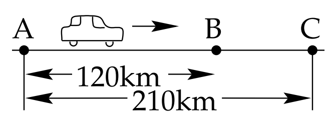
A salmon has a maximum swimming speed of 5 m/s in still water. If it encounters a waterfall while swimming upstream, it will attempt to ascend the fall in one of two ways. If it can swim fast enough, it will swim up the falls. If it cannot, it will jump from the base of the waterfall into the waterfall at a height where the water is moving slowly enough that it can swim up the rest of the fall. Assume that the water is at rest at the top of the falls and at their base.

1. What is the maximum height of a waterfall which a salmon can swim up without jumping? Key 6
2. If the falls are 1 meter high, what is the speed of the fish with respect to the ground when it begins to swim upward at the bottom of the falls? Key 27
3. If the falls are two meters high what is minimum height in the stream to which the fish must jump in order to be able to swim up to the top of the falls? Key 5
4. What must the fish's initial velocity be when it leaves the water in order to reach this height? (Assume that the fish jumps vertically). Key 26

(Helping Questions 12 and 13 may be of use.)

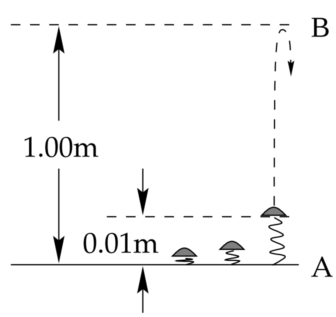
Problem XI: Typical Quiz Problem

Part A. A driver travels with constant speed from A to B in 2 hours, changes speed and then travels again at constant speed from B to C in 3 hours.



1. What is the speed between A and B? Key 3
2. What is the speed between B and C? Key 24
3. What is the average velocity between A and C? Key 2

Part B. A toy cricket is stuck to the floor with a suction cup which after a few moments releases to allow a spring to propel the cricket upward. The expanding spring accelerates the cricket over a distance of 0.01 m to point A, after which the toy travels freely to a maximum height of 1.00 m above the ground (point B).



1. What is the velocity at point A just after lift off? Key 23
2. What is the time off flight between A and B? Key 1
3. What is the net acceleration during the time spring expands? Assume uniform acceleration. Key 22

Helping Questions

1. What is the angle between the lines from the center of the earth through Alexandria and Syrene? Key 19

2. What is the relationship between volume V, mass m and density ρ? Key 16

3. What is the distance from the observer to the center of the earth? Key 37

4. Apply Pythagoras' theorem to the triangle OBA. Key 15

5. Express L in terms of $h/2R$ (which is much less than one). Key 36

6. Draw a diagram showing the relationship between the island of height h, and the lookout of height ℓ at the time when the lookout first spots the island. Key 35

7. What are the distances A and B in the answer to the previous Helping Question? Key 13

8. What is the relationship between distance travelled, d, and the difference between the squares of the final and the initial velocities, v_f and v_i? Key 11

9. How does the distance between the dog and the rabbit depend on the time? Key 10

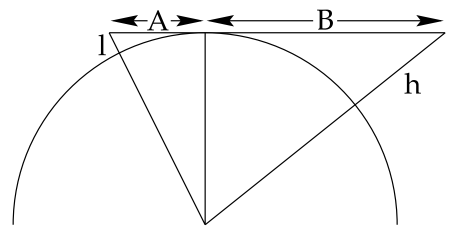
10. How does the distance between the dog and the rabbit depend on the distance covered by the rabbit? Key 31

11. Find the distance D between the lion an the antelope in terms of the acceleration of the lion a_L, the acceleration of the antelope a_A, the initial velocity of the lion v_L, the initial distance D_i between the lion and the antelope and the time t during which the antelope has accelerated. Key 8

12. If the waterfall is h meters high, what is the velocity of the water just before it hits bottom? Key 4

13. If the fish jumps up with initial velocity v_i, at what height above the ground will its velocity be zero? Key 25

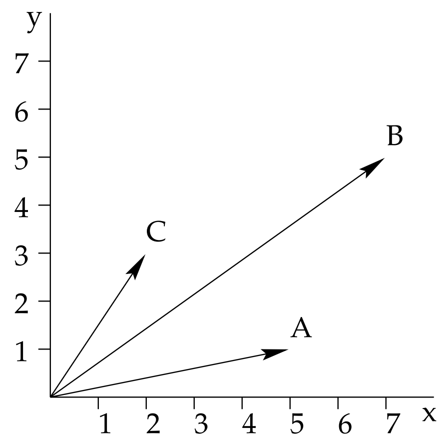
Answer Key

1. 0.45 s
2. 42 km/hr
3. 60 km/hr
4. $v = \sqrt{2gh}$ where g is the acceleration of gravity
5. 0.7 m
6. 1.3 m
7. 8 m/s, 30 km/hr
8. $D = D_1 - v_L t + \frac{1}{2}(a_A - a_L)t^2$
9. 5 s
10. $d = 22 - t$
11. $v_f^2 - v_i^2 = 2ad$ where a is the acceleration
12. $9600\,\mathrm{m/s^2}$ ($980g$)
13. $A = \sqrt{2R\ell}$; $B = \sqrt{2Rh}$
14. 98 km
15. $R^2 + L^2 = (R+h)^2$
16. $m = \rho V$
17. 1.8×10^{-10} m, a typical atomic dimension
18. 1.4×10^6 km
19. 1/50th of a circle (7.2°)
20. $50 \times 830\,\mathrm{km} = 41,500\,\mathrm{km}$
21. 4.0009×10^4 km
22. $970\,\mathrm{m/s^2}$
23. 4.4 m/s
24. 30 km/hr
25. $h = \dfrac{{v_0}^2}{2g}$
26. 3.7 m/s
27. 0.6 m/s
28. 4 m/s; 15 km/hr
29. 170 s
30. 37.5 m
31. $d = 22 - \dfrac{x}{15}$, where x is the distance covered by the rabbit
32. No.
33. $1.5\,\mathrm{m/s^2}$ or 0.16 g; 36 s
34. $96\,\mathrm{m/s^2}$ ($9.8g$, where g is the acceleration of gravity)
35. 

36. $L = (2Rh)^{1/2}\left(1 + \dfrac{h}{2R}\right)^{1/2}$
37. $R + h$
38. 1.6×10^{29} carbon atoms per cubic meter
39. $6.25 \times 10^{-30}\,\mathrm{m^3}$
40. 3500 km
41. That the sun–earth distance is much larger than the radius of the earth, so that the rays of the sun reaching Syrene are almost exactly parallel to those reaching Alexandria.
42. 0.0002

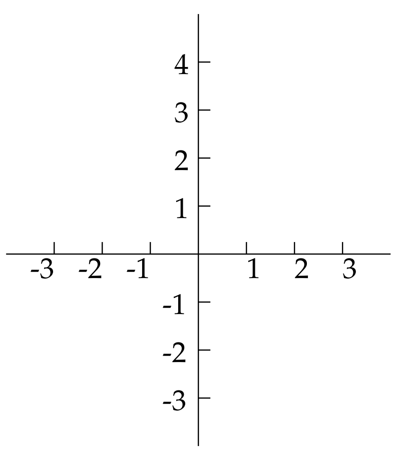
Learning Guide 2

Problem I: Vectors



1. What are the components along the x- and y-axes of the three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$? Key 32
2. What are the lengths of the three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$? Key 64
3. What are the components and the lengths of $\vec{A}+\vec{B}$; Key 11
   $\vec{B}-\vec{C}$? Key 43

Problem II



An ant has position vector $\vec{R}=(a+vt)\hat{x}+(b+vt)\hat{y}$. Here $a=1\,\mathrm{cm}$, $b=.5\,\mathrm{cm}$, $v=1\,\mathrm{cm/s}$ and t is the time in seconds.

1. Draw its path on these axes. Key 75
2. Where is it when $t = 0$ secs? Key 22
3. Where is it when $t = 1.5$ secs? Key 54
4. What is its speed? Key 1

(Try Helping Question 1 if necessary.)

Problem III

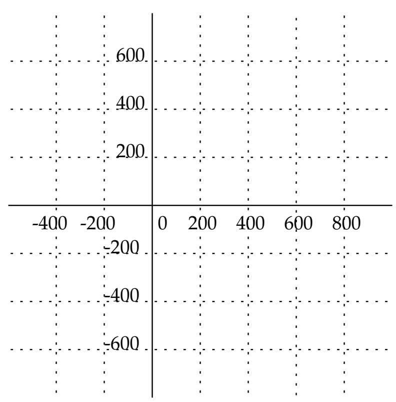
Two speedboats (A and B) have position vectors (in meters)

$$\begin{aligned}\vec{A} &= (350\,\mathrm{m})\hat{x} - (12\,\mathrm{m/s})\hat{y}t, \qquad \text{and} \\ \vec{B} &= (600\,\mathrm{m})\hat{x} + (150\,\mathrm{m})\hat{y} - [(10\,\mathrm{m/s})\hat{x} + (6\,\mathrm{m/s})\hat{y}]t,\end{aligned}$$

respectively (t is in seconds).

Answer questions 1 through 4 using graphical methods.

1. Plot their paths on the scaled axes here. Key 65



2. What are the speeds of the two speedboats in kilometers per hour? Key 12
3. How far apart are the two boats when $t = -15$ seconds? Key 44
   (If you need help, try Helping Question 2).
4. Where do the two paths intersect? Do the two boats collide? Key 23

   Now consider the problem using algebraic methods.

5. Find the length of the vector $\vec{A}(t) - \vec{B}(t)$ as a function of time. Evaluate at $t = -15$ seconds and compare your answer to 3. Key 55

   Assume that speedboat A arrives at the point of intersection of the two paths at t_A and that speedboat B passes through this point at t_B. Then, at the point of intersection of the two paths: $\vec{A} = 350\hat{x} - 12\hat{y}t_A = \vec{B} = 600\hat{x} + 150\hat{y} - (10\hat{x} + 6\hat{y})t_B$. This equation is equivalent to two simultaneous equations (one for the x components and one for the y components) which together determine t_A and t_B.

6. Write down these two equations. Key 2

7. Solve them for t_A and t_B, and check that $\vec{A}(t_A)$ and $\vec{B}(t_B)$ are equal to one another and to the position vector which you gave in your answer to 4. Key 34

Problem IV

The Dot Product of Two Vectors: $\vec{A}\cdot\vec{B} = |A||B|\cos\theta = A_xB_x + A_yB_y$. Two vectors are $\vec{A} = 4\hat{x} + 2\hat{y}$ and $\vec{B} = 3\hat{x} + 7\hat{y}$.

1. What are the lengths of $\vec{A}$ and $\vec{B}$? Key 66

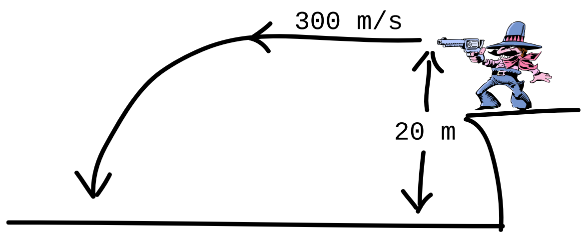
2. What are the angles α and β, which $\vec{A}$ and $\vec{B}$, respectively, make with the x axis? Key 13

3. What is the angle θ between $\vec{B}$ and $\vec{A}$? Key 45

4. Verify $|A||B|\cos\theta = A_xB_x + A_yB_y$.

Problem V: Trajectories

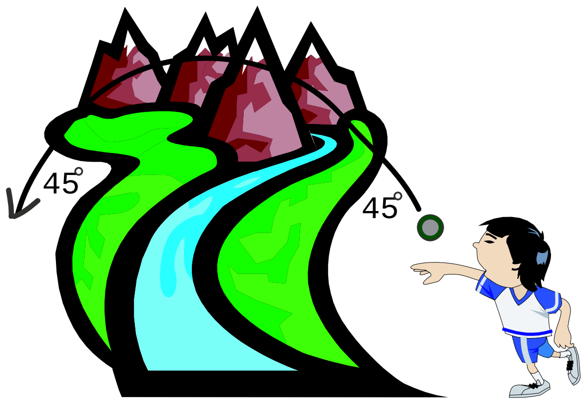
(Air resistance may be ignored.)



Clint Eastwood fires a bullet at 300 m/s horizontally from a saloon window 20 m above the flat ground. How far has it travelled horizontally when it reaches the ground? Key 77
(Do Helping Question 3 if you need to.)

Problem VI

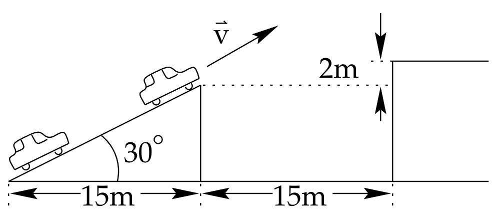
(Air resistance may be ignored.)



A child wishes to throw a ball across a river 20 m wide. If she throws the ball at 45° to the horizontal, how fast must it be traveling when it leaves her hand if it is just to reach the other bank? Key 56

(If you need assistance, go to Helping Question 4.)

Problem VII: The Stunt Driver



This question is taken from a previous exam. It should take about 15 minutes do to.

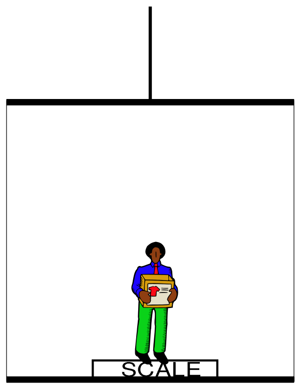
A stunt driver wishes to determine if he can drive off the incline and land on the platform. (See the figure.)

1. On the incline, the stunt driver can go from 0.0 m/s to 10.0 m/s in 2.0 s. Assuming constant acceleration, what speed can he achieve by the end of the incline if he starts at the bottom with speed $v = 0.0$ m/s? Key 67

2. Show whether this speed is sufficient for him to make it. Key 14

Problem VIII

A 75 kg person is standing on a bathroom scale in an elevator. The scale is calibrated to read in Newtons.

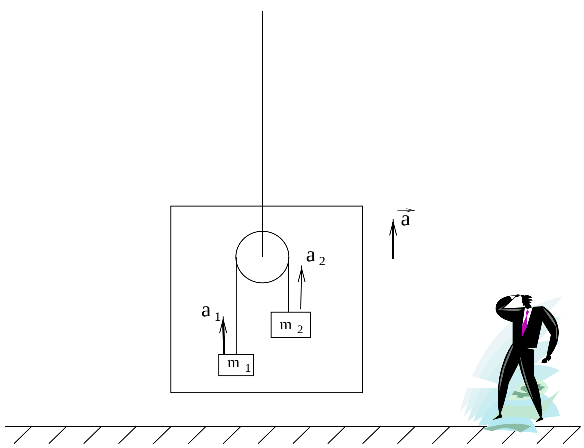


1. The elevator is at rest. What is the reading on the scale? Key 46

2. The elevator is moving upward at a constant speed. What is the reading on the scale? Key 78

3. The elevator is accelerating upward at $135\,\mathrm{m/s^2}$. What is the reading on the scale? What apparent value of g does the man give? Key 25
   (Trouble, see Helping Question 5.)

   Two masses m_1 and m_2 are attached by a light rope passing over a massless pulley. This contraption is put inside an elevator. The elevator accelerates upward with acceleration a. Let T be the tension in the rope suspending the masses.



4. Express $a_1 + a_2$ in terms of a. Key 36

5. Write down the equations of motion for m_1 and m_2. Key 68

6. The answers to (4) and (5) give three equations for the three unknowns T, a_1 and a_2. Solve them: that is, express T, a_1 and a_2 in terms of a, g, m, m_1 and m_2. Key 15

7. What is the tension in the cable holding the pulley to the roof of the elevator? Key 47

Problem IX

1. Neil Armstrong, the first man on the moon, has a mass of 75 kg. What is his weight on earth (in Newtons). Take $g = 9.8\,\text{m/s}^2$. Key 79

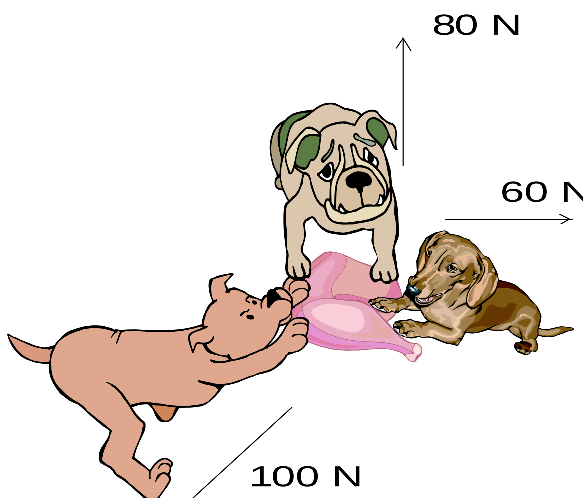
2. What is his weight on the moon?

Data:		
	Mass of Earth	5.98×10^{24} kg
	Mass of Moon	7.35×10^{22} kg
	Radius of Earth	6.38×10^{6} m
	Radius of Moon	1.74×10^{6} m

Key 26

(Go to Helping Question 6 if you have trouble.)

Problem X



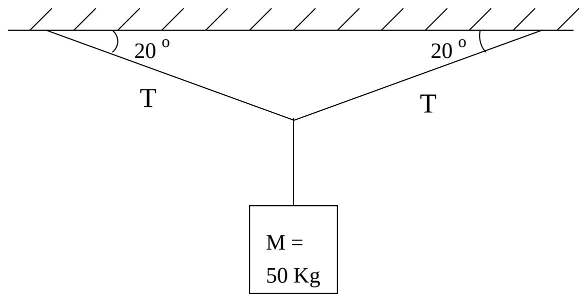
Three dogs battling over a piece of meat pull with forces of 80 N due north, 60 N due east and 100 N 37° west of south. Does the piece of meat move? Key 5
(Do Helping Question 7 if you didn't get this right.)

Problem XI

A baseball player throws a 1 kg baseball at the angle that gives maximum range. The ball lands 70 m away. If the ball is accelerated uniformly in the player's hand over a distance of one meter then released, what is the force exerted on the ball by the player while he is throwing it? Key 16
(If you need help, do Helping Questions 8 and 9.)

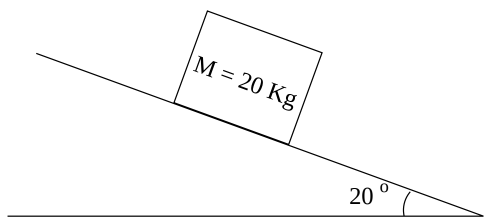
Problem XII

What is the tension in each of the two strings? Key 38



(If you cannot start, go to Helping Questions 10 and 11. Otherwise, continue.)

Problem XIII



The box has a mass of 20 kg and the coefficients of static and kinetic friction between the box and the surface are 0.5 and 0.3, respectively. What force must be applied parallel to the slope in order to:

1. Start the box moving up the slope? Key 49
2. Keep it moving up the slope at a constant speed? Key 81
3. Start it from rest moving down the slope? Key 28
4. Keep it moving down the slope at a constant speed? Key 60
5. How much should the angle of the slope be increased in order for the box just to start slipping spontaneously from rest? Key 7

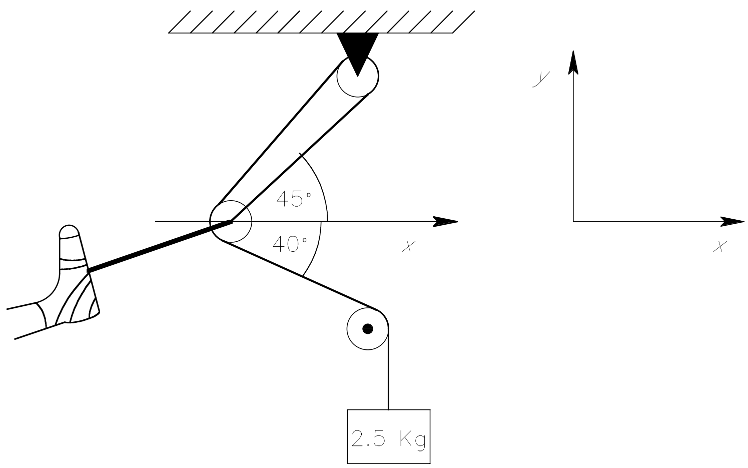
(If you had difficulty with parts 1, 2, 3 and 4 do Helping Questions 12 and 13. If you didn't get the right increase in the slope, do Helping Question 14.)

Problem XIV: In Traction

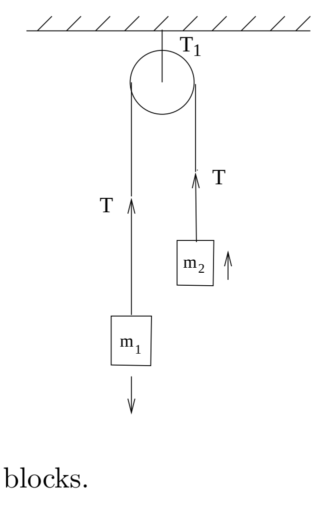
A patients leg is held in traction as shown in the diagram. The top and bottom pulleys are fixed, while the central pulley is not, but has come to rest at a position determined by the various forces acting on it. Note that one single rope runs between the mass and the pulleys and is fixed to the middle pulley, while a separate strap connects the foot to the center pulley. What is the magnitude and direction of the force on the patient's foot produced by the traction appartus shown in the diagram? Key 50

(Go to Helping Questions 15, 16 and 17 if you have difficulty.)

Also, express this force as a vector (using the coordinate system shown). Assume that all three pulleys are frictionless. Key 8



Problem XV: Atwood's Machine



1. Draw force diagrams for both blocks. Key 40

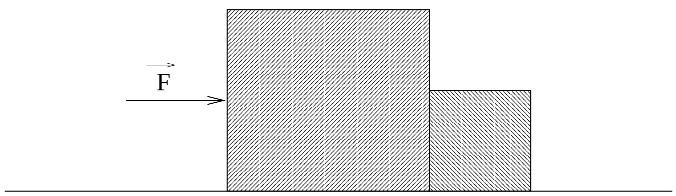
2. Use Newton's second law to write down two equations (one for each mass) giving their accelerations in terms of their masses, g, and the tension T in the string. (The string does not stretch.) Use the convention that up is positive. Key 72

3. Eliminate T between your two equations to find the acceleration a. What is a if m_1 equals 5.0 kg. and m_2 equals 3.0 kg? Key 19

4. What is the tension in the string in this case? Key 51

5. What is the tension T_1 in the string supporting the pulley? Key 83

Problem XVI



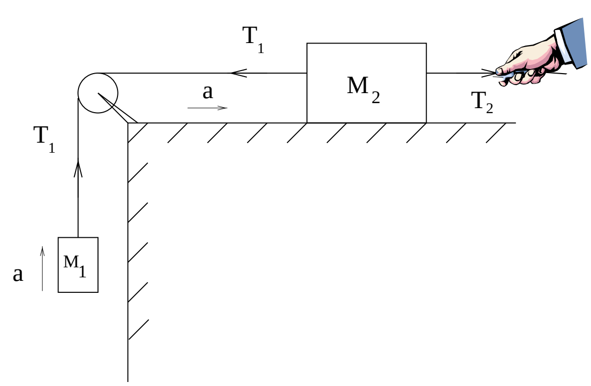
The blocks, in contact on a frictionless table are pushed by a horizontal force applied to one block, as shown in the figure.

1. If $m_1 = 4.0\,\text{kg}$, $m_2 = 2.0\,\text{kg}$ and $F = 6.0\,\text{N}$, find the force of contact, F_c, that each block exerts on the other. Key 30
   (See Helping Question 18 if you are having difficulty starting.)

2. Show that if the same force is applied to m_2 rather than to m_1, the force of contact between the blocks is 4.0 N, which is not the same as the value derived in (1). Explain. Key 9

Problem XVII

Two masses, $M_1 = 0.50\,\text{kg}$ and $M_2 = 0.20\,\text{kg}$ are attached by a massless inextensible string which passes over a massless pulley, as shown. A second string attached to M_2 is pulled horizontally to the right to accelerate the masses at $1.0\,\text{m/s}^2$.

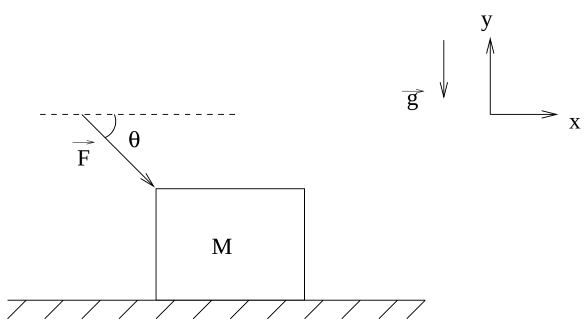


Neglecting frictional forces, determine the following:

1. The tension T_1 in the string connecting M_1 and M_2. Key 41
2. The tension T_2 in the string which is pulled. Key 73
   (If you have trouble with 1 or 2, See Helping Questions 19 and 20.)

   Suppose now there is friction between M_2 and the table with coefficient of sliding friction $\mu_k = 0.1$
3. If the tension T_2 is the same as in part 2, what is the new acceleration? Key 84
4. What is T_1? Key 31

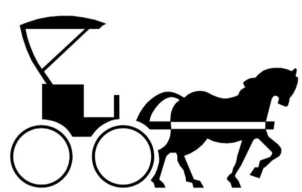
Problem XVIII



A block of mass $M = 3.0\,\text{kg}$ lies on a frictionless horizontal surface. A force F of magnitude $10\,\text{N}$ acts on the body at an angle of $\theta = 45°$ with respect to the x axis. Express all answers in terms of M, F, θ and g and give their numerical values.

