PRINCETON UNIVERSITY

Physics 101 Lab Manual

(Revised September 2009)

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Contents

Lab 1

Newton's Laws: Motion in One Dimension

1.1 Background

In this lab you will study the motion of bodies moving in one dimension. To minimize unwanted forces on the test object, you will use an air track (see Fig. 1.1). The glider floats on a cushion of air above the track, eliminating most of the friction between the glider and the track. The glider can move freely with no horizontal force, or under the influence of a constant horizontal force when attached via a tape to a weight suspended off one end of the air track (see Fig. 1.2). By measuring the time, t, at which the glider passes several positions, x, you can test Newton's laws and measure g.
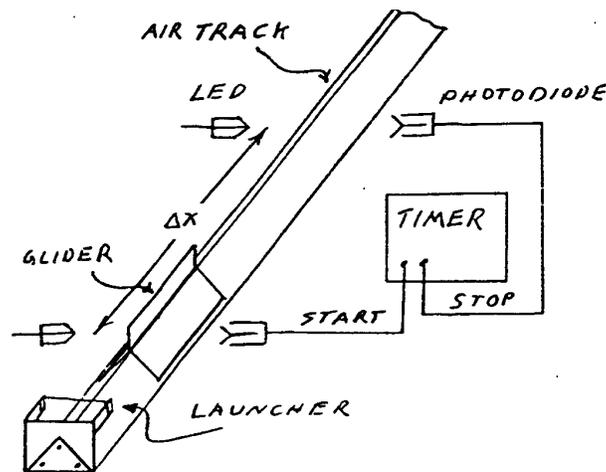

Figure 1.1: An air track with two photogates.

Figure 1.2: Glider of mass m on the horizontal air track, connected by a tape to mass M that falls vertically under the influence of gravity.

1.1.1   Newton's First Law

Newton's first law states that an object will move at constant velocity if no forces are imposed on it. Thus, defining the x axis to be along the direction of the velocity, its motion obeys:

$$x = x_0 + vt, \tag{1.1}$$

where x is the position of the object at time t, x_0 is its position at time t_0, and v is its velocity. You can test this equation by giving the glider a push (so that it has some nonzero velocity) and seeing whether x increases linearly in time.

1.1.2   Newton's Second Law

Newton's second law states that the acceleration a of an object of mass m is proportional to the net force F applied to it,

$$\sum_i F_i = ma. \tag{1.2}$$

If the net force is constant, then the acceleration a is constant and the motion of the object obeys:

$$x = x_0 + v_0 t + \frac{at^2}{2}, \tag{1.3}$$

with v_0 the initial velocity.

To test this, you need to impose a constant force on the glider. You can do this by attaching a hanging mass m to the glider (which itself has mass M) via a pulley, as in Fig. 1.2. Let T be the tension in the tape connecting the masses, and g be the acceleration due to gravity. The only horizontal force on the glider is the tension from the tape, so Newton's second law gives

$$T = Ma. \tag{1.4}$$

The hanging mass m feels both gravity (downwards) and tension T (upwards), so the second law for its vertical motion is

$$mg - T = ma, \tag{1.5}$$

noting that the downward vertical acceleration a of the hanging mass is the same as the horizontal acceleration of the glider. Substituting eq. (1.4) for T into eq. (1.5) gives

$$mg - Ma = ma, \tag{1.6}$$

or, upon rearranging terms,

$$g = a\,\frac{M+m}{m}. \tag{1.7}$$

By measuring the acceleration a and the masses M and m, you can find gravitational acceleration g.

1.2 Specific Instructions

1.2.1 Setup of the Air Track

Start air flowing into the track by plugging in the air pump and/or turning on the switch on the power strip.

The air track will probably already be set up, but you should check the following things:

- The four photogates (enclosed in U-shaped plastic tubing) should be set up so that each one is blocked (and unblocked) <u>twice</u> every time the glider passes by. The system will record the time when each gate is blocked, and not the times when they become unblocked. To check the operation and positioning of the photogates, watch the red lights on the photogate adapter box, which turn on when a gate is blocked.

- The distance between adjacent photogates should be longer than the length of the glider (so the glider never blocks more than one photogate at a time).

- In the acceleration part of this lab, you will work with a glider, a hanging mass, and a tape, connected as shown in Fig. 1.2. The tape slides along the curved bit of the air track with minimal friction. Then, the glider will be accelerated by the hanging mass. Briefly set this up now, and make sure the tape is long enough that the glider can start behind photogate 1, and that the glider will get through photogate 4 before the hanging mass hits the floor.

1.2.2 Timing Software

The photogate timing is measured by a program called Princeton Timer A. Double click on its icon (in the Physics 101 folder of your lab computer) to open the program.

Once the program is open, you can click on Collect to start data collection, and on Stop to end a run. The system records the times of each successive blocking of a photogate, starting with $t = 0$ at the first event. That's all there is to your data collection. Try the system a few times, blocking the optical beams with your hand repeatedly to watch data being collected.