1. What is the magnitude of the force exerted by the surface on the block? Key 63
2. What is the magnitude of the acceleration of the block? Key 10

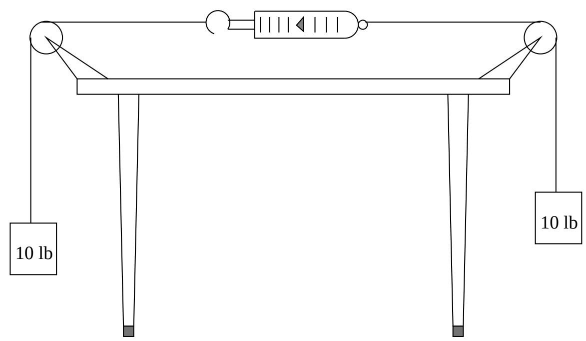
Problem XIX



1. A very lazy (if eloquent) horse, hitched to a cart at rest makes the following defense for his refusal to strain at the bit: “Having completed Physics 101, I know that by Newton’s third law, my pull on the cart can never exceed the cart’s pull on me. Ergo, it is quite impossible for us to move.” Answer the horse and cast doubt on his having received a passing grade. Key 42

2. An aircraft of mass 1000 kg requires a speed of 160 km/hr to safely take off from a runway. Suppose the length of the runway is 1000 m and the place uses it all. How much constant thrust (force) must the engines provide to achieve a satisfactory takeoff? Key 74

3. An automobile has a mass of 1200 kg and has brakes on all four wheels. What must be the average horizontal force between each wheel and the ground if the car is to be brought to rest from a speed of 22 m/s in 10 s? Key 21



4. What does the spring scale in the configuration shown read? Key 53

Helping Questions

1. What is the velocity of the ant? Key 33
   (Try part 4 again)

2. What is the vector separation of the two boats when $t = -15\,\mathrm{s}$? Key 76

3. What is the height h of the bullet above the ground as a function of time? Key 24

4. Write down an equation for the horizontal component of the position vector of the ball as a function of the time t since it left the child's hand and another equation for the vertical component of the position vector. Key 3
   At what time after it was thrown is the height of the ball again $0\,\mathrm{m}$? Ignore the height of the child. Key 35

5. What are the forces on the man? Key 57
   What does Newton's Second law tell you? Key 4

6. What is the relation between the acceleration due to gravity on the surface of the moon, g_m, and g? Key 58

7. What are the northwards components of each of the three forces? Key 37

   What are the eastwards components of each of the three forces? Key 69

8. At what angle to the ground is the baseball's initial velocity for maximum range given with initial speed? Key 48
   How far (in terms of v and g) does the ball go if it is thrown at this angle? Key 80

9. What is the relation between the acceleration a of the baseball in the player's hand, the distance d which it travels in his hand and the speed v at which is leaves his hand? Key 27

   Use this relation and your answer to the second part of Helping Question 10 to find an expression relating a, d, g and x, the distance which the baseball goes when it is thrown at optimum angle. Key 59

   What is the relation between the acceleration of the baseball while it is in the player's hand the force which he is exerting on it while it is in his hand? (You may ignore the acceleration due to gravity for the short time that the ball is being accelerated in the player's hand.) Key 6

10. Draw a diagram showing all the forces acting on the mass. Key 70

11. What does Newton's second law tell you about the components of the forces in the vertical direction? Key 17

12. Draw a force diagram showing all the forces on the box. Key 39

13. Which way does the frictional force act in parts 1 and 4? Key 71
    Now use Newton's second law to find the required forces.

14. What is the condition on the angle at which the box starts to slide spontaneously? Key 18

15. Is there another force in the problem which is equal to the force on the foot? Key 82

16. What is the tension in the rope? Key 29
    Now apply Newton's second law to the center pulley to find the force.

17. How does the rope function in this system of pulleys? Key 61

18. Draw a free-body diagram for each block separately. Key 62

19. Draw free-body diagrams for both M_1 and M_2. Key 20

20. What are the equations which express Newton's second law for each body? Key 52
    Now find T_1 and T_2.

Answer Key

1. $\sqrt{2}\,\text{cm/s} = 1.41\,\text{cm/s}$

2. $350 = 600 - 10t_B$ (x component equation), $-12t_A = 150 - 6t_B$

3. $x = vt\cos 45^\circ = 0.707vt$, $y = vt\sin 45^\circ - \dfrac{gt^2}{2} = 0.707vt - 4.9t^2$

4. $F = s - mg = ma$

5. No

6. $F = ma$ where F is the force and m is the mass of the baseball.

7. Increase the slope by 6.6° to 26.6°.

8. $(53.4\,\text{N}\hat{x} + 18.9\,\text{N}\hat{y})$

9. The force of 6.0 N acts on the two blocks together to yield an acceleration of $1\,\text{m/s}^2$. Thus, the net force on the 4.0 kg block must be 4.0 N and the net force on the 2.0 kg block must be 2.0 N. Once the force of 6.0 N is applied, the contact force must adjust itself so that the appropriate net force acts on each block.

10. $a = F\cos\theta/M = 2.4\,\text{m/s}^2$

11. $\vec{A} + \vec{B} = 12\hat{x} + 6\hat{y}$, $|\vec{A} + \vec{B}| = \sqrt{180} = 13.4$

12. speed of $A = 43.2\,\text{km/hr}$, speed of $B = 42.0\,\text{km/hr}$

13. $\alpha = 26.6^\circ$, $\beta = 66.8^\circ$

14. No, it isn't

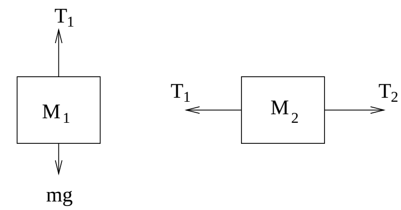
15. $T = \dfrac{2m_1m_2}{m_1+m_2}(g + a)$; $a_1 = \dfrac{(m_2 - m_1)g + 2m_2a}{m_1+m_2}$; $a_2 = \dfrac{(m_1 - m_2)g + 2m_1a}{m_1+m_2}$

16. 343 N

17. $2T\sin 20^\circ = M_g$

18. The largest force of friction possible just balances the component of the weight down the slope, i.e. $\tan\theta = \mu_s$.

19. $a = \dfrac{(m_1 - m_2)}{(m_1 + m_2)}g$ and, if $m_1 = 5.0\,\text{kg}$ and $m_2 = 3.0\,\text{kg}$, then $a = 2.4\,\text{m/s}^2$.

20. 

21. $6.6 \times 10^2\,\text{N}$

22. $r_x = 1$, $r_y = 1.5$

23. No

24. $h = 20 - 4.9t^2$

25. $8.5 \times 10^2\,\text{N}$, $g_{\text{app}} = 11.3\,\text{m/s}^2$

26. 121 N

27. $v^2 = 2ad$

28. $mg(-\sin 20^\circ + 0.5\cos 20^\circ) = 25\,\text{N}$

29. $mg = 24.5\,\text{N}$ everywhere

30. 2.0 N

31. 5.3 N

32. $A_x = 5$, $A_y = 1$, $B_x = 7$, $B_y = 5$, $C_x = 2$, $C_y = 3$

33. $\vec{v} = \dfrac{d\vec{R}}{dt}$

34. $t_A = 0\,\text{s}$, $t_B = 25\,\text{s}$

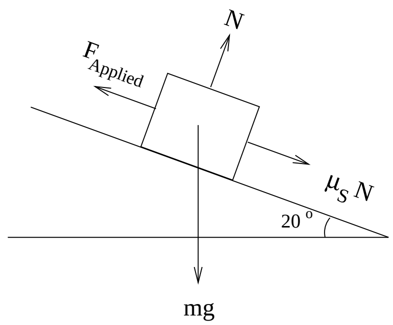
35. $t = \sqrt{2}v_0/g$

36. $a_1 + a_2 = 2a$

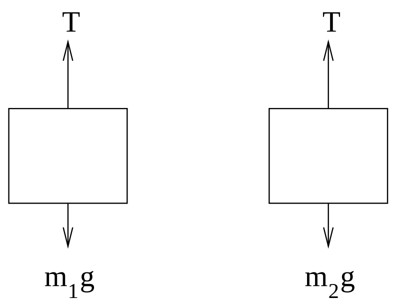
37. $80\,\text{N},\ 0,\ -100\,\text{N}\cos 37^\circ = -80\,\text{N}$

38. $\dfrac{mg}{2\sin 20^\circ} = 7.2\times 10^2\,\text{N}$

39.



40.



41. 5.4 N

42. The horse and cart are not a closed system. The horse-cart system interacts with the Earth. If it applies a force to the Earth, as it does when the horse starts forward, the Earth, by Newton's third law applies an equal and opposite force to the horse-cart system, which therefore moves with respect to the Earth as it is supposed to. The internal forces in the horse-cart system are balanced, but that is as it should be. We don't want the cart to accelerate away from the horse.

43. $\vec{B} - \vec{C} = 5\hat{x} + 2\hat{y}$, $|\vec{B} - \vec{C}| = \sqrt{29} = 5.4$

44. 404 m

45. $\beta - \alpha = \theta = 40.2^\circ$

46. $7.4\times 10^2\,\text{N}$

47. $2T = \dfrac{4(a+g)m_1m_2}{m_1+m_2}$

48. 45°

49. $mg(\sin 20^\circ + 0.5\cos 20^\circ) = 159\,\text{N}\ (= 1.6\times 10^2\,\text{N})$

50. 56.7 N; 19.5° above the horizontal

51. 37 N

52. $T_2 - T_1 = m_2a$; $T_1 - m_1g = m_1a$

53. 10 lb

54. $\hat{r} = 2.5\hat{x} + 3\hat{y}$

55. $|\vec{A} - \vec{B}|(t) = 2\sqrt{(125-5t)^2 + (75+3t)^2}$, 404 m

56. 14 m/s

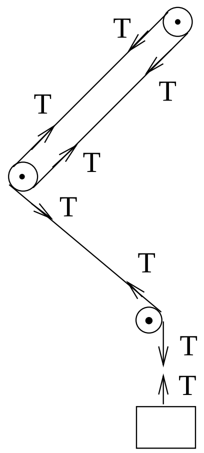
57. The scale's normal force, s upward; and weight, mg downward

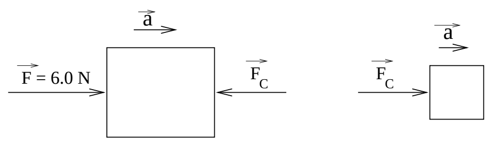
58. $g_m = \dfrac{M_{\text{moon}}}{(R_{\text{moon}})^2}\dfrac{(R_{\text{earth}})^2}{M_{\text{earth}}}g$

59. $2da = gx$

60. $-12\,\text{N}$ (The minus sign means this force acts up the slope.)

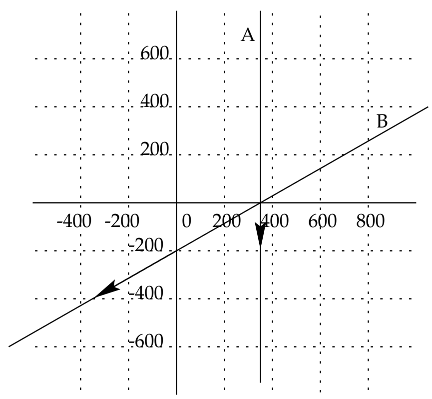
61. The rope pulls with a tension T at every point to which it is attached.



62. 

63. $Mg + F\sin\theta = 36\,\mathrm{N}$

64. $|\vec{A}| = \sqrt{26} = 5.1$, $|\vec{B}| = \sqrt{74} = 8.6$, $|\vec{C}| = \sqrt{13} = 3.6$

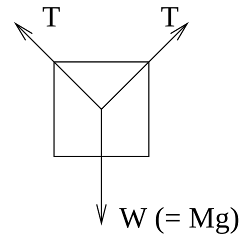
65. 

66. $|\vec{A}| = \sqrt{20} = 4.47$, $|\vec{B}| = \sqrt{58} = 7.61$

67. $13.2\,\mathrm{m/s}$

68. $T - m_1 g = m_1 a_1$; $T - m_2 g = m_2 a_2$

69. 0, $60\,\mathrm{N}$, $-100\,\mathrm{N}\sin 37° = -60\,\mathrm{N}$

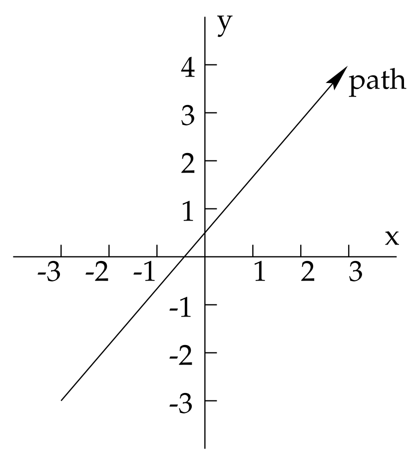
70. 

71. $2T\sin 20° = M_g$

72. $T - m_1 g = -m_1 a$; $T - m_2 g = m_2 a$. In writing these equations, the convention has been used that the acceleration of m_1 is $-a$ and that of m_2 is $+a$.

73. $5.6\,\mathrm{N}$

74. $9.9 \times 10^2\,\mathrm{N}$

75. 

76. $\vec{B} - \vec{A} = 400\hat{x} + 60\hat{y}$ at $t = -15\,\mathrm{s}$

77. $606\,\mathrm{m}$

78. $7.4 \times 10^2\,\mathrm{N}$

79. $735\,\mathrm{N}$

80. v^2/g

81. $122\,\mathrm{N}$

82. Yes, the reaction force on the pulley nearest the foot.

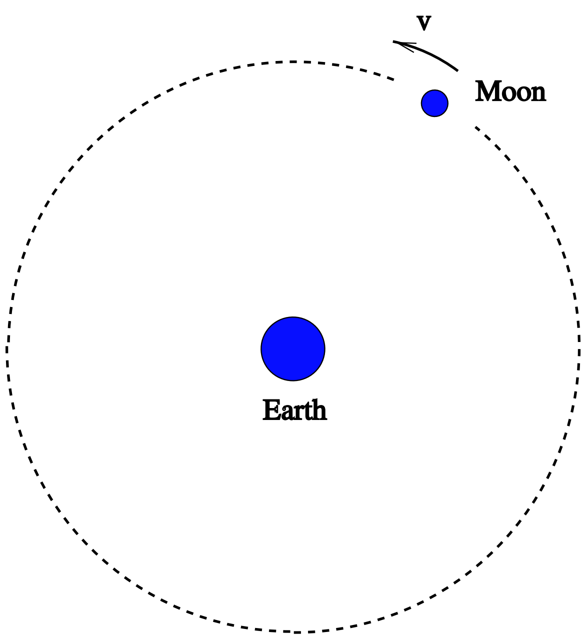
83. $74\,\mathrm{N}$

84. $a = 0.72\,\mathrm{m/s^2}$

Learning Guide 3

Problem I

The moon orbits the earth at a distance of 3.8×10^5 km and a complete orbit requires 2.4×10^6 s (about 27 days).



1. Find the speed with which the moon moves in its orbit around the earth. Key 31

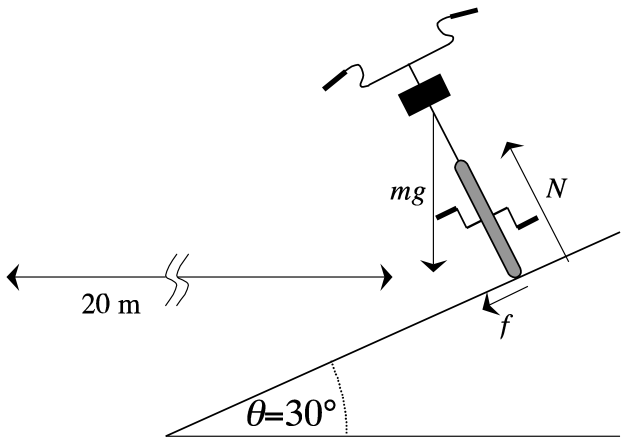
2. What is the acceleration which the moon experiences. Key 23

3. Calculate the mass of the earth from your information about the moon's orbit around the earth. Key 15

(For help see Helping Question 1.)

Problem II

Consider a bicyclist riding around a circular track of radius 20 m. The track is banked at angle of 30°. We will only concern ourselves with the motion of the center of mass and the forces acting on it.



1. Suppose the motion is circular; the center of mass moves in a circle of radius 20 m at constant speed v. Give the magnitude and direction of the acceleration in this case. Key 30

2. What are the magnitudes and directions of the forces acting on the bicycle? Assume no friction. Key 22
(Try Helping Question 2)

3. Write down the horizontal and vertical components of $\vec{F} = m\vec{a}$ and solve these equations to find the speed at which circular motion can be maintained without friction. Key 6

4. Now suppose the track has a coefficient of static friction $\mu_s = 0.2$. Repeat (1) and (2) including the friction and find the minimum and maximum speeds the bicycle can travel without slipping off the track. Key 37

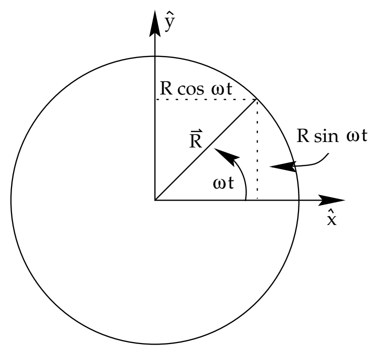
(See Helping Question 3 for a hint.)

Problem III

An object has position vector $\vec{r} = R(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$ where t is the time in seconds and ωt is measured in radians (not degrees).

1. What are the dimensions and units of the constant ω? Key 21
(If you didn't get it right, do Helping Question 4.)

2. Assume that $\omega = 0.5\,\mathrm{s}^{-1}$ and plot the position of the object at $t = 0$, 1, 2, 3, 4, 5, 7, 9 and 11 seconds (take $R = 5\,\mathrm{cm}$). Key 5



(You will need to use your calculator to work out the sines and cosines.)

3. Show *algebraically* that the object moves in a circle of constant radius R about the origin. (Go to Helping Question 5 if you need to.)

4. What is the angle θ which the position vector of the object makes with the x axis at time t (in radians). Key 28

5. What is the rate of change of this angle (i.e. $\Delta\theta/\Delta t$)? This is conventionally called the angular velocity. Key 20

6. What distance along the perimeter of the circle has the particle traveled in time t? Key 12

7. What is the speed v of the particle in terms of R and ω? Key 4

8. By differentiating $\vec{r}$ with respect to time, find the velocity *vector* $\vec{v}$ of the particle. Check that the magnitude of this vector is equal to your answer to 7. Key 35

9. Differentiate $\vec{v}$ with respect to time, to find the vector acceleration $\vec{a}$. Key 27

10. What is the direction of $\vec{a}$? Key 19
(Go to Helping Question 6 if this causes you difficulty.)

11. What is the magnitude of $\vec{a}$? Express your answer in terms of ω and R, and also in terms of v and R. (a is called the "centripetal acceleration.") Key 3

12. Explain in words how an object can have an acceleration even though its speed v is always constant. What relation between the vector velocity and the vector acceleration must hold if the speed is never to change. Key 34

13. If $R = 15 \times 10^{10}\,\text{m}$ and $w = 2.0 \times 10^{-7}\,\text{s}^{-1}$, $\vec{r}$ can represent the position vector of the earth in its orbit around the sun. What is the centripetal acceleration of the earth towards the sun? Key 26

14. The mass of the earth is $6.0 \times 10^{24}\,\text{kg}$. What is the force between the earth and the sun, assuming that this force is responsible for producing all of the centripetal acceleration which you calculated in part 13. Key 18

15. Check that this force is equal to the gravitational force between the earth and the sun. ($G = 6.7 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$, $M_{\mathrm{sun}} = 2.0 \times 10^{30}\,\mathrm{kg}$, $R = 15 \times 10^{10}\,\mathrm{m}$ and $M_E = 5.98 \times 10^{24}\,\mathrm{kg}$). Key 10

Problem IV: Quickies

1. If the coefficient of static friction is 0.5 for tires on a road, how fast can a car go around a curve of radius 100 m on flat ground? Key 2
2. If a car can negotiate a curve of radius 200 m at 30 m/s without any frictional force being required, what is the banking angle? Key 33
3. How much work is required to change the speed of a 0.5 kg ball from 0 to 20 m/s? Key 25
4. How much work must be done to bring a 10^{-5} C charge from infinity to 0.01 m from a second identical charge? Key 17
5. What power is required to raise a 60 kg man vertically at a rate of 0.5 m/s Key 9 .

Problem V: The Spy Satellite

1. Is it possible for a satellite to stay above the same point on the Earth's surface all the time? Key 1
   If so, what is the radius of its orbit? Key 32
   ($g = 9.8\,\mathrm{m/s^2}$ and the radius of the earth is $R = 6378\,\mathrm{km}$.)
   (If you need assistance, go to Helping Questions 7 and 8)
2. The United States would obviously like to keep a spy satellite above Moscow all the time, but instead it is contented to have several satellites, which pass above Moscow in turns. Why is this? Key 8

Helping Questions

1. You should be able to write down two expressions for the same force: What is Newton's second law for the centripetal force on the moon? Key 7
   What is Newton's law of universal gravitation for the earth moon system? Key 38

2. Draw a free-body diagram for the bicyclist. Key 14

3. Draw a free-body diagram for the cyclist (include friction). Key 29

4. What are the dimensions of the product ωt? Key 13

5. What is $|\vec{r}|$ in terms of R, ω and t? Key 36

6. Express $\vec{a}$ in terms of ω and $\vec{r}$. Key 11

7. The period T of a satellite moving in a circular orbit of radius r at speed v is $T = \dfrac{2\pi r}{v}$.
   Express the centripetal acceleration formula $a = \dfrac{v^2}{r}$ in terms of T and r, instead of in terms of v and r. Key 24

8. The acceleration due to gravity must be equal to the centripetal acceleration. What relation must hold between T, r, g and R if this is to be true? Key 16

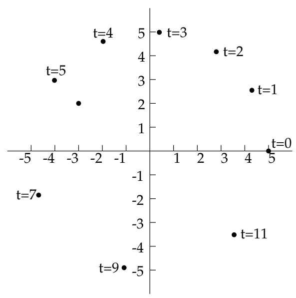
Answer Key

1. Yes, under certain circumstances.

2. $v = 22.1\,\mathrm{m/s}$

3. $a = \omega^2 R = \frac{v^2}{R}$

4. $v = R\omega$

5. 

6. $v = \sqrt{Rg\tan\theta} = 11\,\mathrm{m/s}$

7. $F = M_m a$

8. The centripetal acceleration of an object in orbit around a planet is always towards the center of the planet, but the center of the circle made by a satellite permanently above Moscow is on the line between the center of the earth and the North Pole.

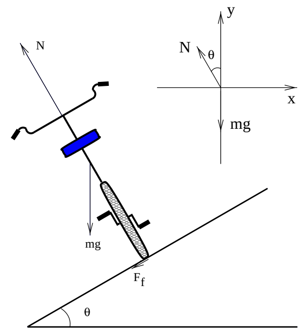
9. $P = mgv = 294w$

10. $3.6 \times 10^{22}\,\mathrm{N}$

11. $\vec{a} = -\omega^2 \vec{r}$

12. $R\theta = R\omega t$

13. ωt must be dimensionless, since many different powers of ωt are added together in $\cos\omega t$ or $\sin\omega t$.

14. 

15. $5.6 \times 10^{24}\,\mathrm{kg}$

16. $\dfrac{GM_{\mathrm{earth}}}{r^2} = g\left(\dfrac{R}{r}\right)^2$

17. $w = U - U_o = 90\,\mathrm{J}$

18. $3.6 \times 10^{22}\,\mathrm{N}$

19. $\vec{a}$ is always directed inwards towards the center of the circle.

20. $\dfrac{d\theta}{dt} = \dfrac{d(\omega t)}{dt} = \omega$

21. $[\omega] = T^{-1}$, s^{-1}

22. The gravitational force has magnitude mg and is directed downward. The normal force has magnitude $N = mg/\cos 30^\circ$ and is directed normal to the track surface.

23. $v = d/t$

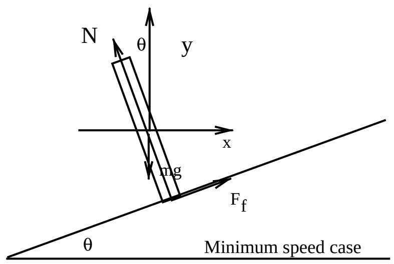
24. $a = \dfrac{(2\pi)^2}{T^2} r$

25. $w = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = 100\,\mathrm{J}$

26. $6 \times 10^{-3}\,\mathrm{m/s^2}$

27. $\vec{a} = -R\omega^2(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$.

28. $\theta = \omega t$

29. 

30. $\dfrac{v^2}{R}$ radially inward

31. 9.9×10^2 m/s

32. 4.2×10^4 km

33. $\theta = 27°$

34. The object is accelerating because the direction of $\vec{v}$ is changing. If the speed is constant, the instantaneous acceleration is always perpendicular to the instantaneous velocity.

35. $\vec{v} = R\omega(-\hat{x}\sin\omega t + \hat{y}\cos\omega t)$

36. $|\vec{r}| = R\sqrt{\cos^2\omega t + \sin^2\omega t} = R$

37. $v_{\text{max}} = \sqrt{\dfrac{Rg(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}} = 13.1\,\text{m/s}$

$v_{\text{min}} = \sqrt{\dfrac{Rg(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}} = 8.1\,\text{m/s}$

38. $F = \dfrac{GM_eM_m}{r_{\text{cm}}^2}$

Learning Guide 4

Problem I

1. The escape velocity from the surface of a star or planet is the speed an object must have as it leaves the surface of the star or planet if it is to be able to escape the gravitational attraction of the star or planet completely. What is the escape velocity from the earth? Key 31

   What is the escape velocity from the sun? Key 8
   ($M_{\rm sun} = 2.0 \times 10^{3}0\,{\rm kg}$, $R_{\rm sun} = 7.0 \times 10^{8}\,{\rm m}$ and $G = 6.7 \times 10^{-11}\,{\rm Nm^2kg^{-2}}$
   (If you need help, go to Helping Questions 1, 2 and 3)

2. In 1798 the French mathematician and physicist Laplace pointed out that the very largest stars might be so heavy and dense that their escape velocity is faster than the speed of light, so that no light would escape from them, and they would be invisible! Using Newton's laws, to what size would the sun have to be compressed if the escape velocity from its surface is to be equal to the speed of light, which is $3.0 \times 10^{8}\,{\rm m/s}$? (Newton's laws are not accurate at such great densities, but the order of magnitude of the answer is correct.) Key 24

Problem II

The physicist's definition of work differs from the everyday meaning people usually attach to the word "work." In the following questions "work" is used in the physicist's sense.

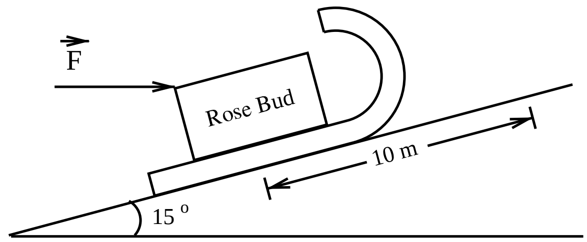
1. A man pushes against a wall for 10 seconds. The wall does not move. How much work does the man do on the wall? Key 1
   How much work did the wall do on the man? Key 32
   (See Helping Question 4)

2. A backpacker gets up in the morning, carries a 20 kg pack 6 km along a seashore, and then sits down for a break. How much work has he done on the pack? Key 40
   (See Helping Question 5)

3. A hoist used in car repair applies a constant force to a 200 kg engine to lift it 1.5 m. How much does the hoist do on the engine? Key 48
   (See Helping Question 6)

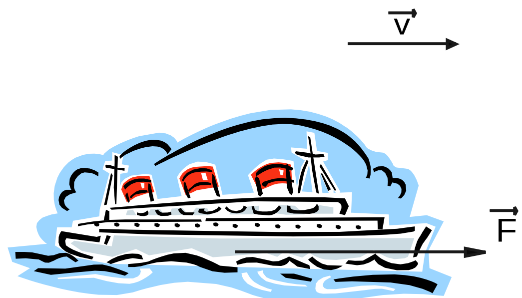
4. A weight lifter jerks a 200 kg weight to chest height (1.5 m). The force the weight lifter uses fluctuates significantly as the weight is jerked upward. How much work does the weight lifter do on the weight? Key 2 (See Helping Questions 7 and 8)

5. A child pushes a sled 10 m up a snowy slope. He accomplishes this using a force of 30 N in the direction shown in the diagram. The slope is 15° from the horizontal. How much work does the child do on the sled? Key 41



Problem III

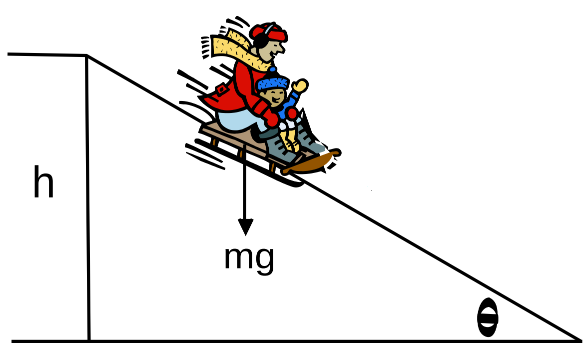
A 3.5×10^7 kg ship is travelling at 7.0 m/s. The ship is accelerated forward with a force of 4.5×10^5 N over a distance of 2.0 km.



1. What is the final speed of the ship? Key 18
2. How long does it take the ship to traverse the 2.0 km? Key 49
3. What is the average power which the engines had to produce to accelerated the ship in part 1? Key 26
   (For assistance see Helping Questions 9 and 10)

Problem IV

A toboggan run of height $h = 60$ m has a slope of $\theta = 30°$ above the horizontal. A toboggan loaded with several people has a total mass $m = 250$ kg and starts to slide from rest at the top of the slope.



1. What is the speed of the toboggan at the bottom of the hill if there is no friction between the toboggan and the snow? Key 11
   (If you have any trouble, try Helping Question 11.)

2. What would be the speed of the toboggan in part (1) if the slope were $\theta = 45°$? The height h is still 60 m. Key 19

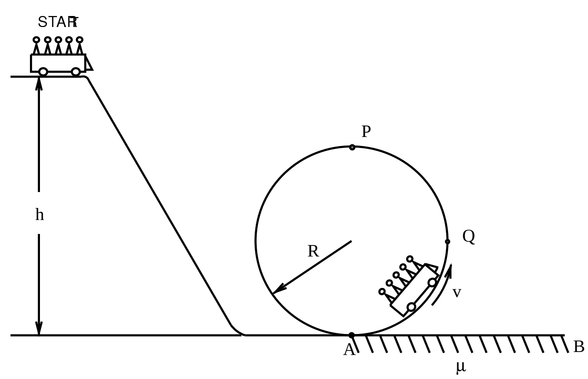
3. Consider again the original slope of $\theta = 30°$ and suppose there is a coefficient of friction $\mu = 0.2$ between the toboggan and the snow. By analyzing the forces acting on the toboggan calculate the toboggan's speed at the bottom of the hill. Key 50
   (If you cannot start, see Helping Question 12. Still stuck? Try Helping Questions 13, 14, 15, 16 and 17.)

4. Repeat part 3 using energy concepts. Key 51
   (Try Helping Questions 18 and 19 if you are unable to start)

5. Suppose the slope is $\theta = 45°$ and the coefficient of friction is $\mu = 0.2$. What will the toboggan's speed at the bottom of the hill now be? Key 36

Problem V: Loop the Loop

A roller coaster with 5 passengers (total mass of 500 kg) travels on the track shown above. At point A the coaster enters a loop the loop segment of track in the form of a circle of radius $R = 20$ m. There is no friction between the coaster and the track *except* for the track segment A–B where the coefficient of friction is $\mu = 0.40$.



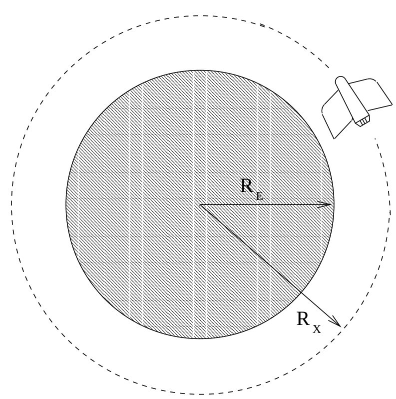
1. From what minimum height, h, must the car start in order to just barely stay on the track at P? (Use this value of h in parts 2 and 3.) Key 13
   (If you cannot get there, see Helping Questions 20, 21, 22 and 23.)

2. How many times heavier than his own weight does each passenger feel when the coaster is at point Q? Key 6
   (If you're stuck, see Helping Question 24.)

3. What is the speed of the coaster the first time it passes point A? Key 14
4. What is the speed the second time the coaster passes point A, i.e. what is the speed after it has looped the loop? Key 45
5. How far from point A will the coaster travel before coming to rest? Key 22

Problem VI

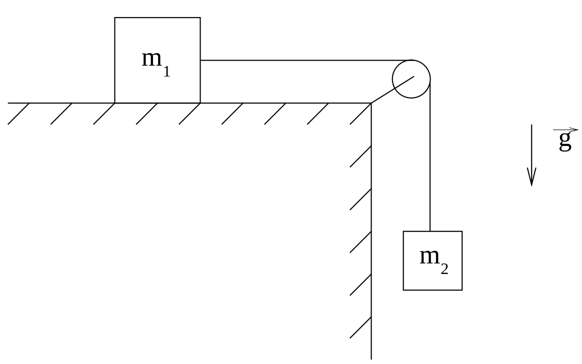
A rocket of mass m is to be boosted from the surface of the earth (radius R_E) into a stable circular orbit of radius R_x about the center of the earth. Assume that the rocket gains sufficient energy to achieve this orbit during lift-off, and that all subsequent boosts are applied so as to change the direction of the rocket only, and not its speed. With what speed v must the rocket leave the launching pad? Key 53



(If you need guidance, go to Helping Question 25. If you still need help, try Helping Questions 26, 27 and 28.)

Problem VII: Blocks and Rope

In the accompanying diagram two masses, $m_1 = m_2 = 10\,\text{kg}$, are attached via a light rope which passes over a light, well-oiled pulley. The coefficient of friction between m_1 and the table is 0.10.



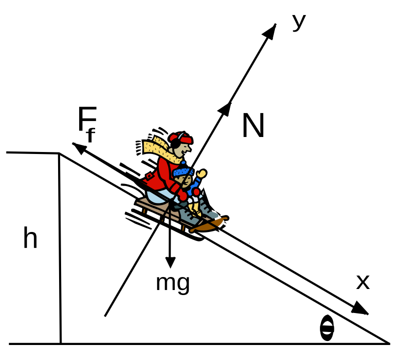
1. Calculate the energy lost to friction as m_1 slides 1.0 m along the table. Key 46
2. Assuming that the masses start from rest, use energy concepts to calculate the speed of m_1 after it has moved 1.0 m. Key 23

Helping Questions

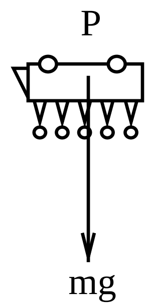
1. What is the gravitational potential energy of a mass m at the surface of the star or planet? (Be careful ot get the sign right.) Key 39
2. What is the total energy at the surface of the star or planet? Key 16
3. What is the total energy of an object which barely managed to escape the star's or planet's gravitational field? Key 47
4. How is work related to force and displacement? Key 9
5. What is the mechanical energy of the pack initially and finally? Key 17
6. What is the force the hoist applies to the engine? Key 25
7. What is the change in potential energy of the weight? Key 33
8. Where does this change in potential energy come from? Key 10
9. What is the definition of power? Key 3
10. What is the work done on the ship by the engines? Key 34
11. What is the total energy of the sled when at the top of the hill? At the bottom of the hill? Key 42
12. What are the forces acting on the toboggan? Key 27
13. Draw the free-body diagram for the toboggan. Key 4
14. What is the force equation in the $\hat{y}$-direction? Key 35
15. What is the force equation in the $\hat{x}$-direction? Key 12
16. Now find the acceleration down the slope. Key 43
17. What remaining piece of information do you need in order to find the speed at the bottom of the slope? Key 20
18. Is there a non-conservative force in this problem? Key 28
19. What is the equation relating the energy of the toboggan at the top of the hill, the energy at the bottom of the hill and the work done by the non-conservative forces? Key 5
20. What would happen if the coaster were going a little faster or a little slower than the slowest speed required to remain on the track? Key 44
21. Draw a force diagram for the coaster at P. Does the track apply any force to the coaster? Key 21

22. What is the force equation for the coaster when it is at P? Key 52
23. What is the relation between the energy of the coaster at the start and the energy at point P? Key 29
24. Draw a diagram showing different accelerations to which the passenger is subjected. Key 37
25. What is the gravitational force on the rocket at a distance x from the center of the earth? Key 30
26. What is the potential energy of the rocket at radius R_x? Key 7
27. What is the kinetic energy of the rocket at radius R_x? Key 38
28. What is the total energy of the rocket at radius R_x? Key 15

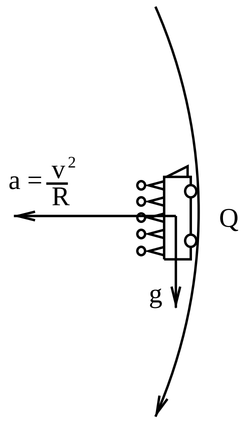
Answer Key

1. 0
2. $2.9 \times 10^3\,\mathrm{Nm}$
3. $P = \frac{W}{\Delta t}$
4. 

5. $E_{\text{top}} - W_{\text{N.C.}} = E_{\text{bottom}}$
6. 3.0 times heavier
7. $\frac{-mM_E G}{R_x}$
8. $6.2 \times 10^5\,\mathrm{m/s}$
9. $w = fs\cos\theta$
10. The work is done by the weightlifter
11. $34\,\mathrm{m/s}$
12. $mg\sin\theta - F_f = ma$
13. $50\,\mathrm{m}$
14. $31\,\mathrm{m/s}$
15. $E = -\frac{mM_e G}{2r_x} = \frac{mv^2}{2}$
16. $\frac{1}{2}mv^2 - GMm/R$
17. $E_i = E_f = -GM_E(20\,\mathrm{kg})/R_E$
18. $10\,\mathrm{m/s}$
19. $34\,\mathrm{m/s}$
20. The length of the slope
21. There is no track force.



22. $1.2 \times 10^2\,\mathrm{m}$
23. $3.0\,\mathrm{m/s}$
24. $3.0\,\mathrm{km}$
25. $\vec{F}_H = |m\vec{g}|$ upward
26. $3.9 \times 10^6\,\mathrm{W}$
27. weight, normal force, frictional force
28. Yes: friction
29. $mgh = \frac{1}{2}mv^2 + mg(2R)$
30. $\frac{mM_E G}{R_x^2}$
31. $1.1 \times 10^4\,\mathrm{m/s}$
32. 0
33. $\Delta PE = mgh$
34. $9.0 \times 10^8\,\mathrm{J}$
35. $N - mg\cos\theta = 0$
36. $31\,\mathrm{m/s}$



37.

38. $\dfrac{mv^2}{2} = \dfrac{mM_E G}{2R_x}$

39. $-\dfrac{GMm}{R}$

40. 0

41. $2.9 \times 10^2\,\mathrm{J}$

42. mgh, $\frac{1}{2}mv^2$

43. $3.2\,\mathrm{m/s^2}$

44. Faster: The track would push on the coaster at P.
Slower: The coaster would fall of before reaching P.

45. $31\,\mathrm{m/s}$

46. $9.8\,\mathrm{Nm}$

47. 0

48. $2.9 \times 10^3\,\mathrm{Nm}$

49. $2.3 \times 10^2\,\mathrm{s}$

50. $v = \sqrt{2gh(1 - \mu\cot\theta) = 28\,\mathrm{m/s}}$

51. $28\,\mathrm{m/s}$

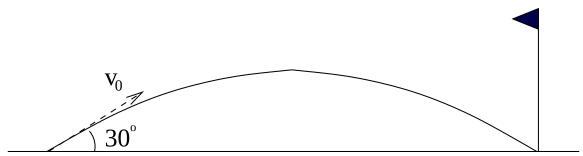
52. $\dfrac{mv^2}{R} = mg$

53. $v^2 = M_e G\Big(\dfrac{2}{R_E} - \dfrac{1}{R_x}\Big) = gR_E\Big(2 - \dfrac{R_E}{R_x}\Big)$

Learning Guide 5

Problem I: Hole in one!

A golf ball (mass of 4.7×10^{-2} kg) hit by Tiger Woods drops exactly into a hole 100 m away. It is observed that the angle between the initial velocity vector and the horizontal plane is 30°.



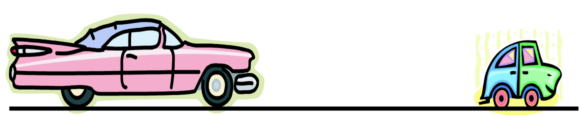
1. What is the magnitude of the initial velocity? Key 31

2. The typical impact time when a driver hits a golf ball is 1.3×10^{-3} s. What is the average force experienced by the ball? Key 23
(If you don't understand, See Helping Questions 1 and 2.)

3. The mass of the driver used by Tiger is 2 kg. If the speed of the driver just before hitting the ball was 51 m/s, what is the velocity right after hitting the ball? Key 38
(Trouble? See Helping Question 3.)

Problem II: Highway Collision

A Lexus (mass of 2100 kg) travelling at 80 km/hr collides front to back with a Volkswagen (mass of 800 kg) travelling at 50 km/hr in the same direction. The resulting mess sticks together.



1. What is the velocity of the cars right after the collision? Key 22

2. What is the change in the velocity experienced by the passengers in each car? Key 14

3. The collision lasted 0.5 seconds. The Volkswagen has a head-support in the driver's seat. Assuming that the head of the driver weighs 8 kg, compute the average force on the head during the collision. Key 6
(See Helping Question 4 for a hint)

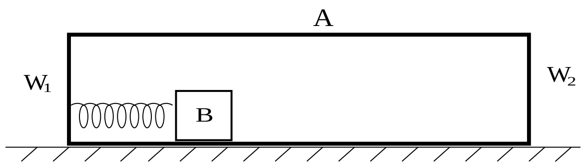
4. Is the kinetic energy conserved? If not, by how much does it change? Key 29

5. Describe and explain the motion of the center of gravity of the system before and after the collision. Key 21

6. Suppose the collision is elastic, namely the two cars do not stick together, but move off after the collision, conserving the total kinetic energy. What are the final velocities of the two cars? Key 13
(See Helping Questions 5 and 6 for assistance.)

Problem III

A cylinder A of mass M (with both ends closed) is placed on a horizontal frictionless plane. Inside the cylinder is a solid cylinder B of mass m and there is no friction between A and B. Initially, the cylinders A and B are at rest with the plane and then cylinder B is accelerated by a spring attached to the wall W_1 of A to a velocity v with respect to the cylinder A. (Neglect the spring's mass.)

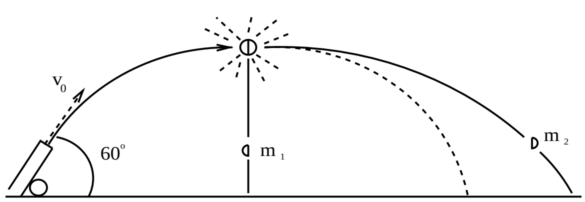


1. At what velocity u_A does the cylinder A move? Key 28
(If you can't do this, see Helping Question 7.)

2. Eventually the cylinder B collides with the wall W_2. Assuming the collision to be elastic, determine the velocities u'_A and u'_B of the cylinders A and B after the collision. Key 12
(See Helping Questions 8 and 9 if you are stuck.)

Problem IV

A steel ball of mass $M = 20\,\text{kg}$ is shot at the angle of $60°$ above the horizon, with an initial velocity $V_0 = 200\,\text{m/s}$. Just as it reaches its maximal height it explodes and breaks into two pieces. One piece of mass $m_1 = 15\,\text{kg}$ stops and then falls vertically to the ground.



1. What is the velocity, V, of the second piece immediately after they break up? Key 35
(See Helping Questions 10 and 11 for aid.)

2. What energy is released by the explosion? Give the least possible figure. Key 11
(Trouble? See Helping Question 12.)

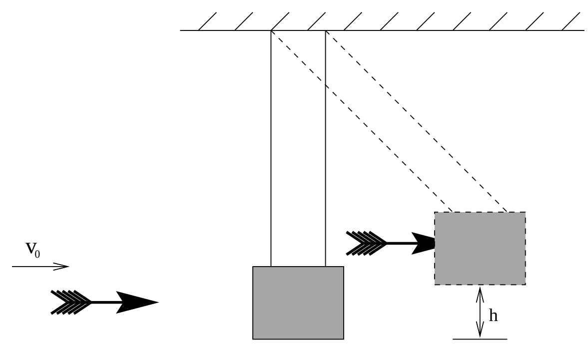
3. Which of the two pieces will be the first to reach the ground? Key 26

4. What is the position of the center of mass after both pieces reach the ground? Answer without using the results of 1. Key 18
(If you don't understand, do Helping Question 13.)

5. At what distance from the cannon would the second piece fall to the ground? Give two derivations, one based on the answer to 4 and the other based on the answer to 1. Key 2

Problem V

A dart of mass $m = 20\,\text{g}$, which is moving horizontally with velocity $v_0 = 12\,\text{m/s}$, strikes and is embedded in a wooden block of mass $M = 0.4\,\text{kg}$. The wooden block hangs on a long string.



1. What will be the velocity, v, of the block immediately after the impact? Key 33

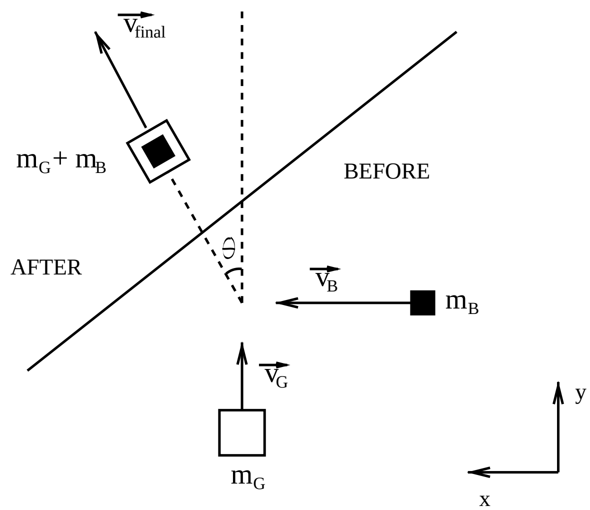
2. How much kinetic energy and how much momentum are lost in the collision? Key 25

3. After the impact the block with the dart swings upward to a maximum height h. Find h. Key 17

4. How much mechanical energy (kinetic plus potential) and how much momentum are lost by the system of block and dart in the swing? Key 9

Problem VI

Grandma Moses is moving in her wheelchair on an icy parking lot with the speed $V_G = 1.5\,\text{m/s}$. Her grandson runs up to her with a velocity $V_B = 3.0\,\text{m/s}$ perpendicular to his Grandma's motion. He jumps into her lap. The mass of the grandma is $M_G = 70\,\text{kg}$ and the mass of the boy is $M_B = 35\,\text{kg}$. Neglect friction.

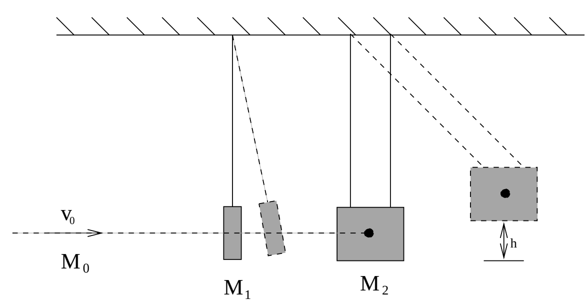


1. What is the final velocity of the wheelchair? Key 1
   (If you have trouble, see Helping Question 14)

2. Find the force, $\vec{F}$, that the boy experienced assuming it was constant and lasted for 0.1 s. Key 24
   (See Helping Question 15 if necessary.)

Problem VII: An Example of an Exam Problem from a Previous Year

A bullet of mass $M = 0.01\,\text{kg}$ travels with a speed of $v_0 = 300\,\text{m/s}$. As shown in the figure, it passes through the center of the plate suspended as pendulum 1 and comes to rest at the center of a sandbag suspended as pendulum 2. The pendulum 1 has a mass $M_1 = 0.1\,\text{kg}$ and the pendulum 2 has a mass $M_2 = 0.99\,\text{kg}$. The center of mass of pendulumm 2 rises a height $h = 0.02\,\text{m}$ as a result of the impact. What is the velocity of the plate of pendulum 1 just after the bullet passes through? Key 8



Helping Questions

1. What is the total change in momentum ΔP experienced by the golf ball? Key 15
2. What is the relation between ΔP and F (think about the impulse)? Key 7
3. What equation expresses conservation of momentum for this problem? Key 30
(If you don't see where this comes from, review K & S section 7.2.)
4. Solve the momentum equation for v_{Lexus} in terms of $v_{\text{Volkswagen}}$. Key 37
Now substitute into the energy equation and solve the resulting quadratic equation.
5. Write down the two equations which express conservation of momentum and energy for the system. What are they? Key 5
Now you have two equations and two unknowns.
6. What was the impulse on the head? Key 36
(If you still can't do this problem, carefully review K & S section 7.1.)
7. What is the total momentum of the system? Key 20
8. Write down the momentum and energy conservation equations for the system. Key 4
9. Use the momentum equation to find u'_A in terms of u'_B; then solve the energy equation by substitution.
10. How does the explosion affect the motion of the center of mass? Key 27
11. What are the kinetic energies of the ball just before and just after the explosion? Key 19
12. What are the momenta of the ball's pieces before and after the explosion? Key 3
Is the momentum conserved, and why? Key 34
13. What is the ball's velocity (speed and direction) just before the explosion? Key 10
14. What is the conserved quantity? Key 32
15. How is the force experienced by the boy related to the change of his momentum? Key 16

Answer Key

1. $\vec{V}_{\text{final}} = (1.0\hat{x} + 1.0\hat{y})\,\text{m/s}$ or $|\vec{V}_{\text{final}}| = 1.4\,\text{m/s}$ and $\theta = 45°$

2. $8.8 \times 10^3\,\text{m}$

3. Before $10^5\,\text{J}$, after $4 \times 10^5\,\text{J}$

4. Momentum: $0 = mu_B + Mu_A = mu'_B + Mu'_A$, $(u_B = v + u_A)$. Energy: $\frac{1}{2}mu_B^2 + \frac{1}{2}Mu_A^2 = \frac{1}{2}m{u'_B}^2 + \frac{1}{2}M{u'_A}^2$

5. Momentum: $(2100\,\text{kg})(80\,\text{km/h}) + (800\,\text{kg})(50\,\text{km/h}) = (2100\,\text{kg})v_{\text{Lexus}} + 800\,\text{kg}\,v_{\text{Volkswagen}}$.
   Energy: $\frac{1}{2}(2100\,\text{kg})(80\,\text{km/h})^2 + \frac{1}{2}(800\,\text{kg})(50\,\text{km/h})^2 = \frac{1}{2}(2100\,\text{kg})v_{\text{Lexus}}^2 + \frac{1}{2}(800\,\text{kg})v_{\text{Volkswagen}}^2$

6. $98\,\text{N}$

7. $F = \dfrac{\Delta P}{\Delta t}$

8. $23.7\,\text{m/s}$

9. $\Delta E = 0$; $\Delta P = 0.24\,\text{kg m/s}$

10. $100\,\text{m/s}$, horizontal

11. $3 \times 10^5\,\text{J}$

12. $u'_A = +\dfrac{m}{M+m}v$, to the right; $u'_B = -\frac{M}{M+m}v$, to the left

13. Final $v_{\text{Lexus}} = 63\,\text{km/h}$; final $v_{\text{Volkswagen}} = 93\,\text{km/h}$

14. $\Delta v_{\text{Volkswagen}} = 72\,\text{km/h} - 50\,\text{km/h} = 22\,\text{km/r}$; $\Delta v_{\text{Lexus}} = 72\,\text{km/h} - 80\,\text{km/h} = -8\,\text{km/h}$

15. $mv - mv_0 = 1.6\,\text{kg m/s}$

16. $\vec{P}_{\text{boy}}^{\text{after}} - \vec{P}_{\text{boy}}^{\text{before}} = \tau\vec{F}$ $(\tau = 0.1\,\text{s})$

17. $h = \dfrac{v^2}{2g} = 1.7\,\text{cm}$

18. The distance from the cannon is $\dfrac{2v_0^2}{g}\sin\theta\cos\theta = 3.5 \times 10^3\,\text{m}$

19. Before: $(15\,\text{kg} + 5\,\text{kg})100\,\text{m/s} = 2,000\,\text{kg m/s}$. After: $(15\,\text{kg})(0) + (5\,\text{kg})V$

20. Zero, since it was initially at rest and no external force acts.

21. Since the momentum of the center of mass is not changed by an internal force like a collision, it moves with the same velocity $v_{\text{CM}} = 72\,\text{m/s}$ before and after.