Note: the data collection system takes a second or so to organize itself after the Collect button is pushed. If you get strange (generally negative) times in the first few data entries, it is probably because a photogate was interrupted before the system was actually ready to take data. You can avoid this problem by pausing for a second after pushing the Collect button, before starting the action.

1.2.3 Events

Because of the way your measurements will be made, it will be useful to think of yourself as measuring the times and positions of eight "events" which occur as the glider moves from one end of the air track to the other:

Event 1: the 1st photogate is blocked by the first "flag" on the glider .
Event 2: the 1st photogate is blocked by the second "flag" on the glider .
Event 3: the 2nd photogate is blocked by the first "flag" on the glider .
Event 4: the 2nd photogate is blocked by the second "flag" on the glider .
Event 5: the 3rd photogate is blocked by the first "flag" on the glider .
Event 6: the 3rd photogate is blocked by the second "flag" on the glider .
Event 7: the 4th photogate is blocked by the first "flag" on the glider .
Event 8: the 4th photogate is blocked by the second "flag" on the glider .

You will measure the positions x and times t of each of these events, and fit these data to eqs. (1.1) and (1.3).

1.2.4   Measuring x Positions

You need to measure the position x of the front edge of the glider at each of the events described above. You can do this as follows:

- Turn the air off (this makes the measurements a bit easier).

- Note in which direction the glider will be moving, and move the glider to the point where its first "flag" is just starting to block photogate 1. By watching the lights on the photogate box to see when the photogate is blocked, you should be able to do this very accurately. Check that the photogate heights are set so that they are blocked by each of the two flags, and not blocked when the glider is at in-between positions.

- Once the glider is at this point, measure the position of the front edge of the glider using the scale attached to one side of the air track. Note, you could equally well use the back edge of the glider – just be consistent during these measurements. In fact, the absolute position of the scale on the air track in both eqs. (1.1) and (1.3) affects only the constant term x_0, which is relatively unimportant; it does not affect the value of the constant velocity in your first measurement, or the value of the acceleration g in your second measurement.

- Make a measurement of the position of the front edge of the glider when its second "flag" is just starting to block photogate 1. Then, repeat this procedure for the other three photogates. Record all your measurements in your lab book.

1.2.5   Motion with Constant Velocity

As your glider moves through the four photogates, the system will record the times of the eight "events" discussed above. From your notebook, you know where the glider was at the time of each event. So you can make a plot of position versus time. To do this, use the program "Excel with WPtools", whose icon is in your Physics 101 folder. (Excel is a standard spreadsheet and graphing program. WPtools (Workshop Physics Tools) is an add-on capability which simplifies graphing and curve fitting. See Appendix A for additional details.)

First, let's test Newton's first law. If a tape is connected to the glider, disconnect it. Give the glider a push, and let it pass through all four photogates. Start collecting data before it gets to the first gate, and stop after it leaves the fourth. If it bounces back and retriggers some of the gates, just use the first eight times recorded and ignore the later ones.

Without closing down the timer program, double click on Excel with WPtools to open it. Answer No to a question about something already being open, and then click on File and New and OK to bring up a blank spreadsheet.

Transfer your eight time values from the timer program into a column in Excel. You can transcribe the measured values of time manually, or you can use the standard Copy and Paste method.

Click on the first value and hold the mouse button down while "swiping" the cursor down to the last value. Then click Edit/Copy *in the timer program, and go back to* Excel. *Click on one* Excel *cell to highlight it, and use* Edit/Paste *to put the data in a column below the highlighted cell.*

Enter your data on the eight x positions in the next Excel *column, with each value adjacent to the corresponding value of t. Add a title to the box above the data in each column. (*Time *or* t *to one column,* Position *or* x *to the other.)*

To make a graph of position versus time, you have to tell Excel what to plot in the vertical and horizontal directions. Swipe your cursor vertically across the title and data cells corresponding to the horizontal variable (time). Then, while holding down on the Control key, swipe the cursor across the title and data cells corresponding to the vertical variable (position).

With the horizontal and vertical data selected, click on WPtools / Scatter Plot to make your graph. Do the points lie close to a straight line? The slope of the line should be the velocity of the glider. Estimate the value of the slope from the printout of the graph, and write it down in the proper S.I. unit. Does the number seem reasonable?
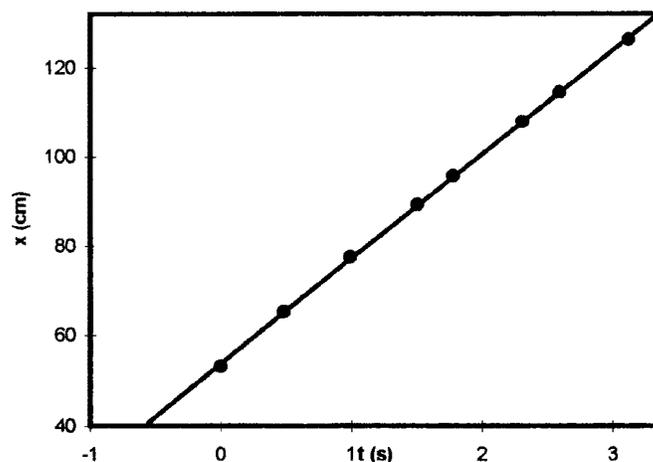

Figure 1.3: Sample plot of distance versus time for zero net force.