22. $72\,\text{km/h}$

23. $1200\,\text{N}$

24. $F_x = -700\,\text{N}$, $F_y = 350\,\text{N}$

25. $\Delta E = 1.4$ Joules; $\Delta P = 0$

26. Neither

27. It doesn't.

28. $u_A = \dfrac{-mv}{M+m}$ to the left

29. No: $\Delta KE = 1.5 \times 10^4\,\text{J}$

30. $M_{\text{driver}}V_{\text{driver}}^{\text{before}} + 0 = M_{\text{ball}}V_{\text{ball}} + M_{\text{driver}}V_{\text{driver}}^{\text{after}}$

31. $34\,\text{m/s}$

32. The total momentum $\vec{P}_{\text{tot}} = \vec{P}_{\text{grandma}} + \vec{P}_{\text{boy}}$

33. $0.57\,\text{m/s}$

34. Yes; The external impulse $(F\Delta t)$ is negligible since the explosion's duration, Δt, is extremely short and the external force, Mg, is not very large.

35. $400\,\mathrm{m/s}$

36. $F\Delta t = \Delta P_{\mathrm{head}} = M_{\mathrm{head}}\Delta v$

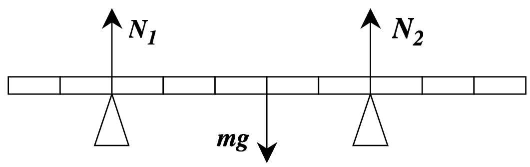
37. $v_{\mathrm{Lexus}} = 99\,\mathrm{km/h} - .38 v_{\mathrm{Volkswagen}}$

38. $50\,\mathrm{m/s}$

Learning Guide 6

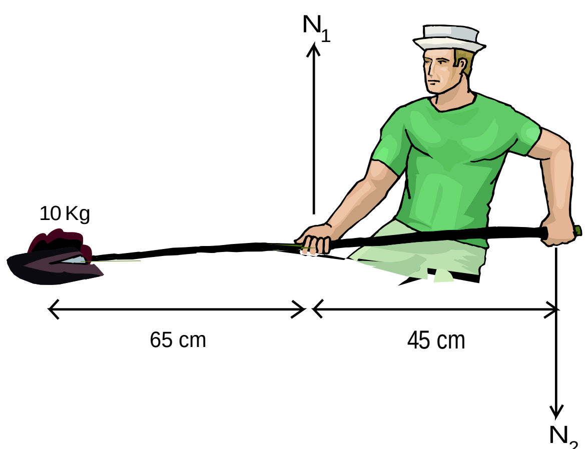
Problem I

1. A 0.1 kg meter rule is supported by two pivots, one at the 20 cm mark an one at the 70 cm mark. What are the reaction forces in the two pivots? Key 32



(Go to Helping Question 1 if you cannot begin.)

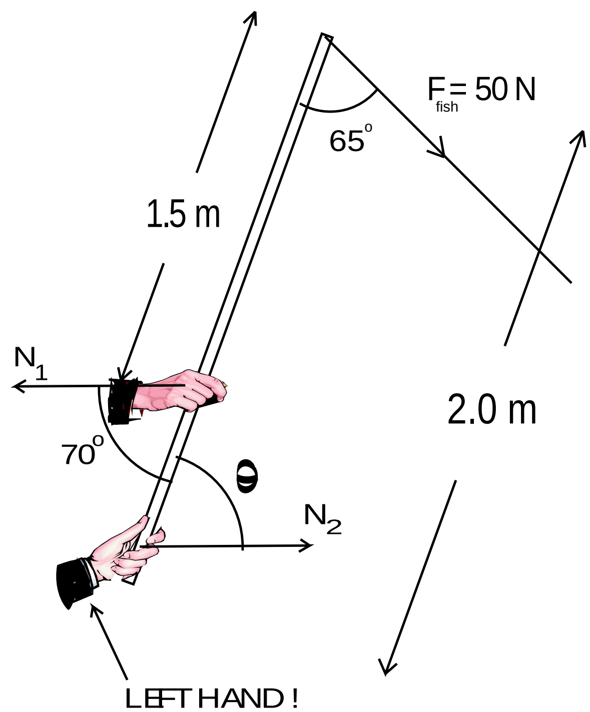
2. The man is holding a 10 kg shovel full of gravel horizontally. (Assume that the shovel has no mass.) What is the torque condition about the man's right hand? What is the force exerted downward on the shovel by the man's left hand? Key 11



3. What is the torque condition about the man's left hand? What is the force exerted upward on the shovel by the man's right hand? Key 43

Problem II

A fish exerts a 50 N force along the fishing line as shown in the diagram.

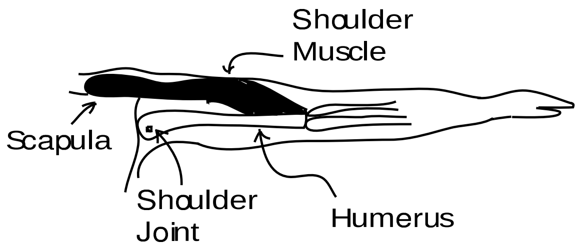


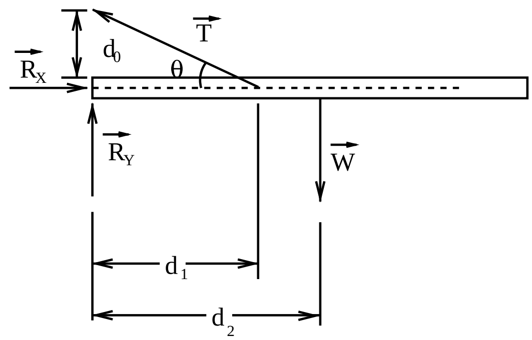
1. Write down the torque condition with the left hand as the pivot. Key 75
   Use this equation to calculate the magnitude of the force N_1 exerted by the right hand on the rod. Key 22
2. Write down the equilibrium condition for the components of the forces along the rod. Key 54
3. Write down either
   (a) The torque condition with the right hand as pivot. Key 1
   or
   (b) The equilibrium condition for the components of the forces perpendicular to the rod. Key 33

   And, use your answer to 2 in conjunction with your answer to either 1 or 2 to find the angle θ, Key 65
   and the magnitude of the force of the force N_2. Key 12

Problem III: Old Exam Problem

Consider the problem of achieving static equilibrium of the arm when extended in the horizontal position. Acting on the arm are the force of tension $\vec{T}$ in the shoulder muscle, the reaction force $\vec{R}$ at the shoulder joint (with components R_x and R_y) and the weight of the arm $\vec{W}$. As indicated in the figure, the shoulder muscle is connected to the humerus at a distance d_1 from the shoulder joint and to the scapula at a distance d_0 from the shoulder joint. The center mass of the arm is at a distance d_2 from the shoulder joint.





1. Where does the weight of the arm act? Key 44

2. Write down the force equation for equilibrium of the arms in both vertical Key 76
   and horizontal Key 23
   directions. Express your answers in terms of the forces and θ.

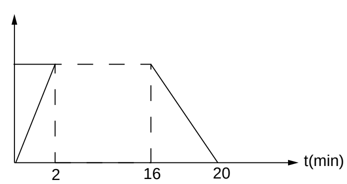
3. Write down the torque equation for the arm using an axis through the shoulder joint. Express your answer in terms of the forces, d_1, d_2 and θ. Key 55

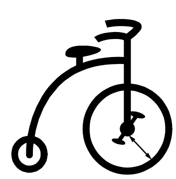
4. If $d_0 = 3.0\,\text{cm}$, $d_1 = 15\,\text{cm}$, and $d_2 = 30\,\text{cm}$, and $W = 60\,\text{N}$, calculate the tension T in the shoulder muscle. Key 2

5. Calculate R_x and R_y Key 34

Problem IV: An Old-Fashioned Bicycle

Mr. A. has an old-fashioned bicycle with two wheels of different radii (0.80 m and 0.15 m). Every morning he rides this bicycle for 20 minutes to his office 12 km away from his home. The velocity of the bicycle is depicted below:



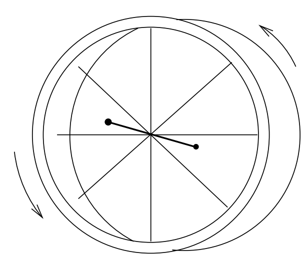


1. After travelling the first 10 meters, what are the rotation angles of the two wheels in degrees and radians? Key 66
   (For a hint see Helping Question 2.)
2. From the velocity graph above, compute the cruising velocity V_c. Key 45
3. During the cruising period, what are the angular velocities of the two wheels? In which direction do these vectors point? Key 77
4. During the cruising period (minutes 2 to 16), what is the radial acceleration at the edge of the bigger wheel? Key 24
5. During the first two minutes, what are the angular accelerations of the two wheels? In which direction do these vectors point? Key 56
   (Trouble? See Helping Question 3.)
6. What is the direction of the angular acceleration vectors of two wheels during the last four minutes? Key 67

Problem V: Moment of Inertia

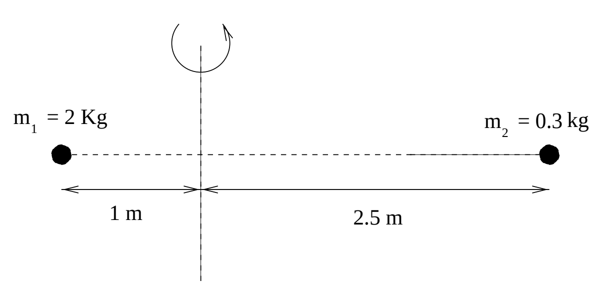
Compute the moment of inertia of the following objects:

1. Spinning wheel with radius 0.4 m, width 7 cm, thickness 0.3 cm and density $\rho = 0.3\,\mathrm{g/cm^3}$. Assume the spokes are massless. Key 14

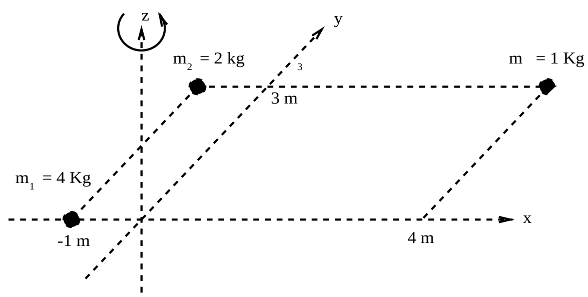


   (See Helping Question 4 if you have trouble)

2. Two balls. What is the moment of inertia? Key 57



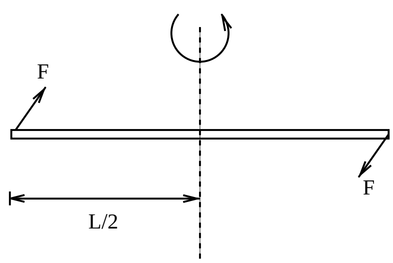
3. Three balls. What is the moment of inertia? Key 36



4. Earth. Assume it to be a perfect sphere of uniform density with radius 6.4×10^6 m and mass 6.0×10^{24} kg. Key 15

Problem VI: Spacecraft Maneuver

Imagine a spacecraft of unrealistic but traditional shape, floating in space. Its mass m is 1.0×10^4 kg and the length ℓ is 1.0×10^2 m. The pilot desires to execute a 90° turn by firing two small steering rockets placed at the craft's nose and tail at right angles to its length. Assume for simplicity that the ship can be approximated by a thin rod of mass m and length ℓ with the steering rockets placed at the ends of this rod. Each steering rocket can produce a thrust of 250 N.



1. Assume that the rocket is at rest relative to the observer O, that is, it does not move relative to O. What is the angular acceleration α produced by the steering rockets when fired? Key 47
   (See Helping Question 5 if necessary.)

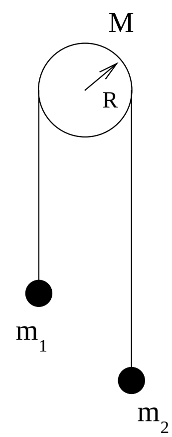
   The pilot decides to perform the maneuver by firing the steering rockets until a 45° swing has been completed, then by firing the opposing rockets (equal thrust) to decelerate the turn for the last 45°.

2. How long does the full 90° turn take? Key 5
   (For a help, recheck Helping Question 6.)

3. What is the maximum angular speed attained during the turn? Key 16
   (For a hint, seek out Helping Question 5, as above.)

Problem VII: Atwood Machine

Two masses, m_1 and m_2 ($m_2 > m_1$), hang on a pulley of radius R and mass M. The pulley is a solid cylinder and rotates without friction.



1. Write down the equation of motion for m_1 and m_2 taking into account that the two strings suspending m_1 and m_2 have different tensions T_1 and T_2. Key 48

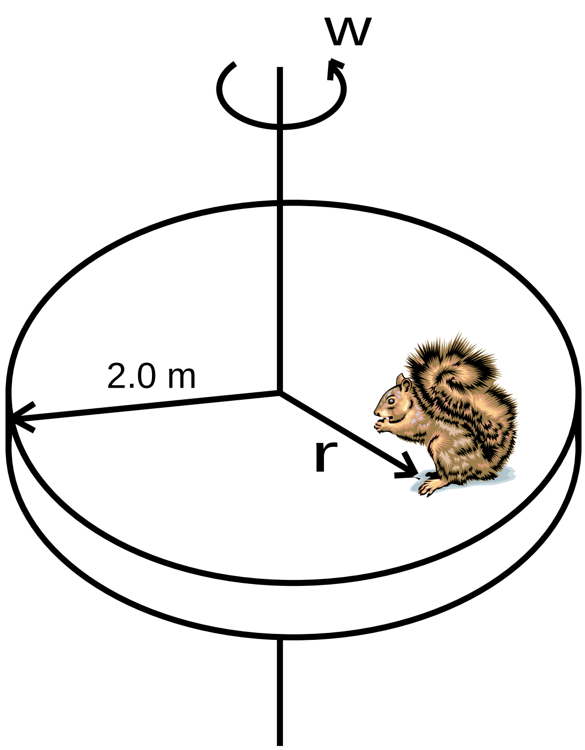
2. What is the equation of motion for the pulley? Key 80
   (Trouble? See Helping Question 7.)

3. What is the relation between the angular acceleration of the pulley and the acceleration of m_2? Key 6

4. Express the angular acceleration of the pulley in terms of m_1, m_2, M, R and $g = 9.8\,\mathrm{m/s}$. Key 38
   (See Helping Question 8 for relief.)

Problem VIII: Hungry Squirrel

A hungry squirrel of negligible size (mass $m = 2.0\,\mathrm{kg}$) is sitting on the edge of a circular disk of radius $R = 2.0\,\mathrm{m}$ and mass $M = 3.0\,\mathrm{kg}$. The disk is rotating frictionlessly with a rotational velocity $\omega_0 = 1.0\,\mathrm{rad/s}$. At the center of the disk is placed a walnut and the squirrel tries to reach the center to get it.



1. Suppose the squirrel moves radially toward the center. When he comes to the distance $r = 1.0\,\mathrm{m}$ from the center, he stops. What is the angular velocity of the disk? Key 17
   (Scrutinize Helping Questions 9 and 10 if necessary.)

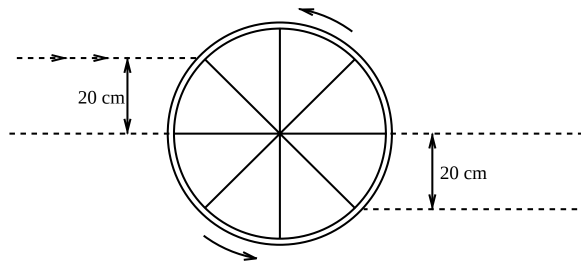
2. At $r = 1.0\,\mathrm{m}$, what is the centripetal acceleration of the squirrel? Key 60

3. Suppose the static coefficient of friction between the squirrel and the disk is $\mu_S = 0.50$. At $r = 1.0\,\mathrm{m}$, does the squirrel slip? Key 7

4. (Strictly optional) With $\mu_S = 0.50$, what is the maximum value of ω_0 so that the squirrel gets the walnut? Key 39
   (See Helping Questions 11 and 12 for hints.)

Problem IX: The Wagon Wheel

A thin wooden wagon wheel of radius 50 cm is initially rotating counter clockwise about a fixed axis at its center with angular velocity 8 rad/s. Assume that the mass of the wheel (40 kg) is concentrated entirely at is rim. Two guns fire an equal number of bullets of mass 10 g and velocity 1000 m/s into the wheel (see figure) at an impact parameter $r = 20\,\mathrm{cm}$. The bullet sticks to the rim at the point of impact.



1. What is the initial angular momentum of the wheel (before the guns start firing)? Key 50

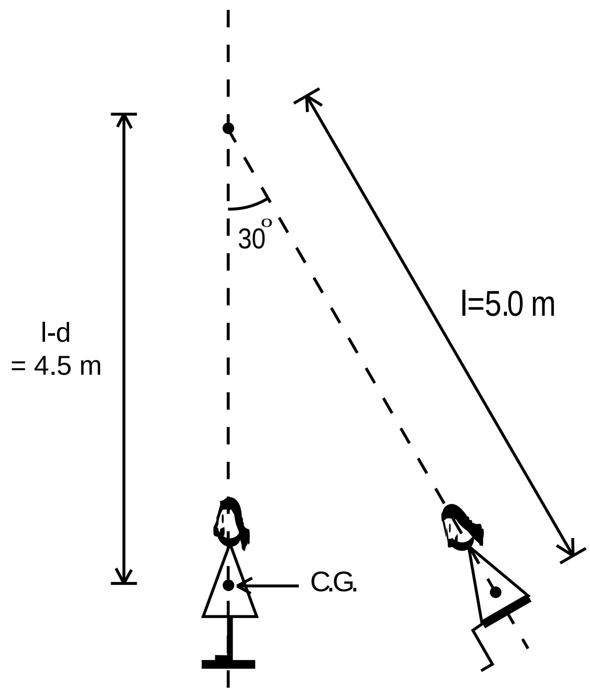
2. On impact, what is the component tangent to the wheel of each bullet's velocity? On impact, what is the angular momentum of each bullet with respect to the center of the wheel? Key 82

3. What is the total number N of bullets which stop the rotation of the wheel? Key 29
(Stumped? Why not see Helping Question 13?)

4. If the impact parameter r had been 50 cm, what would the angular velocity of the wheel be after hit by N bullets? Key 8
(Use the value of N you just calculated in part 3.)

Problem X: The Swing

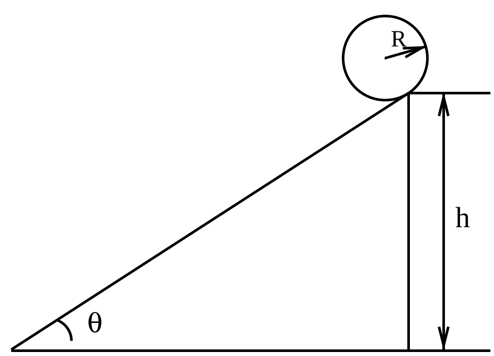
A girl of mass 40 kg is sitting in a massless swing. In this position the distance from her center of mass to the swing's pivot is $\ell = 5.0\,\mathrm{m}$. She is now let go from an initial angle $\theta = 30^\circ$ without a push. In order to "pump" herself higher, just as swing passes through the vertical, she suddenly stands up, raising her center of mass a distance $d = 0.5\,\mathrm{m}$.



1. What are the linear velocities of the girl just before and just after she stands up? (If you can't start, See Helping Question 14) Key 40
2. To what new maximum angle will the girl rise if she remains standing? Key 19
3. If she now (at the top of the first swing) sits down again an repeats the same procedure, to what maximum angle will she rise on the next swing? Key 51
4. Where does the extra energy come from on each swing? Key 83

Problem XI: Rolling down a Hill

A solid cylinder with mass M and radius R is placed on the edge of a slope of height h and inclination θ.



1. Suppose the cylinder rolls down without slipping. What is the angular acceleration α of the cylinder in terms g, θ, R and M? Key 30
   (You may need Helping Questions 15 and 16.)

2. How long does it take the cylinder to roll down to the bottom? Key 73

3. Replace the solid cylinder by a hollow one with the same mass M. Which one reaches the bottom faster, the solid one or the hollow one? Key 20

Problem XII: Typical Exam Questions

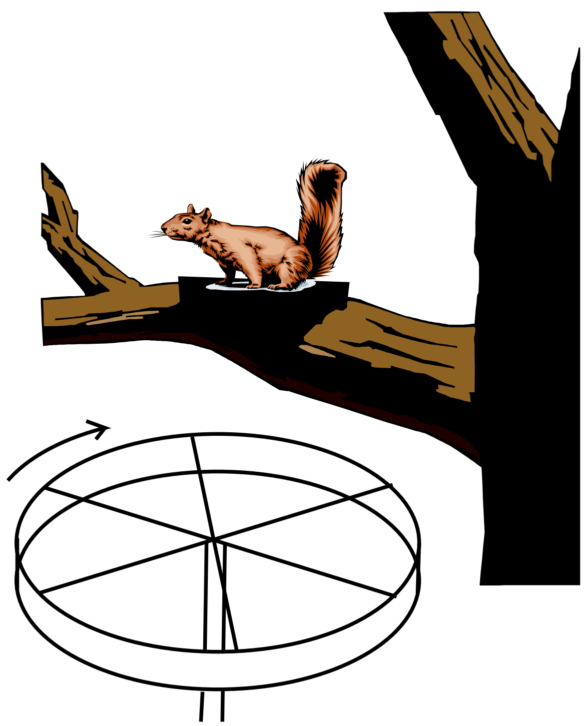
Answer the following questions:

1. The moment of inertia of a wheel about its axle is $5\,\text{kg}\,\text{m}^2$. What torque must be applied to accelerate the wheel at $1.5\,\text{rad/s}^2$? Key 52

2. A cyclist moving straight at a constant speed suddenly applies a torque at the handle bar to make a right turn. How does the front wheel react? Key 84

Problem XIII: More Typical Exam Questions

A squirrel, of mass $2\,\text{kg}$ and neglible size, sits in a tree above a wagon which is turned on its side. One of the wheels is rotating clockwise with angular velocity $\omega = 1.2\,\text{rad/s}$. The wheel has a moment of inertia of $10\,\text{kg}\,\text{m}^2$ and radius $1\,\text{m}$ (see figure).

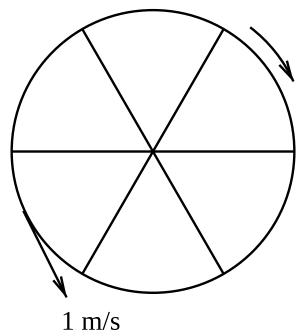


1. The squirrel drops to the center of the wheel. What is ω now?

2. The squirrel walks out to the rim. What is ω now? Key 31

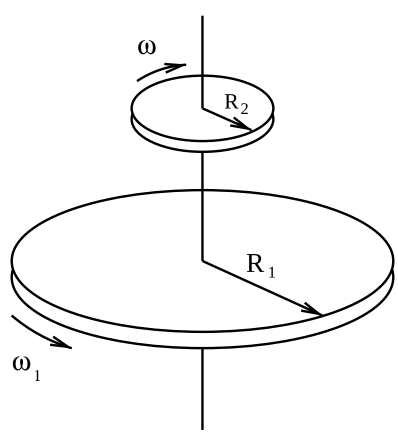
3. The squirrel runs counter clockwise around the rim with an angular velocity of 0.5 rad/s relative to the ground. What is ω now? Key 63
(See Helping Question 17 if you are stumped.)

4. The squirrel jumps off the rim in the direction opposite to the rotation with a velocity tangent to the rim of 1 m/s, relative to the ground (as shown). What is ω now? Key 42



Problem XIV: Still More Typical Exam Questions

Two homogeneous discs of radius $R_1 = 2\,\text{m}$ and $R_2 = 1\,\text{m}$ and mass $M_1 = 4\,\text{kg}$ and $M_2 = 1\,\text{kg}$ rotate with angular velocity $\omega_1 = 3\,\text{rad/s}$ counter-clockwise and $\omega_1 = 14\,\text{rad/s}$ clockwise around the same axis as shown in the figure. They are brought together by lowering the upper disc and after a while due to friction they rotate together as a single body (as shown below).



1. What is the final angular velocity of this compound body? Key 74

2. What is the sense of the rotation, clockwise or counter-clockwise? Key 21

3. What is the difference between initial and final kinetic energy? Key 53
(Recall that the moment of inertia of a homogenous disc is given by $I = \frac{1}{2}MR^2$ where M is the mass of the disc and R is its radius.)