Use the computer to make a graph with a polynomial fit to the data. In terms of the

general equation (1.3) for accelerated motion in one dimension, we expect position x to be a quadratic function of time t,

$$x = a_0 + a_1 t + a_2 t^2. \tag{1.8}$$

The value of the coefficient a_2 is expected to be zero for motion with constant velocity.

Highlight the two data columns as before, and click on WPtools / Polynomial Fit. Then choose Order 2 for the polynomial, and click OK. You get a graph equivalent to the previous one, but with lots of information added about the computer's best fit to your data.

You may want to resize the graph box, or to move some of the fit data boxes around or resize them, to get a good view of the new plot. Your instructor can also show you how to reset the scales on the graph. For now, let's focus on the meaning of the computer's information concerning its fit to your data.

The numbers a_0, a_1 and a_2 are computed values for the coefficients of the equation. The Standard Error (SE) for each coefficient is the computer's estimate of how accurately these coefficients are known, based on how closely your data falls to the fitted line. R^2 is a measure of the goodness of the fit, which need not concern us.[1] The Greek letter sigma (σ) is, however, of interest. It indicates how far a typical data point deviates from the fitted line.

Does the fitted value of a_1 agree with the velocity you calculated manually from the slope of your first graph to within 1-2 times the reported Standard Error on a_1. If not, recheck your procedures.

If the horizontal force on the glider were zero as desired, the fitted value of the coefficient a_2 should be zero to within 1-2 times its Standard Error. If this is not the case, check that the air track is level, and that the air flow is sufficient.

1.2.6   Motion with Constant Acceleration: Measuring g

Attach a tape to the glider and suspend it over the curved bracket so that a hanger can also be attached, as indicated in Fig. 1.2. Add weights to the hanger so that its mass m is between 15 and 25 g. Measure the masses of the glider and the hanging mass using a balance or digital scale.

Release the glider and record times using Princeton Timer A as described above. Transfer these times to Excel and enter the x measurements (if you didn't save them earlier). The displacement x should have a quadratic dependence on t (eq. (1.3)), so use the Polynomial Fit option with Order = 3 to fit the data to an equation of the cubic form

$$x = a_0 + a_1 t + a_2 t^2 + a_3 t^3, \tag{1.9}$$

which should result in a value of a_3 consistent with zero if your data are good. Comparing eqs. (1.3) and (1.9) we see that the acceleration of the glider is $a = 2a_2$. Then, calculate the acceleration g due to gravity from eq. (1.7) as

$$g = 2a_2 \frac{m + M}{m}. \tag{1.10}$$

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[1]If, however, you are curious as to the technical significance of the quantities R^2 and σ, see Appendix D.
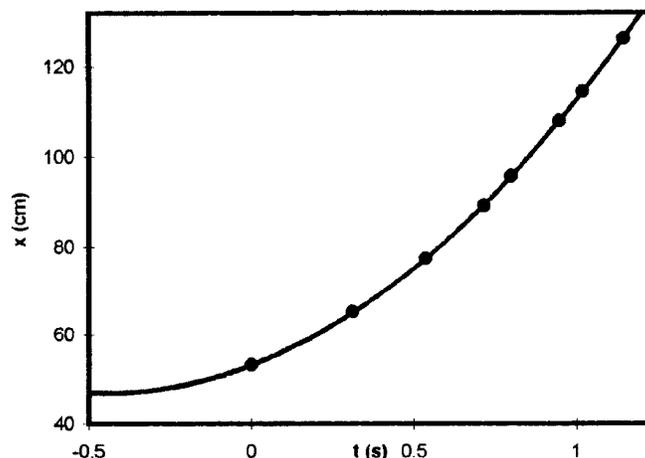
Figure 1.4: Sample plot of distance versus time for accelerated glider.

The corresponding uncertainty σ_g in your measurement of g can be estimated as

$$\sigma_g = 2\,\mathrm{SE}(a_2)\frac{m+M}{m}\,. \tag{1.11}$$

Include a printout of the plot of x *vs.* t and the results of the fit in your lab notebook.

If the fit coefficient a_3 is not zero to within 1-2 times $\mathrm{SE}(a_3)$, your experimental technique needs to be improved.

Is your measurement for g within 1-2 times the estimated uncertainty σ_g of the expected value?

How does the uncertainty $\mathrm{SE}(a_2)$ on the fit coefficient a_2 obtained with the hanging mass compare with that obtained previously when the glider slid freely?

Run the experiment two more times, using different hanging masses, and calculate the value of g twice more. Are the values systematically high or low? If so, can you think of anything that might cause this experiment to go wrong?

From your three measurements of g, you can make another estimate the uncertainty σ_g according to

$$\sigma_g = \frac{\mathrm{Max}(g) - \mathrm{Min}(g)}{2}\,. \tag{1.12}$$

Compare this with your estimate based on eq. (1.11).

The estimate (1.11) of the uncertainty in your measurement of g does not include the effect of the uncertainty in your measurements of the masses m and M. In fact, the uncertainty σ_m on the hanging mass m leads to an additional uncertainty on g of approximately

$$\sigma'_g = g\frac{\sigma_m}{m}\,. \tag{1.13}$$

Is this effect larger or smaller than the uncertainty (1.11) associated with the curve fitting?

Lab 2

Motion in Two Dimensions

This lab extends your exploration of Newton's laws of motion to the case of two dimensions, as described in secs. 2.2 and 2.5. To accomplish this, you will use a video camera and supporting computer software to capture, edit and analyze images of two-dimensional motion. Sections 2.1, 2.3 and 2.4 introduce you to this infrastructure.

2.1 A New Tool

The first goal of this lab is to acquaint you with a new tool. You will use a TV camera to make videos of moving objects, and a computer to digitize the locations of objects in your videos and analyze the motion. It is important that every member of your group learn how to use the computer and analysis software. Be sure that you all take turns operating the system and that, when you are the operator, your partners understand what you are doing.

When your computer is first turned on, the screen should show an icon for a folder named Physics 101 (similar to the folder for Physics 103 shown in the figure). Open this folder, by placing the cursor on it and clicking with the right mouse button and choosing *Open*, or by rapidly clicking twice using the left mouse button. You will then see a screen with icons for the programs that we will use in this lab.

These include:

- **VideoPoint Capture**: this program allows you to capture a series of images, called "frames", from the signal produced by a video camera.

- **VideoPoint**: the program in which you will do your analysis. It allows you to determine the positions of objects in each of the video frames that you captured, and provides ways to generate and analyze graphs of your data.

- **Excel with WPtools**: an extension of the common Excel spreadsheet program, which allows detailed calculations to be carried out on your data. The WP (Workshop Physics) extensions, make it easy to use Excel's sophisticated graphing functions.

- WordPad: a utility word processor, generally used to write and print out simple text documents, or to print out an image of any computer screens that you need to discuss in your write-ups.

- Student Data: a folder with space for you to put your files.

2.1.1 Capturing a Single Still Image

Your first task is to capture a still picture to give to your lab instructor to help him or her remember your names and faces. To get an image from the video camera on your computer screen and make a copy, follow the instructions below:

1. Open VideoPoint Capture, by double-clicking on its icon with the left mouse button.

2. A VideoPoint Capture Preview Screen appears, with a picture of whatever your camera is viewing at the moment. Confirm that your Preview screen is showing live video by waving your hands in front of the camera. If the picture is too dark or too light, rotate the aperture ring on the camera lens to change the amount of light level reaching the camera's sensor. (The lens admits light through a circular opening, or aperture, whose diameter is controlled by rotating the ring. The larger the diameter, the more the light and the brighter the picture.)

3. Position yourselves (re-aiming the camera if necessary) so that your faces pretty well fill the picture on the screen. Then reach over, hold down the ALT key, and hit the Print Screen button near the right end of the top row of keys on your keyboard. This makes a temporary copy of whatever was on the VideoPoint Capture screen when you hit the button.

4. Now, open WordPad by double-clicking on its icon. Then choose Edit→Paste to insert the image that was copied using the Print Screen command. (Please do NOT change any of the computer or display settings in an attempt to make the image larger. Such changes may interfere with your analysis program.)

5. Click on the Printer icon in the WordPad menu to make a copy of your picture. *Write your names on a printout of your picture and give it to your instructor.* You may want to make extra copies for your lab books, but dont let this become your goal for the afternoon.

2.1.2 Stop and Think - What is a Digital Image?

Before trying to capture a video in the computer, let's be sure that we understand what the moving image on a television screen actually consists of. It is a series of individual stationary, or still, pictures, appearing at a rate of 30 pictures, or *frames*, a second. Standard movie films consist of strips of such still pictures, recorded photographically at a slightly lower frame rate. Our videos will generally be taken at 10 frames per second.

Digital computers, and hence our electronic imaging systems, work only in quantized units. Your camera divides the area that it sees into over 76,000 discrete elements, arranged into a grid 320 cells wide by 240 cells high. You will see later that VideoPoint measures x and y positions with twice this resolution, dividing the x-direction into 640 integer steps and the y-direction into 480 steps. In either case, a 1-step by 1-step cell is called a *pixel*, for "picture element."

To determine the coordinates of an object in an image frame using VideoPoint, we will use the mouse to position a cursor over the object, and the system will count the pixel rows and columns to get to the location of the cursor, starting at the lower left of the image on the screen. Then, if we know how many pixels correspond to a meter long object, we can calculate real distances from the pixel counts. This is called "scaling the image". We will return to this shortly.

With real distances known from scaling the images, a nd with the time interval between two chosen frames also known (a multiple of 1/10 sec, say), we can easily calculate such things as velocities and accelerations of moving objects in our videos.