Helping Questions

1. What is the torque condition about the 20 cm mark? Key 64
   Once you find N_2, go on to find N_1

2. What is the relationship between the distance s laid out by a wheel, its radius r and the angle θ (measured in radians) through which it turns? Key 13

3. What is the rotation angle θ_{big} of the big wheel during the first two minutes? Key 3
   What is the relation between θ_{big}, the angular acceleration α_{big} and the final angular velocity ω_{big}? Key 35
   If you still don't follow, reread K & S section 5.3

4. What is the volume of the wheel? Key 46
   What is the mass of the wheel? Key 78
   What is the formula for the moment of inertia of a cylindrical shell of mass M and radius R? Key 25

5. What is Newton's Second Law for rotational motion? Key 79
   What is the moment of inertia I of the rocket, expressed in terms of m and ℓ? Key 26
   What is the torque Γ produced by the thrust $F = 250\,\text{N}$ of the two steering rockets? Key 58

6. What is the relationship between α, the constant angular acceleration of the rocket, the time t for which it accelerates and the total angle θ it traverses in that time? What is the relationship between the final angular velocity ω, α and t? Key 37
   Could you express ω through α and θ? Key 69

7. What are the two forces causing the pulley to rotate? Key 27
   Write down the total torque Γ acting on the pulley? Key 59

8. From your answer to 1, find an expression for $T_2 - T_1$. Key 70
   Now plug this into 2 and use 3 to solve.

9. What quantity is conserved in this problem? Key 49
   Go on to the next Helping Question, if necessary.

10. What is the total angular momentum L_0 of the squirrel and disc before the squirrel walks inward? Key 81
    Write down an expression L_f for the angular momentum *after* he walked inward to $r = 1.0\,\text{m}$. Key 28

11. For a given ω_0, at what point on the radius does the squirrel experience the maximum centripetal acceleration? (i.e. if he makes it past this point without flying off, he makes it all the way to the walnut.) Key 71
    (Hint: write down the formula for the centripetal acceleration as a function of r, and don't forget that ω is a function of r, too. Then find the maximum of this function by differentiating with respect to r.)

12. At this point on the radius, what is the maximum centrifugal force $F_{\max}$ he can feel without flying off? Key 18

13. What is the change in angular momentum caused by one bullet? Key 61

14. Using conservation of energy, find the velocity at the bottom just before the girl stands up. Then write down the equation which expresses the conservation of angular momentum before and after she stands. Key 72

15. What is Newton's Second Law for motion along the incline? Key 62
    What is Newton's Second Law for the rotational motion of the cylinder? Key 9

16. What is the relationship between α and a when the cylinder rolls without slipping? Key 41

17. Write down the equation which expresses conservation of angular momentum for the system. Key 10

Answer Key

1. $(0.5\,\mathrm{m})N_2 \sin\theta - (1.5\,\mathrm{m})(50\,\mathrm{N})\sin 65^\circ = 0$
2. $6.1 \times 10^2\,\mathrm{N}$
3. $\theta_{\mathrm{big}} = \dfrac{v_c}{2} \cdot (2\,\mathrm{min}) \cdot \dfrac{1}{r_{\mathrm{big}}} = 9.0 \times 10^2\,\mathrm{rad}$
4. $1.3\,\mathrm{m}$
5. $4.6 \times 10^1\,\mathrm{s}$
6. $\alpha = a/R$
7. No.
8. $11.9\,\mathrm{rad/s}$ clockwise
9. $\Gamma = F_f R = I\alpha$
10. $I\omega_0 = I\omega - MR^2(.5\,\mathrm{rad/s})$
11. $10 \times g \times .65 = N_2 \times .45$; $N_2 = 1.4 \times 10^2\,\mathrm{N}$
12. $1.6 \times 10^2\,\mathrm{N}$
13. $\theta = s/r$
14. $2.5 \times 10^{-2}\,\mathrm{kg\,m^2}$
15. $9.8 \times 10^{37}\,\mathrm{kg\,m^2}$
16. $6.9 \times 10^{-2}\,\mathrm{rad/s}$
17. $1.75\,\mathrm{rad/s}$
18. $F_{\mathrm{max}} = m\omega^2 R = \mu_s mg$
19. $\theta_1 = 35^\circ$
20. The solid one, with the smaller moment of inertia.
21. Counter clockwise
22. $N_1 = 1.9 \times 10^2\,\mathrm{N}$
23. Horizontal: $R_x - T\cos\theta = 0$
24. $\omega_{\mathrm{big}}^2 r_{\mathrm{big}} = 1.8 \times 10^2\,\mathrm{m/s^2}$
25. $I = MR^2$
26. $I = \frac{1}{12} m\ell^2$
27. T_1 and T_2
28. $L_f = (\frac{1}{2}MR^2 + mr^2)\omega_{\mathrm{final}}$
29. $N = 40$ bullets
30. $\alpha = \frac{2}{3} g \sin\theta / R$
31. $\omega = 1.0\,\mathrm{rad/s}$
32. $N_1 = \dfrac{2}{5} mg = 0.39\,\mathrm{N}$; $N_2 = \dfrac{3}{5} mg = 0.59\,\mathrm{N}$
33. $N_1 \sin 70^\circ - N_2 \sin\theta - (50\,\mathrm{N})\sin 65^\circ = 0$
34. $R_x = 6.0 \times 10^2\,\mathrm{N}$, $R_y = -60\,\mathrm{N}$
35. $\omega_{\mathrm{big}}^2 = 2\alpha_{\mathrm{big}}\theta_{\mathrm{big}}$
36. $49\,\mathrm{kg\,m^2}$
37. $\theta = \frac{1}{2}\alpha t^2$, $\omega = \alpha t$
38. $\alpha = \dfrac{(m_1 - m_1)g}{R(m_1 + m_2 + \frac{1}{2}M)}$
39. $1.26\,\mathrm{rad/s}$
40. $V_{\mathrm{before}} = 3.6\,\mathrm{m/s}$; $V_{\mathrm{after}} = 4.0\,\mathrm{m/s}$
41. $a = \alpha R$
42. $\omega = 1.4\,\mathrm{rad/s}$
43. $10 \times g \times 110 = N_1 \times 45$; $N_1 = 2.4 \times 10^2\,\mathrm{N}$
44. At the center of mass
45. $12\,\mathrm{m/s}$
46. $V = (\text{circumference}) \times (\text{width}) \times (\text{thickness}) = 5.3 \times 10^{-4}\,\mathrm{m^3}$

47. $\alpha = 3 \times 10^{-3}\,\text{rad/s}^2$

48. Let a be the downward acceleration of m_2. Then, $T_1 - m_1 g = m_1 a$ and $m_2 g - T_2 = m_2 a$.

49. Angular momentum, since there is no external torque.

50. $80\,\text{kg m}^2/\text{s}$

51. 39.4°

52. $7.5\,\text{N m}$

53. $68\,\text{J}$

54. $N_2 \cos\theta - N_1 \cos 70^\circ - 50\cos 65^\circ = 0$

55. $d_1 T \sin\theta - d_2 W \sin 90^\circ = 0$

56. $a_{\text{big}} = 1.2 \times 10^{-1}\,\text{rad/s}^2$. $a_{\text{small}} = 6.5 \times 10^{-1}\,\text{rad/s}^2$; again the direction is perpendicular to the place of rotation and given by the right hand rule. In the same direction as the answer for part III.

57. $3.9\,\text{kg m}^2$

58. $\Gamma = 2F \cdot \dfrac{\ell}{2} = F \cdot \ell$

59. $\Gamma = (T_2 - T_1)R$

60. $a_c = \omega^2 r = 3.1\,\text{m/s}^2$

61. $2\,\text{kg m}^2/\text{s}$

62. $Mg\sin\theta - F_f = Ma$, where F_f is the frictional force.

63. $\omega = 1.3\,\text{rad/s}$

64. The net torque is zero; i.e. $-(30\,\text{cm})mg + (50\,\text{cm})N_2 = 0$.

65. $\theta = 57^\circ$

66. $\theta_{\text{small}} = 66.7\,\text{rad}$, or $3.8 \times 10^{3\circ}$
$\theta_{\text{large}} = 12.5\,\text{rad}$, or 720°

67. Again as given by the right hand rule, but in the direction opposite to that found in part V, since the acceleration is negative.

68. $2.6\,\text{m}$

69. $\omega = \sqrt{2\alpha\theta}$

70. $T_2 - T_1 = (m_2 - m_1)g - (m_1 + m_2)a$

71. $r_{\max} = R\sqrt{\dfrac{M}{6m}} = 1.0\,\text{m}$

72. $\ell m V_{\text{before}} = (\ell - d)mV_{\text{after}}$

73. $t = \dfrac{\sqrt{3h/g}}{\sin\theta}$

74. $2\,\text{rad/s}$

75. $(0.5\,\text{m})(N_1 \sin 70^\circ) - (2.0\,\text{m})(50\,\text{N})\sin 65^\circ = 0$

76. Vertical: $T\sin\theta + R_y - W = 0$

77. $\omega_{\text{small}} = V_c/r_{\text{small}} = 80\,\text{rad/s}$; $\omega_{\text{big}} = V_c/r_{\text{big}} = 15\,\text{rad/s}$ perpendicular to the plane of rotation in the direction given by the right hand rule.

78. $M = \rho V = 1.6 \times 10^{-1}\,\text{kg}$

79. $\Gamma = I\alpha$ where Γ is torque, I the moment of inertia around the rotational axis and α is the angular acceleration.

80. $(T_2 - T_1)R = I\alpha$ where $I = \frac{1}{2}MR^2$ is the moment of inertia of the pulley, and α is its angular acceleration (clockwise).

81. $L_0 = \frac{1}{2}MR^2\omega_0 + mR^2\omega_0 = 14\,\text{kg m}^2/\text{s}$

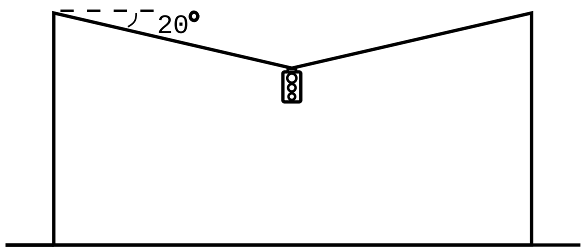
82. $V_{\text{tangent}} = 400\,\text{m/s}$
$L = MV_{\text{tangent}}r = 2\,\text{kg m}^2/\text{sec}$

83. The work done by the girl in pulling herself up.

84. The wheel tilts over to the left; the change in the angular momentum induced is downward.

Learning Guide 7

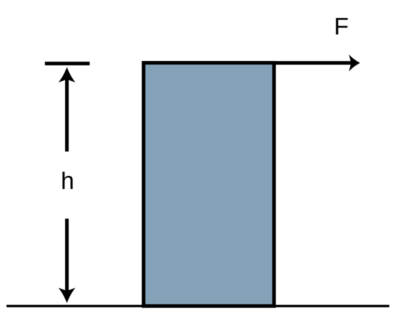
Problem I: The Stoplight



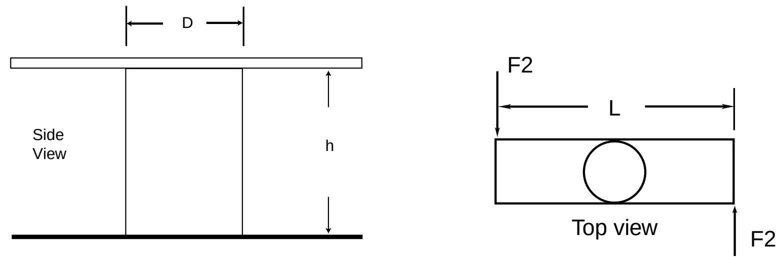
1. An 80 kg stoplight hanging over the center pole of a street is held by two equal steel cables fastened to poles on either side of the street. If the cables make an angle $\theta = 20°$ with the horizontal, by what fraction are the cables stretched because of the weight of the light? (The cross-sectional area of the cables is $.20\,\mathrm{cm}^2$, Young's modulus for for steel is $2 \times 10^{11}\,\mathrm{N/m}^2$, and the ultimate tension strength σ_t is $5 \times 10^8\,\mathrm{N/m}^2$.) Key 52
(If you have trouble see Helping Question 1 and 2.)

2. What is the maximum weight the two steel cables can support without breaking? Key 35
(If you don't understand, See Helping Question 3.)

Problem II: Shear Stress



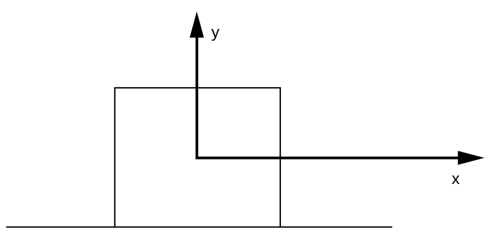
1. A solid aluminum bar of height $h = 2\,\mathrm{m}$ is firmly imbedded in the ground. The bar has a square cross section of area $4\,\mathrm{cm}^2$. If a horizontal force of 5000 N is applied to the top surface of the bar, by how much (δ) is the top displaced with respect to the bottom? Key 64
(If you don't understand how to proceed, See Helping Question 4 and 5.)



2. A solid aluminum cylinder of height $h = 2m$ and diameter $D = 10\,\text{cm}$ is imbedded in the ground and a steel plate of length $L = 50\,\text{cm}$ is welded to the top. If horizontal forces $F_2 = 2500\,\text{N}$ are applied to the ends of the steel plate as shown above, by what angle does the top surface of the cylinder move with respect to the bottom? (The shear modulus for aluminum is $2.4 \times 10^{10}\,\text{N/m}^2$.) Key 47
(See Helping Question 6 7 only if you really need to.)

Problem III: Simple Harmonic Motion

A body oscillates with simple harmonic motion according to the equation $x(t) = 2.0\,\text{m}\cos[(\pi/2)t + \pi/4]$.



1. What are the displacement, the velocity, and the acceleration of the body at $t = 2\,\text{s}$? Key 53
(If you have trouble, See Helping Questions 8 and 9.)

2. What are the frequency f and the period T of the motion? Key 59

3. Suppose the body has a mass M of 1 kg. What is the kinetic energy as a function of time? Key 36
(If you have trouble, See Helping Question 10.)

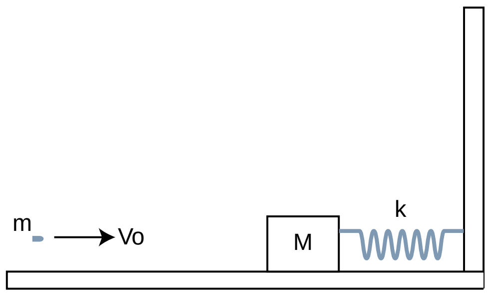
4. If the simple harmonic motion results from a 1 kg mass acted upon by a linear restoring force corresponding to a force constant k, what must k be? Key 65
(For help, Try Helping Question 11)

5. What is the potential energy as a function of time? Key 71
(For help, See Helping Question 12.)

6. Add your answers to parts (3) and (5). What is the significance of the result? Key 25

Problem IV: Ballistics

A block of wood with mass M rests on frictionless surface. The block is attached to the wall through a spring with force constant k. A bullet of mass m and velocity V_0 is fired and is embedded in the block, as shown below.

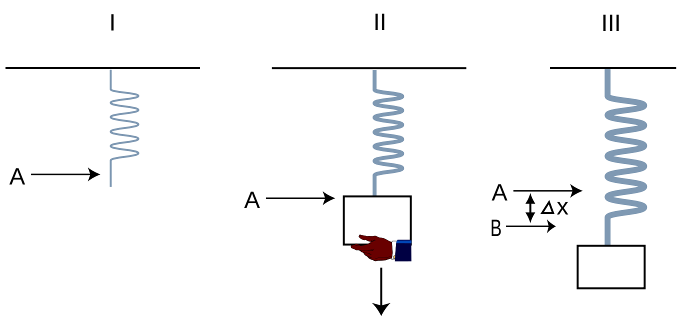


1. What is the amplitude of the resulting oscillation? Key 2
   (For help, Try Helping Questions 13, 14, 15, 16 and 17.)

2. What fraction of the initial kinetic energy of the bullet is stored in the oscillating system? What happens to the difference in the energy? Key 14
   (For help, See Helping Questions 18 and 19.)

Problem V: The Hanging Spring and Mass

A massless spring of force constant $k = 4.9\,\mathrm{N/m}$ hangs unstretched vertically as shown in I. A body of mass $m = .20\,\mathrm{kg}$ is attached to its free end II and then lowered very slowly to its new position III where it is at rest.



1. What is the distance δx the spring has been stretched? Key 20
   (Trouble? See Helping Question 20.)

   If, instead of being lowered slowly, the mass is released instantaneously in (II),

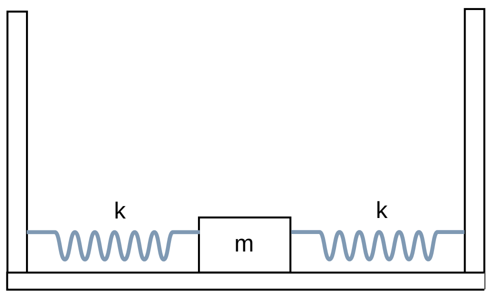
2. How far below point A will the center of the resulting simple harmonic oscillations be? Key 49

3. What is the velocity of m when it passes through the center point? Key 26
   (See Helping Question 21 for a hint.)

4. What is the frequency of the oscillations? Key 55

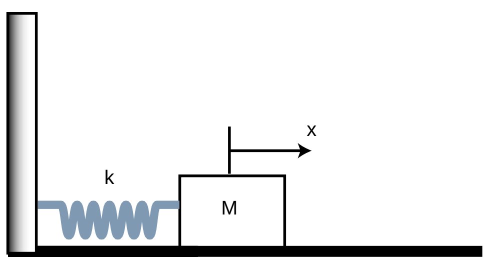
5. What is the amplitude of the oscillations? Key 32
   (Try Helping Question 22 if stuck.)

Problem VI: The Two Spring System



If two springs are attached to m and to fixed supports as shown in the figure, what is the frequency of oscillations? Key 38
(For help, try Helping Question 23 and 24.)

Problem VII: The Damped Oscillator



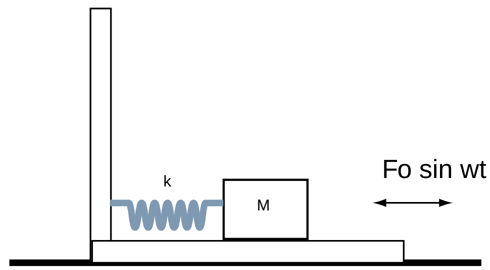
A mass $M = 2\,\text{kg}$ is attached to a wall by a spring (spring constant $k = 8\,\text{N/m}$) and is free to slide along a surface as shown above. The mass is pulled out a distance $d = 20\,\text{cm}$ from its equilibrium position an then released. Because of frictional forces acting between the block and the surface, the mass completes only $6\,\frac{1}{4}$ cycles before coming to rest.

1. Sketch (qualitatively) the displacement (x) of the mass versus time after the mass is released. Key 44

2. How long does it take the mass to stop? Key 21
   (See Helping Question 25 if you need it.)

3. What is the average power lost by the spring-mass system as it slows down? Key 50
   (Trouble? Try Helping Question 26.)

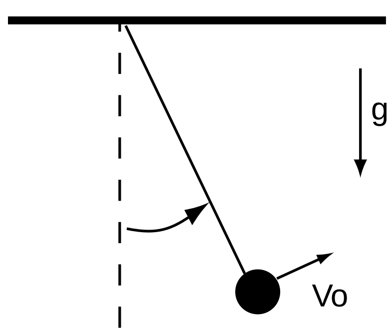
   The same spring and mass system is now placed on a platform and the platform is acted on by a periodic force which causes it to oscillate horizontally at a small amplitude (see figure below).



4. Discuss qualitatively the motion of the mass as the frequency of oscillation of the platform is varied. Make a sketch of the amplitude of M as a function of frequency. Key 4

Problem VIII: The Simple Pendulum

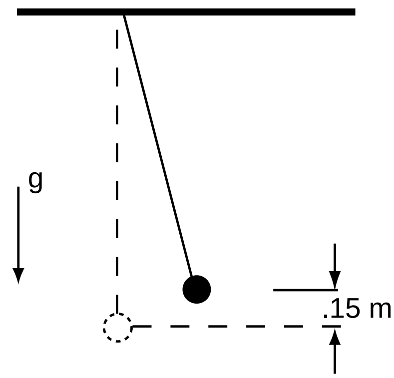
A simple pendulum consisting of a mass $m = 2\,\text{kg}$ at the end of a light string of length $\ell = 1\,\text{m}$ is released with a velocity $v_0 = .50\,\text{m/s}$ at an angle $\theta_0 = 5°$ as shown in the figure.



1. What is the maximum angle that the pendulum makes with the vertical? Key 56
   (See Helping Question 27 for a hint.)
2. How long does it take the pendulum to complete one cycle? Key 10
3. What is the speed of the pendulum as it passes through its lowest point. Key 62
4. How do the answers to parts 1, 2 and 3 change if v_0 is taken in the direction opposite to that shown in the figure? Key 39
   (Try Helping Question 28 if you can't answer this immediately.)

Problem IX: The Stretched Pendulum

A pendulum consists of a 1.1×10^2 kg mass suspended by means of a steel cable 10 m in unstretched length and cross-sectional area $A = .50\,\text{cm}^2$. The pendulum is set swinging in such a way that its maximum height is .15 m above the equilibrium position, as shown. Young's modulus for steel is $E = 2.0 \times 10^{11}\,\text{N/m}^2$.



1. What is the maximum tension in the cable? (Neglect the elongation for this part of the calculation.) Key 68
   (See Helping Questions 29, 30 and 31 if you are having trouble.)

2. What is the maximum elongation of the cable? Key 51

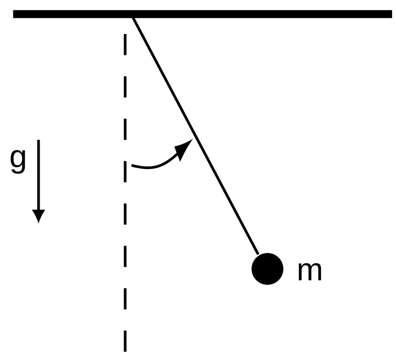
Problem X: Quickies

1. How much work is done in compressing a spring of equilibrium length l m and spring constant $k = 180\,\text{N/m}^2$ to a length of $2/3$ m? Key 28

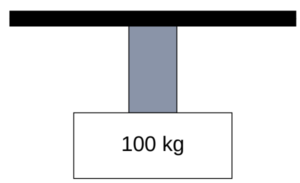
   An object undergoes simple harmonic motion in the x-direction such that its position is given by $x = 3.0\cos(\pi t + \pi/4)$ where x is in meters and t is in seconds.

2. Which of the following statements is (are) false?

   - the amplitude is 3 m
   - the frequency is $\pi/2$ rad/s
   - the period is 1 s Key 5



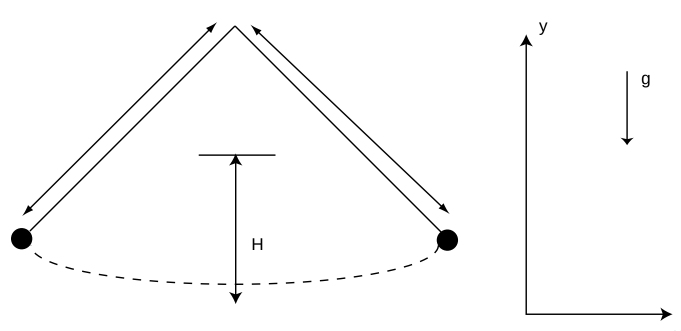
3. At its maximum height, a swinging pendulum makes an angle α with the vertical. If the pendulum consists of a massless cord of length ℓ and a point mass m, what is the magnitude of the tension in the cord and the magnitude and direction of the net force on the mass when it is at its maximum height? Express your answers in terms of m, ℓ, α and g. Key 57



4. A 100 kg mass is hung from the end of a steel bar which is 2 m long and has a cross sectional area of .1 m^2. How much does the bar stretch? (Young's modulus for steel is $20 \times 10^{10}\ \text{N/m}^2$.) Key 34

Problem XI: Sample Exam Problems

Two simple pendula each of the same length ℓ, but different masses, $m_1 > m_2$, are attached to the same point. They are raised a distance H above their lowest positions and simultaneously released. All motions take place in a vertical plane. After the pendula collide, they stick together.



1. Where does the collision occur? Key 11

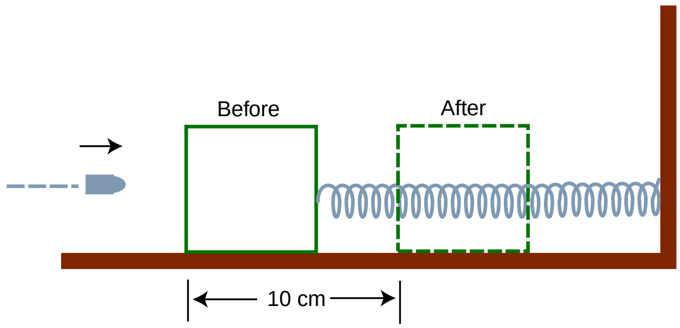
2. What is the velocity (magnitude and direction) of each mass just before the collision? Key 63

3. What is the velocity (magnitude and direction) of the combined system just after the collision? Key 40

4. What is the maximum height, h, above the lowest point reached by the combined system? Key 17

Problem XII: More Sample Exam Problems

A rifle bullet of mass 10 g strikes and embeds itself in a block of mass 90 g which rests on a horizontal frictionless table and is attached to a spring, as shown in the figure. The impact compresses the spring a maximum distance of 10 cm. Calibration of the spring shows that a force of 1 N is required to compress the spring 1 cm.



1. Find the maximum potential energy energy of the spring. Key 69

2. Find the velocity of the block just after the impact. Key 46

3. What was the initial velocity of the bulllet? Key 23

Helping Questions

1. Draw a free body diagram of the stop light. Key 29
   What is the tension in the cables? Key 6
   Retry Problem I.1 and if you still get the wrong answer.

2. What is the stress (σ)in the cables? Key 58

3. What is the strain ϵ in the cables? Key 12

4. What is the shear stress (σ_s) in the aluminum bar? Key 41
   How is the shear strain (ϵ_s) defined? Key 18
   Retry Problem III.1. If you need more help answer the next question.

5. Define the shear modulus (G). Key 70

6. What is the torque (Γ) about the axis of the cylinder? Key 24
   If you are still trouble, go on to the next question.

7. Express the angle (α) that the top surface moves in terms of the height of the cylinder (h) and the torque (Γ). Key 1

8. How are the velocity and acceleration related to the position as functions of time? Key 30
   If you still get this wrong, try the next Helping Question.

9. What is the time derivative of $\cos(\Omega t + S)$, of $\sin(\Omega t + S)$? Key 7

10. What is the kinetic energy for a mass moving with velocity V? Key 13

11. Write down the equation relating the force to the acceleration. Key 42

12. What is the potential energy for a spring of force constant k which is displaced a distance x from equilibrium? Key 19

13. What is the displacement of the mass at $t = 0$ (the time of impact)? Key 48

14. What is the displacement of the mass at $t = 0$? Key 54

15. What is the velocity at $t = 0$? Key 31
    If wrong, try the next Helping Question. Otherwise retry Problem V.1.

16. Is momentum conserved when the bullet strikes the block? Key 8

17. What is the potential energy when the mass is at its maximum displacement? Key 60

18. What is the kinetic energy when the mass is at its maximum displacement? Key 37

19. What is the kinetic energy of the bullet before the collision? Key 66

20. What is the total energy stored in the oscillator? Key 43

21. Write down the equation which describes the condition of static equililbrium at the point B. Key 72

22. If the amplitude and angular frequency are known, what is the maximum velocity? Key 3

23. What is the total energy of the system at the point of maximum extension of the spring? Key 9
    Using conservation of energy find the maximum extension $x_{\max}$. Key 61

24. What force acts on the mass when it is displaced to the left by a distance x? Key 15
    If you still don't follow, try the next question.

25. What "effective spring constant" does the mass feel in its simple harmonic oscillations? Key 67

26. What is the period of the oscillations, in terms of the spring constant k and the mass m? Key 73

27. What is the change in total energy during the course of the motion? Key 27

28. What is the potential energy of the pendulum in terms of the length ℓ and the angle θ, for any θ? Key 33
    Now use conservation of energy.

29. Does the total energy E change if the sign of V changes? Key 16

30. When the pendulum is at an arbitrary point on its swing, what is the expression for the total force in the radial direction (centripetal force), in terms of m, g, θ, ℓ and T, the tension? Key 45

31. Solve for T. At what angle is T at a maximum? (Remember that v varies with θ.) Key 22

32. Now determine v at this angle from energy considerations. Key 74

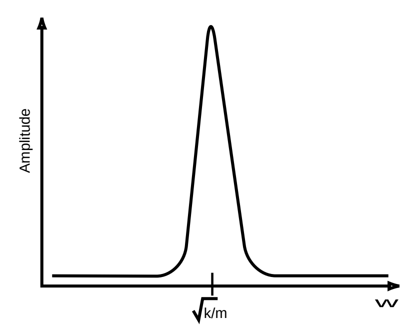
Answer Key

1. $\alpha = \dfrac{h\Gamma}{GI_p}$, where G is the shear modulus and I_p is the polar moment of inertia.

2. $mV_0/\sqrt{K(m+M)}$

3. $V_{\max} = A\omega$ where A is the amplitude and ω is the angular frequency.

4. For either very low or very high frequencies of the driving force, little energy will be transferred into oscillations of the block and amplitudes will be small. At driving frequencies very close to the characteristic frequency of the system ($\sqrt{k/m}$), however, the amplitude of the resulting vibrations will be very large.



5. The first is true, the second and third are false.

6. $T = \dfrac{mg}{2\sin\theta}$

7. $-\Omega\sin(\Omega t + S)$; $\Omega\cos(\Omega t + S)$

8. Yes

9. $E = \frac{1}{2}kx_{\max}^2 - mgx_{max}$, if $x = 0$ at point A

10. $T = 2\pi\sqrt{\dfrac{\ell}{g}} = 2.0\,\mathrm{s}$

11. At the bottom

12. $\varepsilon = \dfrac{\Delta\ell}{\ell} = \sigma/E$, where E is the Young's modulus.

13. (K.E.) $= \frac{1}{2}MV^2$

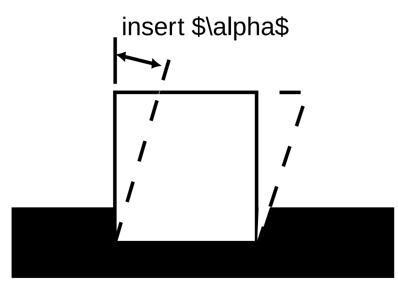
14. $m/(M+m)$; heat and deformation

15. $F = -kx + (-kx) = -2kx$

16. No

17. $h = \left(\frac{(m1-m2)}{(m1+m2)}\right)^2 H$

18. (shear strain) $= \epsilon_s \equiv \tan\alpha$



19. (P.E.) $= \frac{1}{2}kx^2$

20. $\Delta x = \dfrac{mg}{k} = 0.4\,\mathrm{m}$

21. $6\frac{1}{4}$ oscillations $\cdot\, T$ sec/oscillation $= 19.6\,\mathrm{s}$

22. $\theta = 0^\circ$

23. $v = \dfrac{m_{\mathrm{tot}}v_0}{m_{\mathrm{bullet}}} = 100\,\mathrm{m/s}$

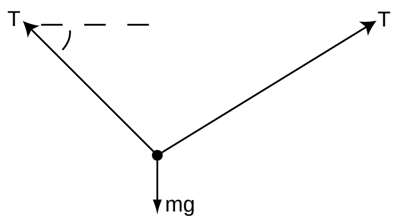
24. $\Gamma = F_2L = (2500\,\mathrm{N})(.5\,\mathrm{m}) = 1250\,\mathrm{N\,m}$

25. (Total energy) = (K.E.) + (P.E.) $= \pi^2/2\,\mathrm{J}$ is independent of time.

26. $v = \sqrt{\dfrac{-k}{m}(\Delta x)^2 + 2g\Delta x} = 2.0\,\mathrm{m/s}$

27. $\Delta E = 0 - \frac{1}{2}k(.2\,\text{m})^2 = -0.16\,\text{J}$

28. $(\text{P.E.}) = W = \frac{1}{2}kx^2 = 10\,\text{J}$



29.

30. $v = \dot{x} = \dfrac{dx}{dt}$; $a = \ddot{x} = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt}$

31. $V(0) = \dfrac{mV_0}{m+M}$

32. $(\text{amplitude}) = \dfrac{mg}{k} = 0.4\,\text{m}$

33. $(\text{P.E.}) = mg(\ell - \ell\cos\theta)$

34. $\Delta L = \dfrac{L \cdot Mg}{AE} = 9.8 \times 10^{-8}\,\text{m}$

35. $mg = 2A\sin\theta\sigma_t = 6.8 \times 10^3\,\text{N}$

36. $(\text{K.E.}) = \pi^2/2\sin^2\left[\dfrac{\pi}{2}t + \dfrac{\pi}{4}\right]\,\text{J}$

37. $(\text{K.E.}) = 0$

38. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{m}}$

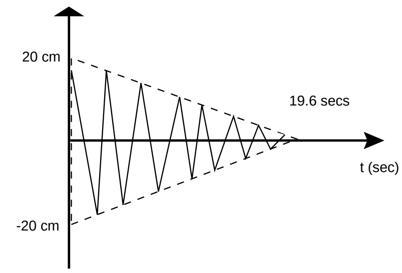
39. They don't change.

40. $\vec{v}_f = \dfrac{(m1 - m2}{m1 + m2)}\sqrt{2gH}\hat{x}$

41. $\sigma_s = F/A = \frac{5000\,\text{N}}{4\times10^{-4}\,\text{m}^2} = 1.25 \times 10^7\,\text{N/m}^2$

42. $F = -kx = Ma = +M\left(-\dfrac{\pi^2}{2}\cos[(\pi/2)t + \pi/4]\right)$

43. $E = \frac{1}{2}kA^2$, where A is the maximum displacement of the oscillator.



44.

45. $T - mg\cos\theta = \dfrac{mv^2}{\ell}$

46. $v_0 = \left(\frac{k}{m_{\text{tot}}}\right)^{1/2} x_{\text{max}} = 1\,\text{m/s}$

47. $\alpha = \dfrac{hF_2 \cdot L}{G\pi R^4/2} = .011\,\text{rad}$

48. $x(0) = 0$

49. $mg/k = 0.4\,\text{m}$

50. $\bar{P} = \frac{\Delta E}{\Delta t} = 8.2 \times 10^{-3}\,\text{watt}$

51. $\Delta\ell_{\text{max}} = \dfrac{\ell T_{\text{max}}}{A \cdot E_{\text{steel}}} = 1.1 \times 10^{-3}\,\text{m}$

52. $\dfrac{\Delta\ell}{\ell} = \dfrac{mg}{2AE\sin\theta} = \dfrac{(80\,\text{kg})(9.8\,\text{m/s}^2)}{2(.2\times10^{-4}\,\text{m}^2)(2\times10^{11}\,\text{N/m}^2)\sin 20^\circ}$
2.9×10^{-4}

53. $x(2) = 2.0\cos[\pi + \pi/4]\,\text{m} = -1.4\,\text{m}$
$v(2) = -2.0\sin[\pi + \pi/4](\pi/2)\,\text{m/s} = 2.2\,\text{m/s}$
$a(2) = -\frac{\pi^2}{2}\cos[2(\pi/2) + \pi/4] = 3.5\,\text{m/s}^2$

54. $x(0) = 0$

55. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = 0.79\,\text{Hz}$

56. $\theta_{\text{max}} = \cos^{-1}\left(\cos\theta_0 - \dfrac{v_0^2}{2gl}\right) = 10^\circ$

57. $T = mg\cos\alpha$; $F_{\text{net}} = mg\sin\alpha$ perpendicular to the string

58. $\sigma = T/A$, where T is the tension in the cables and A is the cross sectional area.

59. $f = \dfrac{\omega}{2\pi} = 1/4\,\mathrm{Hz}$; $T = l/f = 4\,\mathrm{s}$

60. $(\text{P.E.}) = \frac{1}{2}kx_{\max}^2$

61. $x_{\max} = \dfrac{2mg}{k}$

62. $v = \left(v_0^2 + 2g\ell(1-\cos\theta_0)\right)^{1/2} = .57\,\mathrm{m/s}$

63. $\vec{v}_2 = -\sqrt{2gH}\hat{x}$; $\vec{v}_1 = \sqrt{2gH}\hat{x}$

64. $\delta = h\dfrac{F}{A}\dfrac{l}{G} = 1.04 \times 10^{-3}\,\mathrm{m}$

65. $k = \pi^2/4\,\mathrm{N/m}$

66. $(\text{K.E.}) = \frac{1}{2}mv_0^2$

67. $K_{\text{eff}} = 2k$

68. $T_{\max} = \dfrac{mv^2}{\ell} + mg = 1.1 \times 10^3\,\mathrm{N}$

69. $\frac{1}{2}kx_{\max}^2 = .5\,\mathrm{J}$

70. $G \equiv \dfrac{\sigma_s}{\epsilon_s}$

71. $(\text{P.E.}) = \dfrac{\pi^2}{2}\cos^2\left[\dfrac{\pi}{2}t + \dfrac{\pi}{4}\right]\,\mathrm{J}$

72. $-k\Delta x + mg = 0$

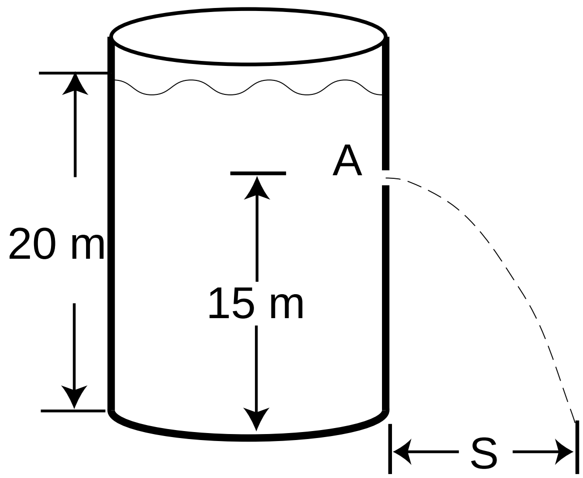
73. $T = 2\pi\sqrt{\dfrac{m}{k}} = 3.14\,\mathrm{s}$

74. $v = \sqrt{2gh} = 1.7\,\mathrm{m/s}$

Learning Guide 8

Problem I: Bernoulli's Equation

A water tank is filled to a height of $h = 20\,\mathrm{m}$ and is open to the air at the top.

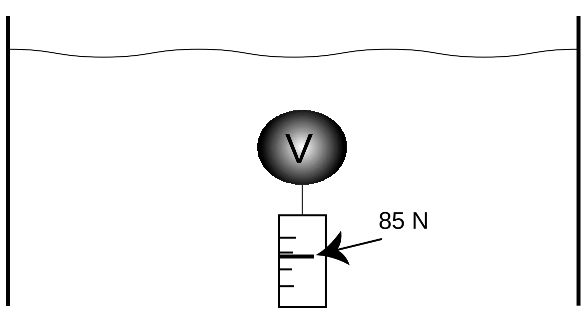


1. What is the pressure at point A ($y = 15\,\mathrm{m}$ above the bottom of the tank)? Key 19
   (If you have difficulty, see Helping Questions 1 and 2.)

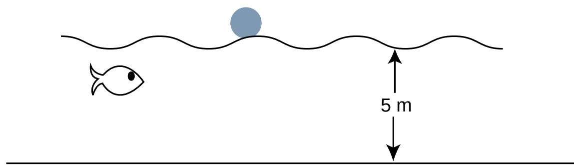
2. Suppose a small hole is punched in the side of the tank near point A. What is the velocity of the stream of water emerging from the tank? Hint: the pressure of the water in the stream is 1 atmosphere. Key 76
   (If you have difficulty, see Helping Question 3.)

3. At what distance y from the edge of the tank does the stream of water hit the ground? Key 35
   (If you have difficulty, see Helping Question 4.)

Problem II: Archimedes' Principle

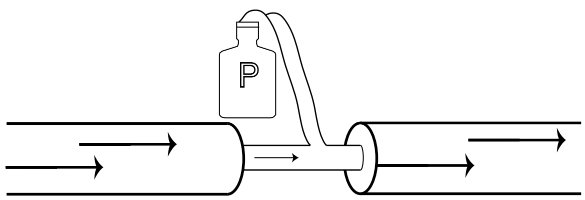


1. A hollow steel ball weighing 5 kg is submerged in a swimming pool. It requires 85 N to keep it submerged. What is the volume of the ball? Key 73
   (If you have trouble, see Helping Question 5 and 6.)

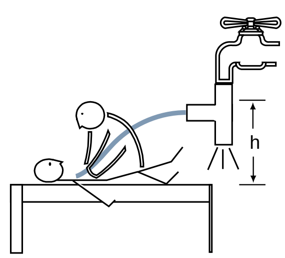


2. A hard rubber ball of radius 0.1 m is dropped at the surface of a pond $h = 5$ m deep. The ball takes $t = 2.5$ s to hit the bottom. If you are told that the ball always sinks so slowly that friction can be neglected (this is true), find the specific gravity of hard rubber. Key 51
   (See Helping Question 14 and 15 7, 8, 9 and 10 for hints.)

Problem III



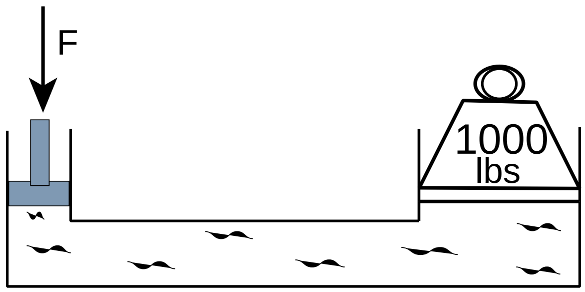
1. The apparatus shown above can be used to obtain a partial vacuum (i.e. pressure less than atmospheric). Suppose water is flowing in a pipe with cross sectional area $6.0 \times 10^{-4}\,\text{m}^2$ at a velocity of 1 m/s. The pressure in the pipe is 1.1 atmospheres. There is a short restriction in the pipe with a cross sectional area of $1.0 \times 10^{4}\,\text{m}^2$. Suppose a small hose connects the restriction to a sealed jar at the same height. What will be the pressure in the jar? Key 67
   (If you have trouble, see Helping Question 11 and 12.)



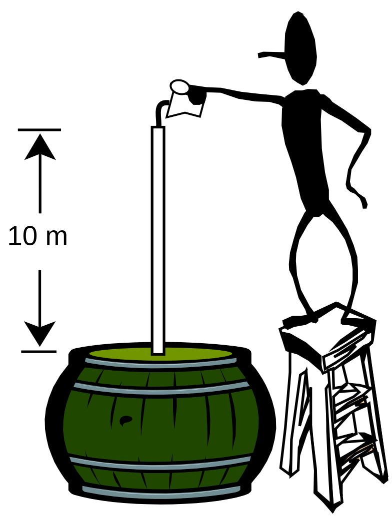
2. In emergency situations doctors have used this method for suctioning patients. A small hose is connected to the side of a water faucet. Because of the low pressure in the faucet, fluid from the patient(at atmospheric pressure, in an open wound, for example) will be sucker up

the hose and out the faucet. Assuming the pressure in the faucet is the same as calculated in part A, how far, h, can the patient lie below the faucet if the device is still to work? Key 45
(If you have trouble, see Helping Questions 13 and 14.)

Problem IV



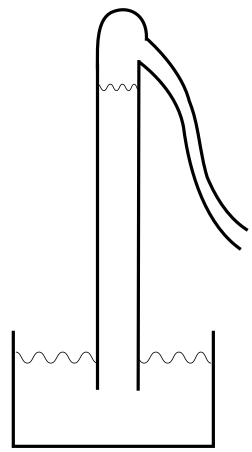
1. Consider the hydraulic jack in the figure above. If the area of the plunger is $10^{-3}\,\mathrm{m}^2$ and the area under the weight is $5 \times 10^{-2}\,\mathrm{m}^2$, how much force must be applied to the plunger to support the weight? Key 23
(If you have trouble, go to Helping Question 15.)



2. A Physics 101 student fills a barrel by pouring water into the top of a 10 m pipe connected tightly to the top of the barrel. Suppose the student fills the barrel and the pipe up to the top. If the barrel has a total area of $3\,\mathrm{m}^2$, what is the total outward force on its surface (top, bottom and sides)? Neglect the height of the barrel, and express your answer in Newtons and pounds. Key 61
(If you have trouble see Helping Question 16 and 17.)

Problem V: Water Barometer

The diagram to the right shows a long tube containing water. The bottom of the tube is beneath the surface of a water tank. Suppose we evacuate the space above the water in the tube.



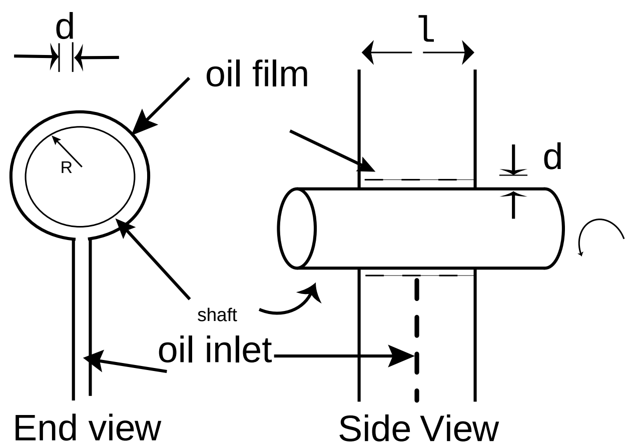
1. If the outside pressure is one atmosphere, how high will the water rise in the tube? Key 39
2. Suppose the water is replaced with mercury (specific gravity of 13.6). How high will the column of mercury be? Also express your answer in inches. Key 58
3. Weathermen use the units "inches of mercury" to indicate barometric pressure. If a warm front moves in, barometric pressure will drop by "one inch of mercury." How far will the water column in part A drop? Key 77

Problem VI: Typical Exam-Quiz Questions

1. Ignoring the viscosity of water, find the acceleration of a ball of plastic of volume $0.1\,\mathrm{m}^3$, when the ball is released $1\,\mathrm{m}$ below the surface. The plastic density is $1.25 \times 10^3\,\mathrm{kg/m^3}$, and the density of water is $1.0 \times 10^3\,\mathrm{kg/m^3}$. Key 17
2. A pipe carrying water at a pressure $p = 2 \times 10^5\,\mathrm{N/m^2}$ and a velocity of $1\,\mathrm{m/s}$ springs a small leak in the top. How high does the water spurt? (Neglect the width of the pipe.) Key 36
3. One gram of ice at $0°\mathrm{C}$ is melted to form $1\,\mathrm{g}$ of water at $0°\mathrm{C}$. If the latent heat of fusion of water is $80\,\mathrm{kcal/kg}$, what is the entropy change of the water? Key 55

Problem VII: Gas Guzzling Bearings

The main bearings in an automobile engine consist of a short section of the crankshaft rotating on a thin film of oil. An oil pump keeps pumping oil into the bearing to make up for the oil which leaks out the ends of the bearing. (The principle of operation is similar to the air tracks you've used in the labs.) Typical dimensions of a main bearing are length $\ell = 3.0\,\text{cm}$, radius of crankshaft $R = 2.0\,\text{cm}$ and oil film thickness $d = 5 \times 10^{-4}\,\text{cm}$. For a good grade of oil the viscosity at $20°\text{C} = 0.1\,\text{Pas}$ and the viscosity at $100°\text{C} = 0.01\,\text{Pas}$.



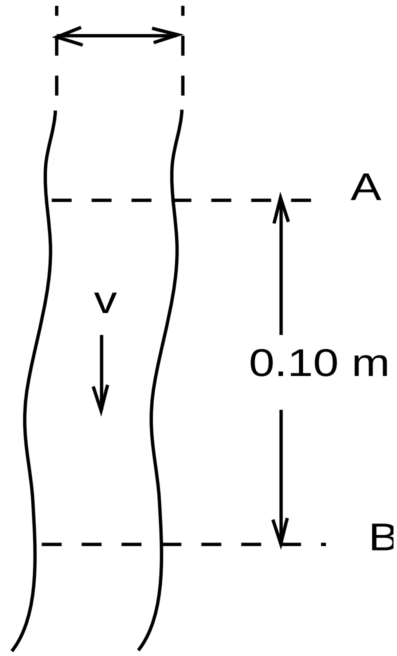
1. Calculate the drag force acting on a crankshaft rotation at 50 revolutions per second with cold (20°C) oil acting as a lubricant. Key 74
   (If you have difficulty, see Helping Questions 18 and 19.)

2. What is the drag force if the oil is hot (100°C)? Key 52

3. How much power is lost to friction in parts A and B above? Key 71
   (If you have trouble, try Helping Question 20.)
   Note: From these numbers you can see one reason why gas mileage is poor for winter and/or short distance driving – a cold engine wastes energy because the oil has a high viscosity.

Problem VIII: Pumping Blood Downhill

Consider an artery of diameter $d = 3.0\,\text{mm}$. The blood is at 37°C and exhibits a laminar flow pattern in which the average speed of the speed blood is $0.3\,\text{m/s}$.



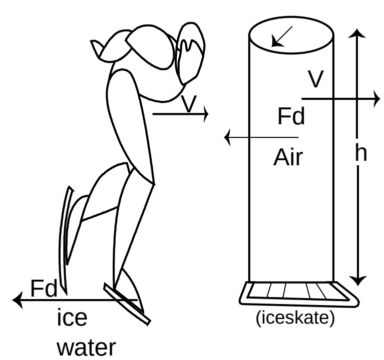
1. What is the maximum speed of the blood in the artery? Key 30

2. What is the flow of the blood? Key 49

3. The artery is vertical and the blood is being pumped downward. What is the pressure difference between points A and B if A and B are 0.1000 m apart, i.e. what is $P_B - P_A$? (The density of blood is $1.0595 \times 10^3\,\mathrm{kg/m^3}$.) Key 68
(If you have difficulty, See Helping Questions 21 and 22.)

Problem IX: Drag on a Skater

In skating one actually melts a thin layer of ice and the skate travels on a film of water.



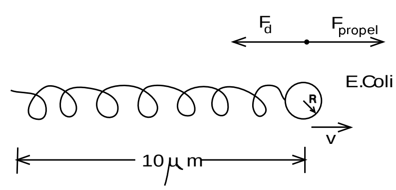
1. Assuming that the film of water is at 0°C and 1.0 μm (i.e. 10^{-6} m) thick, and that each skate blade is 3.0 mm wide and 25 cm long, what is drag force on the skates for a skater coasting at 10 m/s? Key 46

2. Approximate the skater as an upright cylinder of height $h = 1.5$ m and radius $r = 0.20$ m. Calculate the Reynolds number for the skater if the air temperature is 0°C and the air density is $\rho_0 = 1.2\,\text{kg/m}^3$. You may use the formula for sphere of radius r. Key 65

3. What is the air drag force on the skater? Key 5
   (If you're stuck, See Helping Questions 23 and 24.)

Problem X: Drag at Low Reynolds Number

The movement of microscopic creatures through water is typically characterized by a low Reynolds number. For example, the bacterium Escherichia coli can use its flagellum to propel itself through water at a speed of 30 μm/s.



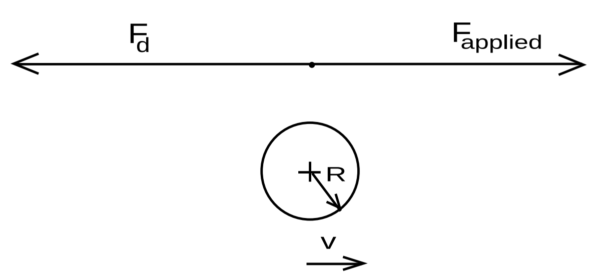
1. If one considers a bacterium to a be sphere of radius $R = 1.0\,\mu$m, what is the Reynolds number appropriate to its motion? (Assume that the water is at 20°C.) Key 62

2. What is the drag force on the bacterium? Key 2
   (Optional – Requires Calculus) Suppose the bacterium stops rotation its tail. How far would the bacterium coast before the drag of the water stopped its motion? Assume the density of the bacterium is the same as the density of water. Key 21
   (If you cannot start, see Helping Questions 25 and 26.)