2.2 Physics Background

In this lab you will analyze the trajectory of a bouncing ball. The ball will be accelerated by gravity in the y direction, but not in the x direction. Its motion should obey the equations

$$x = x_0 + v_{0x}t, \tag{2.1}$$

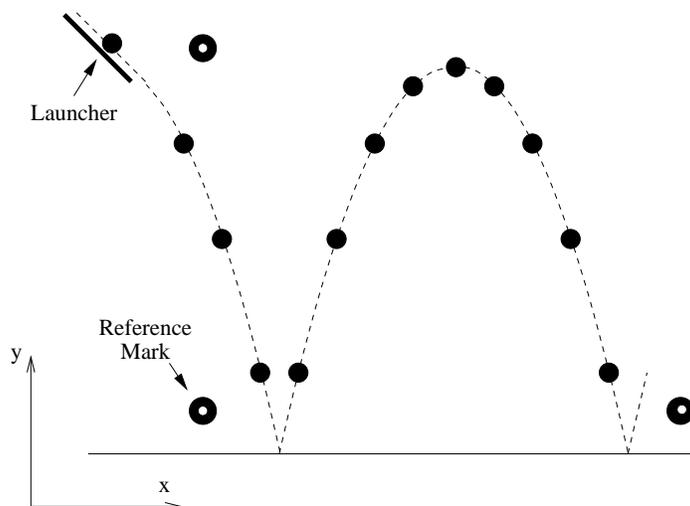$$y = y_0 + v_{0y}t + \frac{gt^2}{2}. \tag{2.2}$$

Figure 2.1: Setup of the experiment.

The experimental setup is sketched in Fig. 2.1. A ball rolls down a fixed rail (the "launcher"). After it leaves the launcher it falls freely, bouncing at least twice on the table

top. The camera takes a series of pictures at fixed time intervals during the motion. By measuring the position of the ball in each picture, you can test eqs. (2.1) and (2.2).

2.3   Acquiring the Data

You are now ready to record a video. If necessary, minimize or close the WordPad window to get the VideoPoint Capture screen back into view.

First, aim the camera to view the bouncing ball, and make a few trial runs, watching the Editing screen to see that the camera catches the motion properly. Include a meter stick in the field of view, for scaling purposes. You may want to mark two or more positions on the meter stick with tape, to give two clearly visible reference marks. Remember, the meter stick should be the same distance from the camera as the bouncing ball will be. Otherwise, your scale factor will be in error.

Clicking on the button at the lower left of the screen gives you the option of recording frames at many different rates. For the moment, set the frame rate at 10 frames per second.

The Preview screen has a Record button to be clicked when you are ready to start recording. To stop the recording, hit the Escape (Esc) key at the upper left corner of your keyboard. (You can also click on one of the Stop buttons that appear, but this doesn't always work.) Try not to record more than 5 to 10 seconds of video at a time. Otherwise, the size of the video files becomes outrageous.

Go ahead and take a video of the bouncing ball. After you have stopped recording, the Editing screen takes a few seconds to organize itself. It comes to a stop displaying the first frame of your recording, with a new control bar appearing at the bottom.

First, click on the right arrow at the left end of the control bar. The system will "play back" your video. Note the round *progress button* below the picture moving across the bottom of the picture as the video goes from beginning to end.

You can manually play the video, backward or forward, by using your mouse to shove the progress button back and forth. You can also advance through the video a single frame at a time, forward or backward, by hitting one of the two buttons at the right of the control bar.

Editing your Video: In general, your original video will contain more frames than you will want to use for analysis. Today, we want to analyze the ball's motion between two consecutive bounces off the table.

First, use the progress and single-step buttons to choose the first frame following a bounce of the ball. Then click the First button at the lower right. The software then eliminates all of the earlier frames from the record, and the display jumps to the last frame that you recorded. Use the progress and single-step buttons until you see the last frame before the next bounce. Then click on the button at the lower right, which now reads Last. Any later frames are now eliminated from your video.

Naming and storing your VideoPoint **capture files**: *The video containing your chosen frames must be named and stored for future reference. We have provided a folder identified by your lab groups meeting time (e.g., "Monday Afternoon Section"). This folder is found in a higher level folder titled* Student Data, *which in turn is located on the hard drive partition labeled* STUDENTS (D:).

Click on the Save *button at the lower right of the* VideoPoint Capture Editing *screen. A screen appears with the title* Save Movie File As.... *You will want to get the section folder for your lab section to show in the screen's top* Save in: *box. If the folder for some other section is shown there, use the yellow icon with the left-and-up arrow to go to the* Student Data *folder, and then double click on the folder for your section. Consult your instructor if you have any difficulty getting your section name in the* Save in: *box – if you don't get this right, you may not be able to find your data later!*
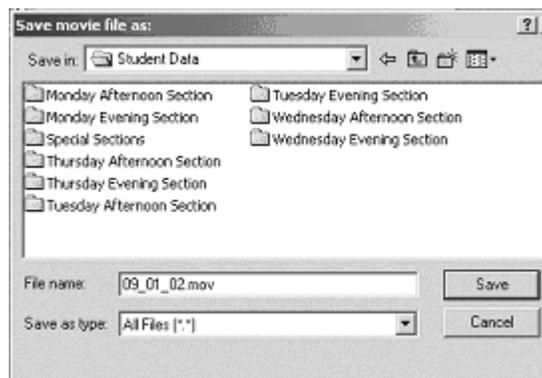
At the bottom of the screen the system has suggested a name such as 07_24_02.mov. The first two numbers (07) and (24) refer to the month of the year and the day of the month. The last two digits are assigned by the computer, from 00 to 99, and starting again. You may want to add more readable information. If your group's last initials are P and Q, for instance, you could use the mouse to insert "PQBounce" in front of the 07_24. Then the file name would be "PQBounce07_24_02.mov," which tells you a lot about the file, and will help you remember what it was.

Click on the Save *button. Your edited video file is now stored at the location and with the name which you specified. It can be opened at any time for analysis, using the* VideoPoint *software described earlier. For now,* VideoPoint Capture *provides a shortcut to allow us to go immediately to analyzing your movie in* VideoPoint.

Click on the Open in VideoPoint *button that appears at the bottom of the* VideoPoint Capture *image editing screen. This automatically opens the* VideoPoint *software package, and loads your video file into it. In the* Number of Points *screen which first appears, say* OK *to* 1, *since there is only one object that we want to analyze (the bouncing ball). If you see an information screen titled* About VideoPoint, *just click on the little* x *at the top right to make it disappear.*

2.4   Data Analysis Using VideoPoint

Three windows appear when VideoPoint is first opened. The Movie window shows the first frame of your edited video, with your file name appearing in its title bar. The Table window shows three columns ready to receive your data. (The left hand column already has the time entered for each of your video frames). The Coordinate Systems window will be used later to manage your data operations. For the moment, simply note that it has a line for Point S1, which is the name which VideoPoint has assigned to the single moving point which you said you wanted to track.

The Movie **Window**

The Movie window in VideoPoint has the same video controls as before. To "activate" the movie window, left-click on its title bar, where the name of your video should appear. The title bar should now appear in blue, which indicates that we have selected that window for our next steps. Try running the video, single-stepping through the frames, and moving back and forth in time by moving the progress button from side to side.

Next, notice that when the cursor is moved into the image area of the movie window, it changes shape and becomes a "cross hair" pointing device and the coordinates of the cross hair's position are read out at the lower left hand corner of the window. These coordinates are expressed in a Cartesian system indicated by the two yellow lines that are superimposed on the image. The numbers shown give the number of pixels you are to the right (x-direction), and the number of pixels you are above (y-direction), the origin of this coordinate system.

The Table **Window**

VideoPoint will load your position data into the table shown in the Table window. By scrolling vertically, you will see that the number of rows is equal to the number of frames in your video, and that the time when each frame was recorded is already shown, in the first column of each row, starting with 0 seconds.

As you will soon see, positioning the cursor on top of a selected position in the Movie window and clicking on the left mouse button will cause the x and y coordinates of any point in each frame (the tip of a finger, say) to be automatically entered into the second and third columns of the table, in the row corresponding to that particular frame.

The Coordinate Systems **Window**

Although we don't need to use it in this experiment, you should know that the first use of the Coordinate System window is to describe an object in which you are interested. By clicking on a Series Name, you can use your mouse to highlight and then change its name - say from Point S1 to the more descriptive Bouncing Ball. You can also assign a mass to an object, by clicking on and editing the value of mass assigned to it in the table. (The system assigns an arbitrary value of 100 grams to every object, until you change it to a real value. For the moment, just leave it at 100 grams.)