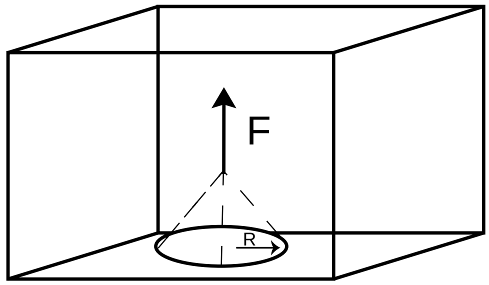
3. Consider now a spherical diving bell of radius $R = 2.0$ m.

4. How fast would the diving bell be moving in order to have the same Reynolds number as the bacterium in part 1 above? Key 78

5. What is the drag force on the diving bell? Key 18



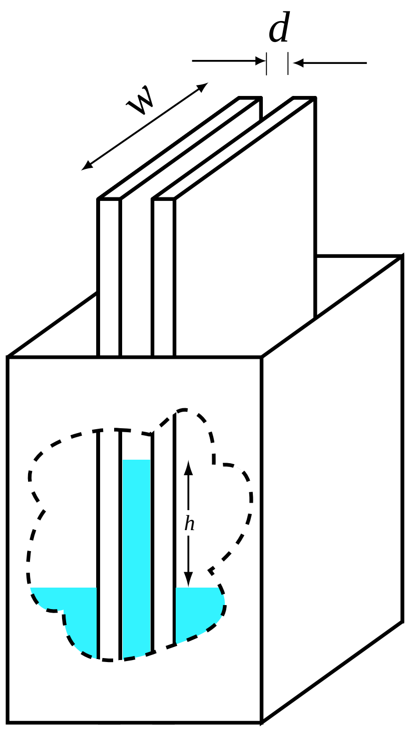
Problem XI: Sticky Water



A circular plastic hoop of radius $R = 5.0\,\text{cm}$ and mass $m = 2.0\,\text{g}$ is floating on water at 20°C. What force is required to lift the hoop free of the water? Key 37
(If you have trouble, see Helping Question 27.)

Problem XII: Capillarity

Consider two glass plates which are separated a distance $d = 0.20\,\text{mm}$ and which protrude 15 cm out of a trough of liquid. The plates have a width $w = 20\,\text{cm}$. The liquid has a surface tension $\Upsilon = 8.40 \times 10^{-2}\,\text{N/m}$, a density of $0.80 \times 10^3\,\text{kg/m}^3$ and a contact angle of 37° with glass.

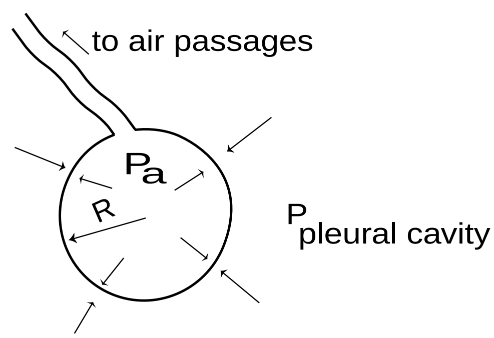


1. What is the height h which the liquid rises between the plates? Key 75
   (If you have a problem, see Helping Questions 28, 29, 30 and 31.)

2. What is the force pulling each plate in toward the other plate? Key 12

Problem XIII: Laplace's Law and the Lungs' Alveoli

Consider an alveolus in the midst of a breathing cycle when it can be approximated as a sphere of radius 0.10 mm.



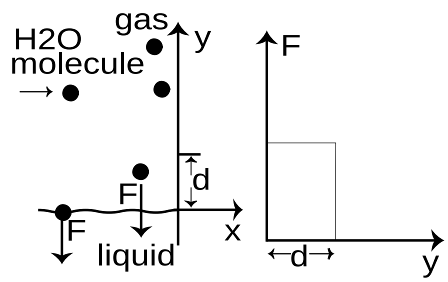
1. If the wall tension of the alveolus is $\Upsilon_w = 6.0 \times 10^{-2}$ N/m, what is the pressure difference across the wall of the alveolus? Key 31
   (See Helping Question 32 if you cannot begin.)

2. After exhaling, the chest muscles expand the pleural cavity, drawing air into the lungs. If the pressure in the pleural cavity is 5.3×10^2 Pa below air pressure and the pressure in the alveoli is 4.0×10^2 Pa below air pressure, what must the wall tension in the alveoli be if its radius is 0.050 mm? Key 69

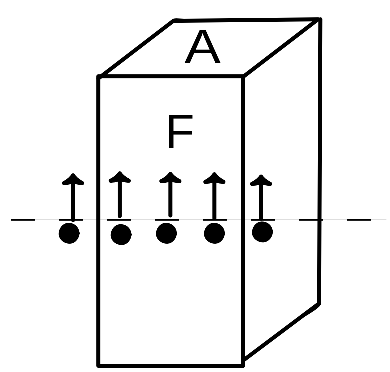
Problem XIV: Tensile Strength of Water

The tensile strength of a material is the force required to pull apart a column of that material divided by the area of the column, i.e. tensile strength is the maximum negative pressure a material can withstand. The tensile strength of water can be calculated from the following simple model.

1. Water is held together in a liquid form by the molecular attraction among the water molecules. How much energy is required to remove one water molecule from the liquid? Key 9
   (If you have no idea how to begin, see Helping Question 33.)



2. The force between water molecules only extends for a distance of the order of a water molecule's diameter, i.e. $d \simeq 3.1 \times 10^{-10}\,\mathrm{m}$. What is the average force which holds a water molecule on the liquid's surface from evaporating? Key 66
(See Helping Question 34 if you have trouble.)



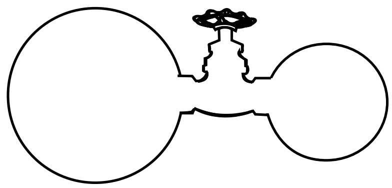
3. Consider a column of water of area $A = 1.0\,\mathrm{m}^2$ with an imaginary plane slicing through it. Each molecule on the surface below the plane is attracted to the water above the plane. What

is the total force holding the liquid below the plane to the liquid above the plane? Key 25
(If you are having trouble with conversions, See Helping Question 35)

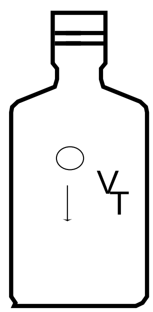
4. What is the tensile strength of water? Key 63

5. Could a column of water conceivably break at a smaller tensile strength? Key 3

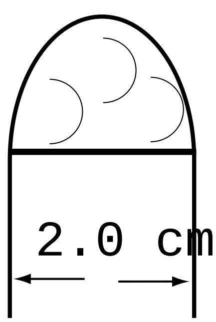
Problem XV: Quickies



1. Two soap bubbles made from the same solution are inflated to two different radii, and then connected to a closed valve as shown. When the stopcock is opened, will the larger bubble (i) increase in size; (ii) decrease in size; (iii) remain the same? Explain. Key 22



2. A 0.5 gram pearl of radius 2 mm is dropped in a bottle of Pert shampoo, $\eta = 9\,\mathrm{Pas}$. Assume the Reynolds Number $N_R < 1$, which it is. What terminal velocity is achieved by the pearl? Assume $\rho_{\mathrm{Pert}} = 2\,\mathrm{gm/cm}^3$. Key 41



3. A soap film can support a 0.5 g wire of length 2 cm in a U-loop configuration as shown. A Physics 101 student uses the film to blow a bubble of radius 5 cm. How much higher than atmospheric pressure is the pressure inside the bubble? Key 60

Helping Questions

1. What is the water pressure just under the surface at the top of the tank? Key 38
   (If you still have difficulty, see the next Helping Question.)
2. How does Bernoulli's equation relate the pressure at A to the pressure at the surface? Key 57
3. How does Bernoulli's equation relate the velocity of the stream to the pressure at point A? Key 16
4. Do you remember how to do projectile motion problems? Key 54
5. In terms of the volume, what is the buoyant force on the ball? Key 13
   (Still can't get it? Try Helping Question 13.)
6. What is the total force on the ball? Key 32
7. In terms of the volume V, what is the buoyant force on the ball? Key 70
8. In terms of the density of hard rubber, ρ_{HR}, and the ball volume V, what is the total force acting the ball? Key 10
   (If you still need help, try the next two Helping Questions.)
9. What is the acceleration of the ball? Key 29
10. What is the relation between time and distance for an object moving with constant acceleration?
    Key 48
11. What is the velocity of the water in the constriction? Key 7
12. Solve Bernoulli's equation for the pressure in the small pipe. Why is this the pressure in the jar? Key 26
13. What is the pressure difference between the top and bottom of a vertical pipe of length h full of fluid? Key 64
14. If the pressure at the top of the vertical pipe is maintained at the pressure found in A, what is the height of the water column that can be supported? Key 4
15. What is the fluid pressure required to support the weight? Key 42
16. How does Bernoulli's equation relate the pressure at the top of the pipe to the pressure on the barrel? Key 1
    (If you are still having trouble, see the following Helping Question.)
17. What is the pressure on the outside of the barrel pushing in? Key 20

18. What is the area of the shaft which is in contact with the bearing? Key 14

19. How fast is the shaft's surface moving? Key 33

20. What is the work done by the drag force in one revolution of the shaft? Key 11

21. If the blood were not moving, what would be the relationship between the pressure at A and the pressure at B? Key 8

22. What pressure differential would be necessary to maintain the blood flow rate if the artery were horizontal? Key 27

23. What is the drag coefficient on the skater? Key 24

24. What is the cross sectional area of the skater? Key 43

25. What is Newton's second law for the bacterium? Key 40

26. Now, multiply both sides of your equation by dt and integrate. Key 59

27. Draw a force diagram for the hoop. Key 56

28. Draw a force diagram for the liquid between the plates. Key 15

29. What is the condition for vertical static equilibrium of the sheet of liquid? Key 34

30. What is the surface tension force F_s? Key 53

31. What is the mass of liquid lifted into the region between the plates? Key 72

32. What is the relation between the radius of sphere and the pressure difference across the surface of that sphere? Key 50

33. When one boils water, how much energy is required to evaporate 1.0 kg of water? Key 28
    How many molecules are there in a kilogram of water? Key 47

34. How much work must be done to remove a water molecule from the liquid? Key 6

35. How may atoms are on a surface on $1\,\mathrm{m}^2$ of water? Key 44

Answer Key

1. $P_{\text{top}} + \rho gh = P_{\text{barrel}}$

2. $F_d = 6\pi\eta RV = 5.7 \times 10^{-13}\,\text{N}$

3. If a bubble forms it will grow by tearing water molecules away from each other. Since at any one time relatively few molecules are involved in increasing the tiny bubble's surface area, the force required to make the bubble grow is much smaller than F_t calculated with this model.

4. $h = \dfrac{\Delta p}{\rho g} = 0.71\,\text{m}$

5. $F_d = \dfrac{C_0 A \rho_0 v^2}{2} = 43\,\text{N}$

6. $W = F_d C_d$

7. $V_{\text{small}} = V_{\text{large}} \dfrac{A_{\text{large}}}{A_{\text{small}}} = 6\,\text{m/s}$

8. $P_A + \rho gh = P_B$

9. $\Delta E = \dfrac{Q}{N} = 6.7 \times 10^{-12}\,\text{J}$

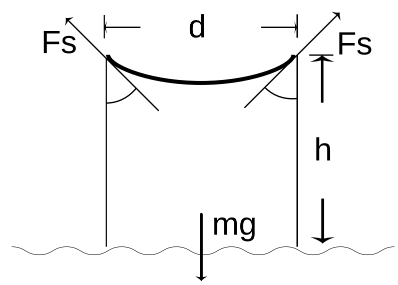
10. $F_{\text{tot}} = B - mg = (\rho_w - \rho_{\text{HR}})gV$

11. $W_d = F_d \cdot 2\pi R = 60\,\text{J}$ at $20°\text{C}$ or $6.0\,\text{J}$ at $100°\text{C}$

12. $F_{\text{inward}} = \gamma w \sin\theta = 1.0 \times 10^{-2}\,\text{N}$

13. $B = \rho_w V g$

14. $A = 2\pi R\ell = 3.8 \times 10^{-3}\,\text{m}^2$



15.

16. $P_{\text{atm}} + \rho gh = P_{\text{atm}} + \frac{1}{2}\rho v^2 + \rho gy$

17. $a = \dfrac{(\rho - \rho_W)g}{\rho} = 2.0\,\text{m/s}^2$

18. $F_d = 6\pi\eta RV = 5.7 \times 10^{-13}\,\text{N}$

19. $P_A = P_{\text{atm}} + \rho g(h - y) = 1.5 \times 10^5\,\text{N/m}^2$

20. $P_{\text{outside}} = 1\,\text{atm}$

21. $\Delta x = \dfrac{2\rho_0 R^2 v}{9\eta} = 6.7 \times 10^{-12}\,\text{m}$. This is much shorter than the bacterium's length and indicates that the force required to overcome the inertia of the bacterium is miniscule compared to the drag force. Could you swim normally at such a Reynolds number?

22. Increase! (Laplace's Law shoes that the smaller bubble has the larger internal pressure; therefore air will be forced through the valve to the other side until the larger one is completely inflated.)

23. $F_1 = A_1 \dfrac{F_2}{A_2} = 88\,\text{N}$

24. $C_D = 1.2$, from Fig. 15.8 of K&S

25. $F_t = N_s F = 2.3 \times 10^9\,\text{N}$

26. Jar and constriction are at the same height.

27. $\Delta P_f = \dfrac{8\eta h\bar{v}}{R^2} = 22.2\,\mathrm{N/m^2}$

28. $Q = Lm = L \cdot 1\,\mathrm{kg} = 2.3 \times 10^6\,\mathrm{J}$

29. $a = \dfrac{F_{\mathrm{tot}}}{\rho_{\mathrm{HR}} V} = \dfrac{(\rho_w - \rho\mathrm{HR})g}{\rho_{\mathrm{HR}}}$

30. $v_{\max} = 2\bar{v} = 6.00 \times 10^{-2}\,\mathrm{m/s}$

31. $\Delta P = 1.2 \times 10^3\,\mathrm{Pa} = 0.012\,\mathrm{atm}$

32. $F_{\mathrm{tot}} = B - mg$

33. $v = 2\pi Rf = 6.3\,\mathrm{m/s}$

34. $2F_s \cos\theta - mg = 0$

35. $s = v_0\sqrt{\dfrac{2h}{g}} = 17\,\mathrm{m}$

36. $h = \dfrac{1}{\rho g}\left[(p - p_{\mathrm{atm}}) + \dfrac{1}{2}\rho v^2\right] = 10\,\mathrm{m}$

37. $F = mg + 4\pi\gamma R = 6.5 \times 10^{-2}\,\mathrm{N}$

38. $1\,\mathrm{atm}$

39. $h = \dfrac{\Delta p}{\rho g} = 10\,\mathrm{m}$

40. $F_d = -6\pi\eta Rv = m\dfrac{dv}{dt}$, where $m = \rho_0\left(\frac{4}{3}\pi R^3\right)$

41. $v = \dfrac{mg}{6\pi\eta R} = 1.25\,\mathrm{cm/s}$

42. $p = \frac{1000\,\mathrm{lb}}{A} = 8.8 \times 10^4\,\mathrm{N/m^2}$

43. $A = (\text{height}) \cdot (\text{width}) = (1.5\,\mathrm{m})(0.40\,\mathrm{m})$

44. $N_s = \dfrac{1.0\,\mathrm{m^2}}{(\text{area of one atom})} = \dfrac{1.0\,\mathrm{m^2}}{(3.1 \times 10^{-10})^2}$

45. $h = \dfrac{\Delta P}{\rho g}$

46. $Fd = \eta A\dfrac{\Delta v}{\Delta y} = 13\,\mathrm{N}$ on each skate

47. No. (Number of molecules) $= \dfrac{6 \times 10^{23}\ \text{molecules}}{18 \times 10^{-3}\,\mathrm{kg}} \cdot 1\,\mathrm{kg} = 3.3 \times 10^25$ molecules.

48. $y = y_0 + \frac{1}{2}a_y t^2$ $(= h + \frac{1}{2}at^2$ here)

49. $Q = \bar{v}A = 2.12 \times 10^{-7}\,\mathrm{m^3/s}$

50. $\Delta P = \dfrac{2\gamma_w}{R}$

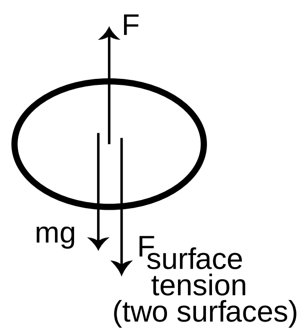
51. $\rho_{\mathrm{HR}} = \left(1 - \dfrac{2h}{gt^2}\right)^{-1}\rho_W = 1.2\rho_W$

52. F_d at $100°\mathrm{C} = \dfrac{\eta_{100°}}{\eta_{20°}}$ (F_d at $20°\mathrm{C} = 47\,\mathrm{N}$)

53. $F_s = \gamma w$

54. If not, review section 2.5

55. $\delta s = \dfrac{L \cdot M}{T} = 3.0 \times 10^{-4}\,\mathrm{kcal/K}$

56. 

57. $P_A + \rho gyk = P_{\mathrm{atm}} + \rho gh$

58. $h = \dfrac{\Delta p}{\rho g} = 760\,\mathrm{mm\,Hg} = 30\,\mathrm{in}$

59. $-\displaystyle\int_{x=x_i}^{x=x_f} 6\pi\eta R\frac{dx}{dt}dt = \int_{v=v}^{v=0} m\,dv$; $6\pi\eta R\Delta x = mv$

60. $\Delta p = \dfrac{4mg}{2\ell r} = 9.8\,\mathrm{Pa}$

61. $F = P_{\text{tot}} \cdot A = (2.0 \times 10^5\,\text{N/m}^2 - 1.0 \times 10^5\,\text{N/m}^2) \cdot 3\,\text{m}^2 = 3.0 \times 10^5\,\text{N} = 6.8 \times 10^4\,\text{lb}$

62. $N_R = \dfrac{\rho_0 V R}{\eta} = 3.0 \times 10^{-5}$

63. $P_{\text{max}} = \dfrac{F_t}{A} = 2.3 \times 10^9\,\text{Pa} = 2.2 \times 10^4\,\text{atm}$

64. $\delta p = \rho g h$

65. $N_R = \dfrac{\rho_0 V R}{\eta} = 1.4 \times 10^5$

66. $\bar{F} = \Delta E/d = 2.2 \times 10^{-10}\,\text{N}$

67. $P_{\text{small}} = P_{\text{large}} + \frac{1}{2}\rho(V_{\text{large}}^2 - V_{\text{small}}^2) = 9.3 \times 10^4\,\text{N/m}^2$

68. $P_B - P_A = \rho g h - \Delta P_f = 1016\,\text{N/m}^2$

69. $\gamma_w' = 3.25 \times 10^{-3}\,\text{N/m}$

70. $B = \rho_w V g$

71. $P_{20^\circ C} = \dfrac{w_d}{t} = 3.0 \times 10^1 0\,\text{erg/c} = 3.0\,\text{kW} = 4.0\,\text{hp}$; $P_{100^\circ\text{C}} = \dfrac{w_d}{t} = 3.0 \times 10^9\,\text{erg/c} = 300\,\text{W} = 0.40\,\text{hp}$

72. $m = \rho d w h$

73. $V = \dfrac{F + mg}{\rho_w g} = 1.4 \times 10^{-2}\,\text{m}^3$

74. $F_d = \dfrac{\eta A v}{d} = 470\,\text{N}$

75. $h = \dfrac{2\gamma \cos\theta}{\rho d g} = 8.6\,\text{cm}$

76. $V = \sqrt{2g(h-y)} = 9.9\,\text{m/s}$

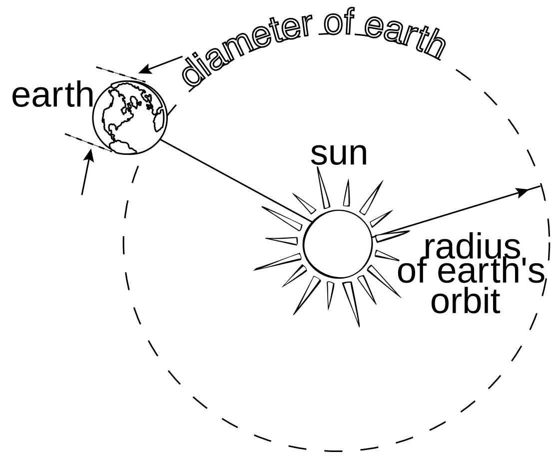
77. $\dfrac{\Delta h}{h} = \dfrac{\Delta p}{p}$, so $\Delta h = (10\,\text{m})\dfrac{1}{30} = .33\,\text{m}$

78. $v = \dfrac{\eta N_R}{\rho_0 R} = 1.5 \times 10^{-11}\,\text{m/s}$

Learning Guide 9

Problem I

Radius of Sun	6.95×10^8 kg
Radius of Earth	6.4×10^6 m
Radius of Earth's orbit	1.5×10^{11} m

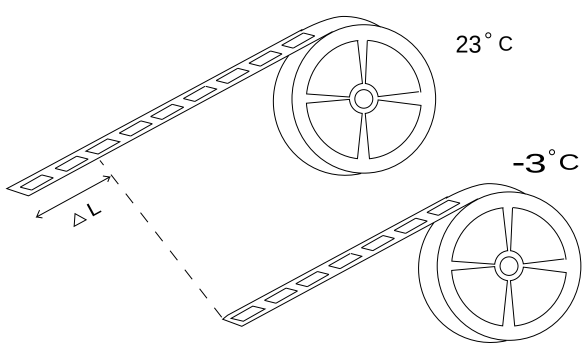


1. The temperature of the sun is $6,000$ K. Assuming that is has an emissivity coefficient of unity, what is the total power radiated by the sun? ($\alpha = 1.1 \times 10^{-4}{}^\circ C^{-1}$) Key 16
   (Trouble? See Helping Question 1.)

2. How much of this power does the earth receive? Key 48
   (Confused? See Helping Questions 2, 3 and 4.)

3. If the emissivity of the earth is also unity, what is the equilibrium temperature of the earth? Key 43

Problem II

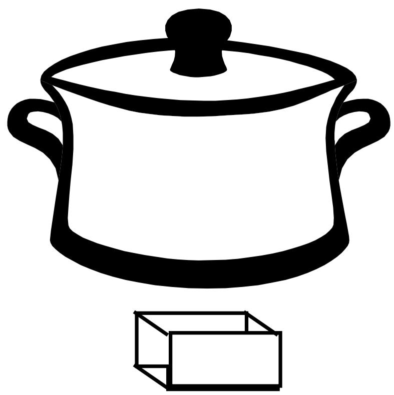


1. A movie film made of celluloid has a length of 1.0×10^3 feet on a cold day, when the temperature is 3°C. How many feet longer is the film when the temperature is 23°C? $\alpha = 1.1 \times 10^{-4}$ Key 59

2. If the film expands isotropically, what will be the percent change in its volume? Key 6
(Trouble? See Helping Question 5.)

Problem III

A 10 kg block of ice at −10°C is heated.



1. How much energy is required to bring this block to the freezing point, 0°C? Key 38
(If you have trouble, see Helping Question 6.)

2. How much to melt the block at 0°C? Key 1
(For a reminder, see Helping Question 7.)

3. How much to raise the temperature of the water to the boiling point? Key 33

4. How much to boil all of the water? Key 49

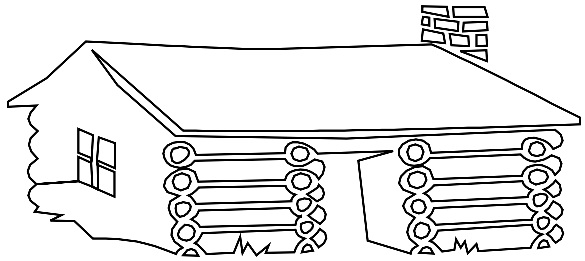
5. How much to heat the resulting steam to 200°C? Key 65

6. What is the total energy $-10°\text{C} \rightarrow 200°\text{C}$? Key 12

7. How high would you have had to lift the original block in order to give it this much potential energy? Key 28

8. If a snowflake leaves a cloud at 0°C on a day when the temperature is 0°C, what is the maximum height the cloud can have for the flake to avoid completely melting before it hits the ground? (Assume the flake achieves terminal velocity immediately, and that one-half of the potential energy lost by falling goes to heating the flake, and one-half to heating the surrounding air.) Key 44

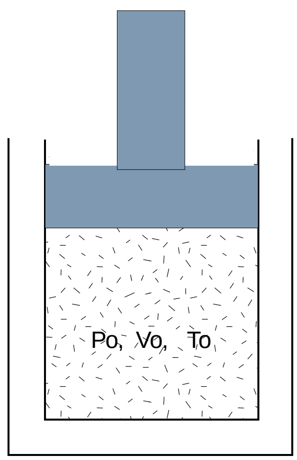
Problem IV



A log cabin has walls and roof about 20 cm thick on the average, and a total surface area of $70\,\mathrm{m}^2$. If the outside temperature is $-20°$C and the inside is $+20°$C, how much heat is lost per second through the walls? ($\kappa_{\mathrm{wood}} = 0.15\,\mathrm{W/M\,°C}$) Key 60

Problem V

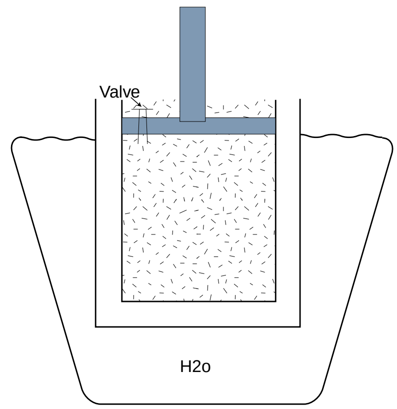
You are given N moles of a gas satisfying the ideal gas law having volume, pressure and temperature V_0, P_0 and T_0 respectively.