Later, you will see that this window is also useful when you want to Select an object in order to change the way its points are plotted on a graph, *etc.* For the moment, just note that clicking on the box to the left of a series name indicates that it has been selected,
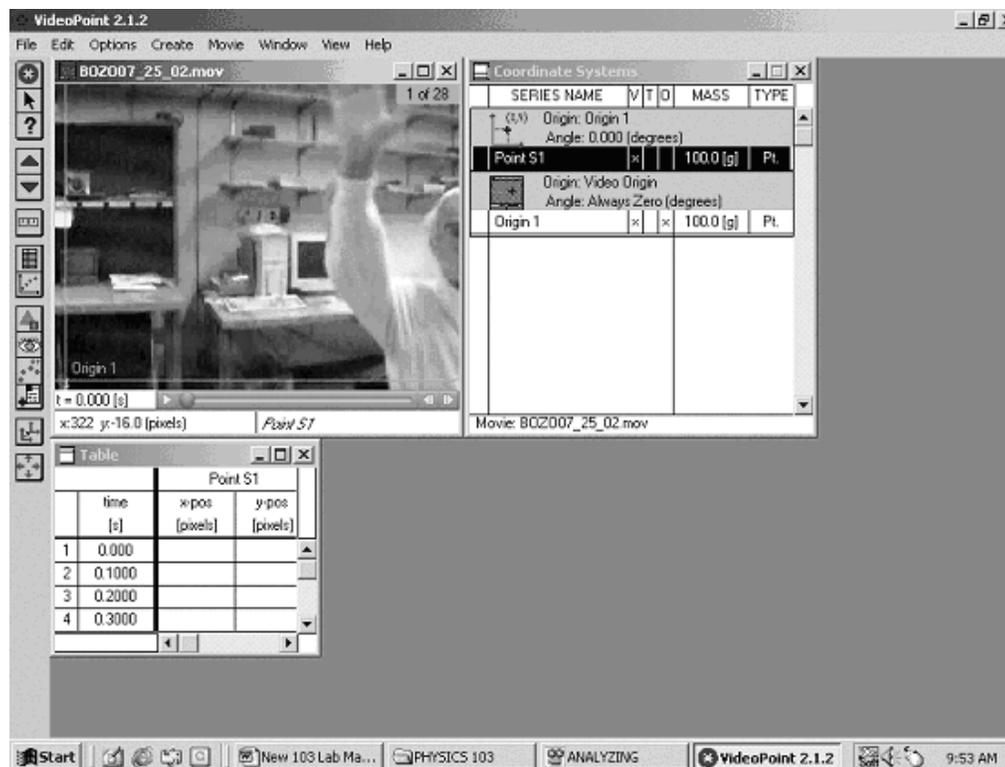
by highlighting the entire row. If you double-click in this box, a dialog box appears which offers an alternate means of changing series names, masses, *etc.*, in addition to more complex variables which we don't need for the moment.

2.4.1   Scaling the Movie

You should have included a meter stick in your video. We'll use it to convert our measurements from pixels to physical units.

First, choose any frame in which the meter stick is clearly visible. Move the cursor to either end of the meter stick, and note the pixel coordinates at the two positions. Calculate the distance between the two points, in pixels. Using the distance in pixels, calculate your movie's scale factor, in units of pixels per meter. Note the result in your lab book.

Now let VideoPoint do an automatic scaling of our movie. With the Movie screen selected, move your cursor to the Movie menu item, and select the Scale Movie option. When the dialog box appears, enter the length of the meter stick, and set the units of that length (cm, m, inches, *etc.*) Then click on Continue. (Ignore the other items on the Scale Movie box.)

Follow the instructions to click first on one end on the meter stick, and then on the other end. Like magic, all of the x and y entries in your data table, and all of the x and y axes of your graphs, are converted from pixel units to whatever physical unit (cm, m, inches,...) you have chosen! Even the cursor position readout at the lower left of the Movie screen is now in physical units.

Compare the value of VideoPoint's scale factor (visible near the top of the Coordinate System screen) with that which you calculated manually. Do they agree within "reasonable" accuracy? Note any comments that you might have.

2.4.2   Picking Off Coordinates

After clicking on the Movie window to again make it the active window, review your video. If you like, you can maximize the video image (*i.e.*, make it full-screen) by clicking on the center box at its top right corner. Repeating this action will make it small again, to let you see the other screens.

Now we will run through an actual coordinate pickoff:

1. Make sure that the Series Name labelle Point S1 is highlighted in the Coordinate Systems window. Then click anywhere on the Movie window to make sure that it is the active screen. Move your video back to its first frame. Then move the cross-hair cursor to center it on the ball. Press the left mouse button, being careful not to move the cross hair off your chosen point as you press the button. This automatically enters the coordinates of the cursor location into the corresponding line of the table window. The video may automatically advance to the next frame. If it does, just continue clicking on your chosen point to track its motion as the frames advance. If the frames don't automatically advance, just go to the Options item on the movie menu bar, and click next to Auto Frame Advance.

2. Continue clicking on the ball for four or five frames. Your movie screen may or may not be showing the positions you have chosen. In any case, stop and go to the Edit movie menu item, and click next to the Leave/Hide Trails option. When the option is clicked, you will see markers appear and disappear for each of the hand positions on which you have clicked. Sometimes these are valuable, and sometimes they get in the way. You can turn them on and off, using the option button. Continue picking off coordinates, letting the frames advance automatically. That's all there is to measuring positions of moving objects in a video!

   If you go back and step through the frames with the Leave Trails option on, you will note that the label moves along the string of position markers. If the series has been selected in the Coordinate Systems window, a second circle will also follow along the trail of position markers.

   The cursor always looks like an aiming circle when you are using it to pick off a points location in any given frame. If the location of the point has already been determined in any given frame, a circle appears around the location, and the cursor looks like an arrow instead of an aiming circle.

3. If you make a mistake in positioning your mouse in a particular frame, you can simply move the point. With the item selected in the Coordinate Systems window, and two concentric circles showing around the previously chosen location, first place the arrow-shaped cursor at the center of the circles. Then, while holding down the left button on the mouse, move the location circle to the new, corrected, position. Releasing the

mouse button completes the change and enters the corrected coordinates into the data table.