1. If the volume and temperature are both doubled, what will the new pressure be? Key 7
   (If you have trouble, see Helping Questions 8 and 9.)
2. If, instead, the temperature and pressure are both doubled, what will the new volume be? Key 55
3. If, instead, the volume and pressure are both doubled, what will the new temperature be? Key 2

Problem VI

An ideal gas is confined to a steel cylinder at 20°C and a pressure of 6 atmospheres.



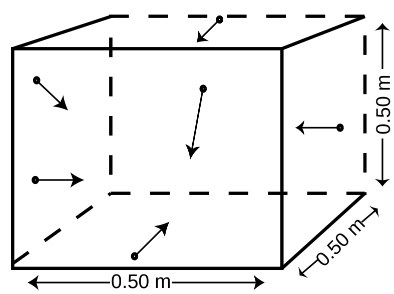
1. If the cylinder is surrounded by boiling water (at sea level) and allowed to come to thermal equilibrium, what will the new pressure be? Key 18
   (If you are having difficulty, review Problem I.)

2. While the gas is kept at 100°C a valve is opened and some of the gas is allowed to escape until the pressure is again 6 atm. What fraction of the original gas (by weight) will escape? Key 34
   (Refer to Helping Question 10 if you need assistance here.)

3. The temperature of the remaining gas in the cylinder is returned to 20°C; what will the final pressure be? Key 13

Problem VII

One mole of He gas at 27°C is contained in a cubical box with sides of length $\ell = .50\,\text{m}$.



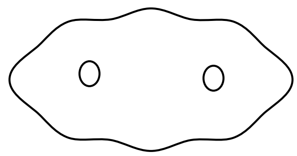
1. What is the pressure in the box? Key 29

2. What is the pressure due to a single He atom? Key 45
   (If you have trouble, see Helping Questions 11 and 12.)

3. What is the average kinetic energy of a single He atom? Key 24 (See Helping Question 13 if you need help.)

Problem VIII

A typical hemoglobin molecule diffuses 1.0 cm in one week in blood.

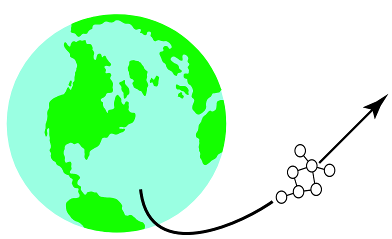


1. What is the diffusion coefficient, D, for hemoglobin in blood. Key 56
2. How long will it take, on the average, for a hemoglobin molecule to diffuse 10 cm? Key 3
3. Is diffusion the method by which hemoglobin is transported throughout the body? Key 19

Problem IX

Useful Information:

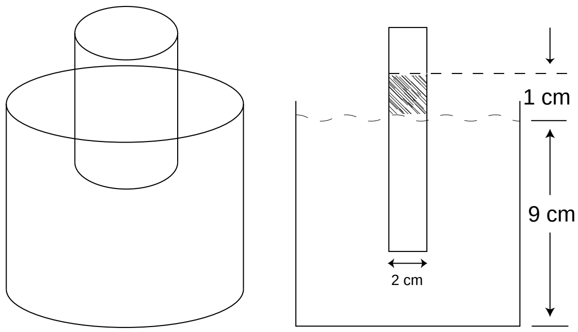
$$\text{Mass of moon} = 7.3 \times 10^{22}\,\text{kg}$$
$$\text{Radius of moon} = 1.7 \times 10^{6}\,\text{m}$$
$$\text{Mass of earth} = 6.0 \times 10^{24}\,\text{kg}$$
$$\text{Radius of Earth} = 6.4 \times 10^{6}\,\text{m}$$



1. Consider an ideal gas whose molecules have mass m. Find the temperature at which the root-mean-square speed is equal to the escape velocity for the earth. Key 35
   (If you still have trouble, see Helping Questions 14 and 15.)
2. Evaluate the result of part (1) for the following gases: hydrogen, oxygen, helium. Key 14
3. Evaluate the corresponding temperature for these same gases escaping from the moon's surface. Key 30
4. If are told that the temperature of the moon varies between −2000°F and 200°F, what would you say its chances were to hold onto an atmosphere over a long period? Key 46

Problem X

A cylindrical tube of radius 1 cm, made out of a material permeable to water but not sugar, is inserted 9 cm into a beaker of pure water. The temperature in the laboratory is 23°C. When an unknown amount of sugar is added to the tube, the column is observed to rise 1 cm.

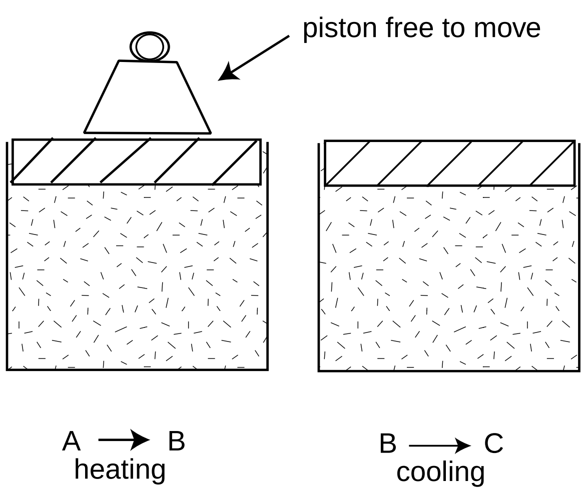


1. What is the osmotic pressure? Key 62
   (Trouble? See Helping Question 16.)

2. How many moles of sugar were added? Key 25
   (For a hint see Helping Question 17.)

Problem XI

Suppose 1 mole of an ideal gas is originally in a state $P_1 = 3\,\text{atm}$ and $V_1 = 1$ liter. It is then allowed to expand at constant pressure to a volume of $V_2 = 3$ liters. Then it is cooled at constant volume until its pressure is $P_2 = 2\,\text{atm}$.



1. Indicate this process on a P-V diagram. Key 57

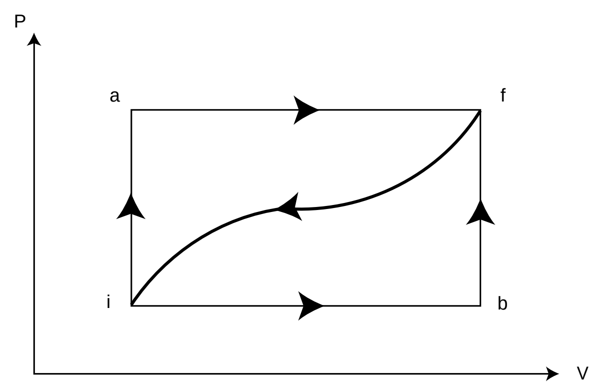
2. Calculate the work done by the gas against the piston. Key 4

3. If the initial internal energy is $U_i = 460\,\text{J}$ and the final internal energy is $U_f = 920\,\text{J}$, how much heat was added in the process? Key 20
   (If you are confused see Helping Question 18.)

4. What is the final temperature of the gas? Key 52
   (See Helping Question 19 for a hint.)

Problem XII

1. Consider the P-V diagram below:



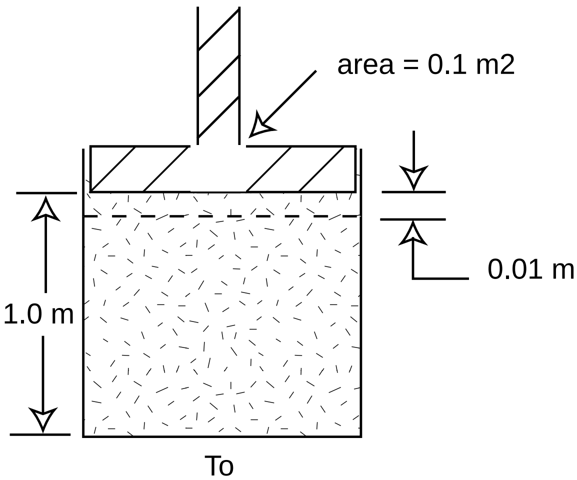
When the system is taken from state i to state f along path iaf, it is found that $Q = 50\,\text{cal}$ and $W = 20\,\text{cal}$. Along path ibf, $Q = 36\,\text{cal}$.

   (a) What is W along path ibf?
   (b) If $W = -13\,\text{cal}$ for the curved path fi, what is Q for this path?
   (c) Take $U_1 = 10\,\text{cal}$. What is U_f?
   (d) If $U_b = 22\,\text{cal}$, what is Q for the process ib? Key 15

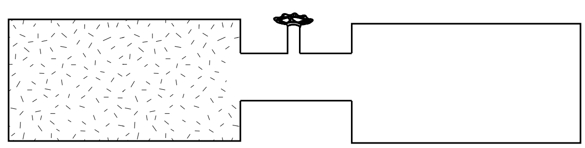
A thermos bottle contains coffee. The thermos bottle is shaken vigorously. Consider the coffee as a system.

   (a) Does its temperature rise?
   (b) Has heat been added to it?
   (c) Has work been done to it?
   (d) Has its internal energy changed? Key 31
   (Trouble? See Helping Question 20.)

Problem XIII: Second Law



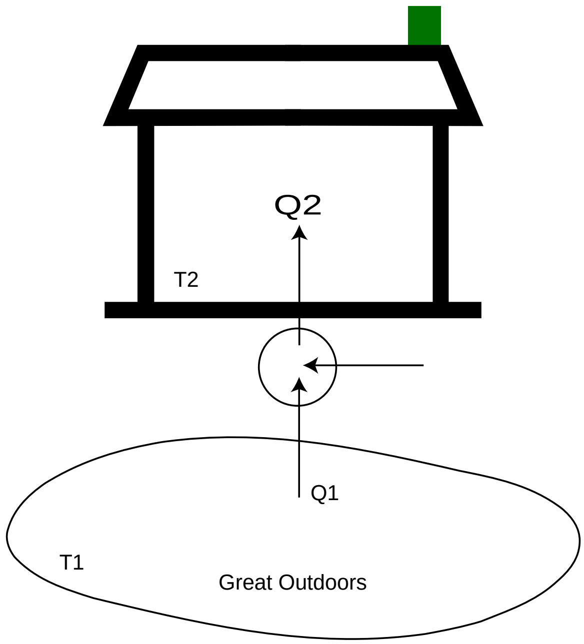
1. The walls of a cylinder are in contact with a heat reservoir of temperature T_0. If one mole of an ideal gas is in the cylinder and the piston is pushed in a distance $\Delta X = .01\,\mathrm{m}$, what is the entropy change of the gas? Key 63
(Hint: Assume the displacement is so small the pressure doesn't change.)
(If you have trouble, see Helping Questions 21 and 22.)



2. Suppose two insulated containers, one evacuated, one with one mole of ideal gas, are connected by a tube which is closed off by a valve. When the valve is opened does the entropy of the gas increase or decrease? What about its internal energy? Key 42
(Confused? See Helping Question 23.)

Problem XIV

Consider the Carnot cycle described in section ?. If we run the cycle i the opposite direction, the arrow in the figures change direction but the first law still holds, i.e. $W = Q_2 = Q_1$, where now W is work is applied to the system, Q_1 is the heat absorbed from the colder reservoir T_1, and Q_2 is heat delivered to the hotter reservoir T_2. (Note: $Q_1/Q_2 = T_1/T_2$ as before)



1. Suppose I want to heat my house (as shown above) with one of these heat pumps (backwards Carnot cycles). If I define heating effeciency as heat delivered to my house Q_2 divided by work done on the cycle W, i.e. $\epsilon = Q_2/W$, what is the efficiency in terms of the two temperatures? Key 5
(If you have trouble see Helping Question 24.)

2. Suppose I have a fuel oil tank with internal energy E_0 which I can burn and convert entirely into heat Q_0 to heat my house. Instead, I use the oil to run an engine at 15% efficiency, i.e. $W = .15E_0$, which I in turn use to run a heat pump. If $T_2 = 20°$C (room temperature) and $T_1 = 0°$C (a chilly day) how much heat Q_2 can I deliver to my house in terms of E_0? Key 37

3. Why don't we use heat pumps to help solve the energy crisis in this country? Key 53

Helping Questions

1. How are the power radiated, the emissivity coefficient, and the temperature related? Key 32
2. What is the area of an imaginary sphere of radius equal to the radius of the earth's orbit? Key 64
3. What is the cross sectional area of the earth? Key 11
4. What fraction of the imaginary sphere area does the earth block out, then? Key 27
Since the sun radiates isotropically, this is also the fraction of the sun's power the earth gets!
5. How is the coefficient of volume expansion β related to the coefficient of linear expansion α? Key 22
6. In terms of the specific heat capacity, α, what is the heat required for a temperature change Δt in a mass m. Key 54
7. How much heat is required to make a substance undergo a phase transition related? Key 17
8. State the ideal gas law for any set of pressure, volume, temperature, and/or other revelant variables. Key 23
9. What is the new pressure in terms of the new volume and new temperature? Key 39
10. What quantity in the ideal gas law is proportional to the amount (weight) of gas present? Key 50
11. In terms of the x-component of the velocity, say, and the length ℓ of the cube, express the average time between collisions of one atom with one wall. Key 66
12. How much does the average x-component, say, of an atom's momentum change during a collision? Key 61
13. What is the force associate with this change in momentum and what pressure does this force produce? Key 8
14. Write down the relationship between the average square of the x-component of the velocity and the average square of the velocity. Key 40
15. Write down the escape velocity for an object of mass m in terms of the useful info given and fundamental physical constants. Key 51
16. Now express the temperature as a function of the root-mean-square speed. Key 67
17. When the system reaches equilibrium, what downward pressure must exactly balance the upward osmotic pressure? Key 9
18. What is the concentration of the sugar in the column? Key 41
19. What is the first law of thermodynamics, in terms of the quantities in the problem? Key 36

20. What is the equation of state of one mole of ideal gas? Key 68

21. Are frictional forces present? Key 47

22. What is the entropy change for a reversible process, in terms of the heat absorbed Q and the temperature T_0 of the reservoir? Key 10

23. Does the internal energy of the system change during the compression? Key 26
(If you still need help, go on to the next two Helping Questions.)

24. Is free expansion a reversible process? Key 58

25. What is the ratio Q_1/Q_2 for a Carnot cycle, in terms of T_1 and T_2? Key 21

Answer Key

1. $Q = 3.3 \times 10^6\,\text{J} = L_{\text{fusion}} \cdot 10\,\text{kg}$
2. $T = \dfrac{(2P_0)(2V_0)}{NR} = 4T_0$
3. $t = 10^2\,\text{weeks} = 1.9\,\text{years}$
4. $W = P_1 \Delta V = (3\,\text{atm})(2\,\text{liters}) = 6 \times 10^2\,\text{J}$
5. $\varepsilon = \dfrac{Q_2}{W} = \dfrac{Q_2}{Q_2 - Q_1} = \dfrac{T_2}{T_2 - T_1}$
6. $\dfrac{\Delta V}{V} = \beta \Delta T = 3\alpha \Delta T = 0.65\%$
7. $P = P_0$
8. $F_x = \dfrac{2m\vec{v}_x^2}{2\ell/\vec{v}_x} = \dfrac{m\vec{v}_x^2}{\ell}$, $P_{\text{He}} = \frac{F_x}{\ell^2} = \frac{m\vec{v}_x^2}{\ell^3}$
9. (weight)/(area) $= \rho g h$
10. $\Delta S = \dfrac{Q}{T}$
11. $A = \pi R_E^2 = 1.3 \times 10^{14}\,\text{m}^2$
12. $\sum Q_i = 3.2 \times 10^7\,\text{J}$
13. $\dfrac{P_{20^\circ\text{C}}}{P_{100^\circ\text{C}}} = \dfrac{293}{373}$ so $P_{20^\circ\text{C}} = \dfrac{293}{373} \times (6\,\text{atm}) =$ $4.7\,\text{atm}$
14. $T_{\text{H}_2} = 1.0 \times 10^4\,\text{K}$; $T_{\text{O}_2} = 16 \times 10^4\,\text{K}$; $T_{\text{He}} = 2.0 \times 10^4\,\text{K}$
15. $W_{\text{ibf}} = 6\,\text{cal}$, $Q_{\text{fi}} = -43\,\text{cal}$, $U_{\text{f}} = 40\,\text{cal}$, $Q_{\text{ib}} = 18\,\text{cal}$
16. $P_{\text{sun}} = 4.5 \times 10^2 6\,\text{W}$
17. A constant amount per unit mass, referred to as the latent heat L.
18. $P = \dfrac{NR(373\,\text{K})}{v} = \dfrac{373}{293} \cdot \dfrac{NR(293\,\text{K})}{v} =$ $1.27 \cdot (6\,\text{atm}) = 7.6\,\text{atm}$.
19. Clearly not; the heart and bloodstream are necessary.
20. $Q = \Delta U + W = 1.1 \times 10^3\,\text{J}$
21. $Q_1/Q_2 = T_1/T_2$
22. $\beta = 3\alpha$
23. $PV = NRT$
24. $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}m\bar{v}_x^2 = \frac{3}{2}P_{\text{He}}\ell^3 = 6.2 \times 10^{-21}\,\text{J}$
25. $N = c(\text{volume of column}) = 1.2 \times 10^{-6}$ moles
26. No; ΔE for an ideal gas is a function of temperature only.
27. 4.6×10^{-10}
28. $h = \dfrac{Q_{\text{tot}}}{mg} = 3.3 \times 10^5\,\text{m}$
29. $P = 2 \times 10^4\,\text{Pa} = 0.20\,\text{atm}$
30. $T_{\text{H}_2} = 4.6 \times 10^2\,\text{K}$; $T_{\text{O}_2} = 7.4 \times 10^3\,\text{K}$; $T_{\text{He}} = 9.3 \times 10^2\,\text{K}$
31. Yes, no, yes, yes
32. By Stefan's Law: $P = e\sigma A T^4$, where $\sigma = 5.67 \times 10^{-8}\,\text{W}\,\text{m}^{-2}\text{K}^{-4}$
33. $Q = 4.2 \times 10^6\,\text{J}$
34. $1 - \dfrac{6.0\,\text{atm}}{7.6\,\text{atm}} = .21$
35. $T = \dfrac{2}{3}\dfrac{mg}{k_B}R_E$
36. $\Delta U = Q - p_1 \Delta V$
37. $Q_2 = 0.15E_0 \dfrac{t_2}{T_2 - T_1} = 2.2E_0$
38. $Q = mc\Delta T = 2.1 \times 10^5\,\text{J}$

39. $P = \frac{NRT}{V} = \frac{NR(2T_0)}{2v_0} = \frac{NRT_0}{V_0}$

40. $3\bar{v}_x^2 = \bar{v}^2$

41. $c = \frac{\rho g h}{RT} = .040\,\text{moles/m}^3$

42. Entropy must increase; internal energy doesn't change.

43. $T = \left(\frac{P_{\text{earth}}}{\sigma A_{\text{earth}}}\right)^{1/4} = 16°\text{C}$

44. $h_{\text{max}} = 6.8 \times 10^4\,\text{m}$

45. $P_{\text{He}} = \frac{P}{N_A} = 3.3 \times 10^{-25}\,\text{atm} = 3.3 \times 10^{-20}\,\text{Pa}$

46. In fact, not very good. Even though the temperatures you calculated are higher than the temperature ever reaches on the moon, since at a given temperature there is a broad distribution of molecular speeds, many will have the energy necessary to escape.

47. Yes

48. $P_{\text{earth}} = 2.1 \times 10^{17}\,\text{W}$

49. $Q = L_{\text{vaporization}} \cdot 10\,\text{kg} = 2.25 \times 10^7\,\text{J}$

50. $N = \frac{PV}{RT}$

51. $v_0 = \sqrt{\frac{2Gm_E}{R_E}} = \sqrt{2gR_E}$, independent of m

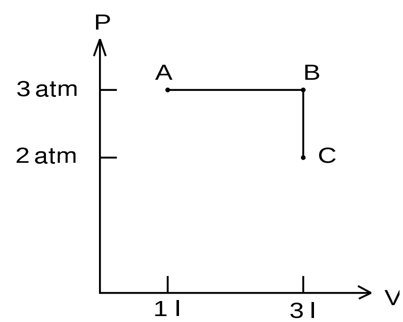
52. $T = \frac{pV}{R} = 73\,\text{K}$

53. Probably because no heat pump can really attain the Carnot efficiency. They are coming into more widespread use, however. The fact that $Q_2 = 2.2E_0$ may surprise you, but note that the first law hasn't been violated: we still have $Q_1 + W = Q_2$.

54. $Q = mc\Delta T$

55. $V = V_0$

56. $D = \frac{x_{\text{rms}}^2}{2t} = 8.3 \times 10^{-11}\,\text{m}^2/\text{s}$

57. 

58. No

59. $\Delta\ell = \alpha\Delta T = 2.2\,\text{ft}$

60. $P = 2100\,\text{J/s}$

61. $2m\bar{v}_x$

62. $P = \rho_w g h = 98\,\text{N/m}^2 \doteq 10^{-3}\,\text{atm}$

63. $S = \frac{p\Delta V}{T_0} = \frac{R\Delta V}{V} = -8.3 \times 10^{-2}\,\text{J/K}$

64. $A = 4\pi r^2 = 2.8 \times 10^{23}\,\text{m}^2$

65. $Q = 2.0 \times 10^6\,\text{J}$

66. $\Delta t = \frac{2\ell}{\bar{V}_x}$

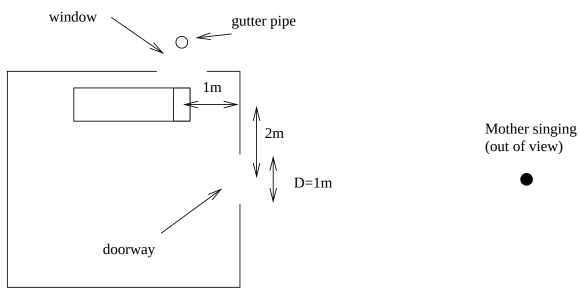
67. $T = \frac{1}{3}\frac{m}{k}v_{\text{rms}}^2$

68. $PV = RT$

Learning Guide 10

Problem I: Afraid of the Dark

A sleeping child is awakened by an electrical storm 1 km away. He sees a flash of lightning and then hears the roar of thunder.



1. How much time separated the lightning and the thunder? Key 7
   (For assistance, see Helping Question 1)

2. He opens the door to let in the light from the living room. Unfortunately, now his mother Barbra's singing practice keeps him awake. Her range spans from 100 Hz to 1000 Hz. He finds the low frequency tones reach him better than the high frequency tones. Why the difference? Key 21
   (If you need help, see 2)

3. What is the "cut-off" frequency; ie, what frequency separates the low frequency tones that reach him relatively well from the high frequency tones that don't reach him very well? Key 11
   (If you're stuck, try Helping Questions 3 and 4.)

4. The rain stops and his mother finally goes to sleep. He opens the window to get some fresh air. He is just drifting off to sleep when the wind picks up, setting up acoustic standing waves in the gutter pipe outside his window. The pipe is 4 m long and is open at both ends. A typical child (who has not yet abused his Walkman!) hears frequencies between about 20 Hz and 20 kHz. What are the lowest and highest frequencies the child hears? Key 8
   How many frequencies does he hear? Key 15
   (See Helping Question 5.)

5. Now he is really fed up. He goes outside and jams a tennis ball into the bottom of the gutter pipe. But this only changes the sound! What are the highest and lowest tones he hears now? Key 5
   How many different frequencies does he hear? Key 12
   (Try Helping Question 6.)

Problem II

A student typing in a computer cluster generates a sound level of 60 dB. What will the dB level the night before junior papers are due when there are 20 equally noisy students working in the cluster?
Key 2

Problem III

Sitting in your dorm room in Spelman you hear the Princeton Dinky sound its horn at a frequency of 550 Hz. You know that the stationary frequency of the horn is 520 Hz.

1. Do you still have time to catch the train? Key 9
2. What is the velocity of the dinky when you hear the 550 Hz sound? Key 16

Problem IV

A wave is described by the following equation:

$$y(x,t) = 15\sin(18t - 6x)$$

1. In what direction is the wave traveling? Key 23
2. What is the maximum amplitude? Key 6
3. At what times is $y(0,t)$ maximal? Key 13
4. What is the period of the wave? Key 20
5. What is the wavelength of the wave? Key 3
6. What is the velocity of the wave in the direction of propagation? Key 10
7. What is the maximum and minimum speed of the wave perpendicular to the direction of propagation? Key 17

Helping Questions

1. What are the speeds of light and sound? Key 14
2. How does the diffraction angle depend on the frequency? Key 4
3. What angle does the position of his head correspond to? Key 18
4. What frequency has a diffraction angle (angle of first diffraction minimum) that coincides with the position of his head? Key 1
5. What is the expression for the natural frequencies of a tube of length L, open at both ends? Key 22
6. What is the expression for the natural frequencies of a tube of length L, open at one end? Key 19

Answer Key

1. $\lambda/D = \sin\theta$
2. $73\,\mathrm{dB}$
3. $\lambda = \pi/3$
4. The diffraction angle is larger for lower frequencies.
5. lowest: $21\,\mathrm{Hz}$; highest: $19,960\,\mathrm{Hz}$
6. 15
7. $2.9\,\mathrm{s}$
8. lowest: $43\,\mathrm{Hz}$; highest: $19,980\,\mathrm{Hz}$
9. Yes, train moving towards you
10. $v = 3$
11. $f = C_{\mathrm{sound}}/(D\sin 63^\circ) = 383\,\mathrm{Hz}$
12. 466
13. $t = (2n+1)\pi/36$
14. light, $3\times 10^8\,\mathrm{m/s}$; sound, $343\,\mathrm{m/s}$.
15. 466
16. 42 miles per hour towards Princeton
17. 270, 0
18. $\tan\theta = 2/1$; $\theta = 63^\circ$
19. $f_n = n(C_{\mathrm{sound}}/4L)$, where n is an odd integer.
20. $T = \pi/9$
21. Because low frequencies are diffracted at larger angles than high frequencies.
22. $f_n = n(C_{\mathrm{sound}}/2L)$, where n is an integer
23. positive x direction