2.5 Graphing and Fitting Your Data Using Excel

While VideoPoint has some built-in graphing and curve-fitting features, it is preferable to transfer your table of values of x, y and t to an Excel spreadsheet.[1] Then, you can apply the methods you learned in Lab 1 to produce (and print out) plots of y *vs. x*, x *vs. t*, and y *vs. t*. In addition, use WPTools to fit the data of x *vs. t* to a Polynomial of Order 2, and to fit the data of y *vs. t* to a Polynomial of Order 3.

According to eqs. (2.1) and (2.2) the coefficient a_2 of the fit to x *vs. t* and the coefficient a_3 of the fit to y *vs. t* should be zero to within 1-2 times their respective Standard Errors. Coefficient a_1 of the fit to x *vs. t* is the horizontal velocity of the bounding ball, and twice the coefficient a_2 of the fit to y *vs. t* should be the acceleration g due to gravity.

Report the x velocity and the y acceleration and their Standard Errors. Is the value of g measured in this experiment within 1-2 times its Error of the expected value?

Repeat the entire process of data collection and analysis so that each member of your lab group does each part of the experiment once.

Is the technique of Lab 2 better or worse than that of Lab 1 in measuring the value of g?

---

[1]While in VideoPoint, click on the empty data cell to the left of the *time* column heading to select all of your data columns, and then click on Edit/Copy data. Then open an Excel workbook, click on the upper left cell, and click on Edit/Paste.

Lab 3

The Pendulum
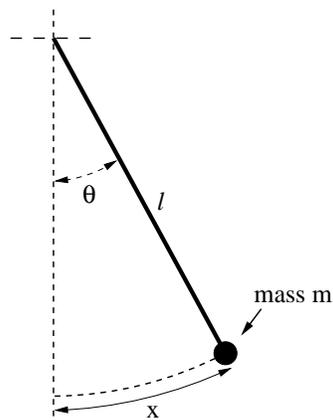
3.1 Background

3.1.1 Pendulum Motion

Figure 3.1: A simple pendulum, consisting of a mass m suspended from a string of length l.

The period T of a simple pendulum moving at small angles θ, as shown in Fig. 3.1, is related to its length l and gravitational acceleration g by:

$$T = 2\pi\sqrt{\frac{l}{g}}. \tag{3.1}$$

This is derived in your textbook and in section 3.4 of this writeup. In this lab you will confirm that the period T is proportional to $l^{1/2}$ and that T is independent of mass m. Also, you will use the pendulum to measure gravitational acceleration g with precision of about 1 part in 1000.

3.1.2 Logarithmic Plots

In this experiment, you will test eq. (3.1). In particular, you should show that T is proportional to $l^{1/2}$ and not l^1 or l^2 or some other power of the length l. For this, you could measure the pendulum period with several string lengths. But then what? You could try fitting curves of T versus $l^{1/2}$ and T versus l^1 and so on, to see which "looks best". However, there is a more quantitative approach to this problem: logarithmic plotting.

Here's the basic idea. You expect that T and l are related by an equation of the form:

$$T = c\,l^{\alpha}, \tag{3.2}$$

where c and α are fixed constants, and you want to find the value of the exponent α (which you expect to be 0.5 for the pendulum). Take the logarithm of both sides of eq. (3.2) to get:

$$\log T = \log(c\,l^{\alpha}). \tag{3.3}$$

The general logarithmic identities:

$$\log ab = \log a + \log b, \qquad \log a^b = b \log a, \tag{3.4}$$

can be used to rewrite eq. (3.3) as:

$$\log T = \alpha \log l + \log c, \tag{3.5}$$

which is the equation for a straight line. For example, if you set $y = \log T$, $x = \log l$, $b = \log c$, and $m = \alpha$, then you obtain the standard form,

$$\underbrace{\log T}_{y} = \underbrace{\alpha}_{m} \underbrace{\log l}_{x} + \underbrace{\log c}_{b}. \tag{3.6}$$

Thus by fitting a straight line to your measurements of $\log T$ versus $\log l$, you can find value and uncertainty of the exponent α. To do this with Excel:

- Enter your l and T measurements into two columns of a spreadsheet.

- Calculate $\log l$ and $\log T$. Use the LOG10 function in Excel (*e.g.*, cell —sf B1 might contain the formula =LOG10(A1)).

- Graph and analyze the $\log l$ and $\log T$ columns using WPtools/Linear Fit.

- The slope a_1 and uncertainty $SE(a_1)$ reported from the fit are the value of α and its uncertainty σ_α.

3.2 Specific Instructions

3.2.1 Setup and Timing of the Pendulum

Set up a photogate at the bottom of your pendulum's motion. Use the program Pendulum Timer to measure the pendulum's period. Start the ball swinging from a fixed angle $\theta \approx 2°$. (Use the protractor near the pivot point to set this angle accurately.) Then click on Collect.

The program Pendulum Timer actually measures the time between successive blockings of the photogate by the swinging ball, and reports twice this value as the period. Hence, it is important that the photogate be directly below the point of suspension of the pendulum to get an accurate measure of the period.

Pendulum Timer makes an entry in its data table, and also plots a point on the graph. It continues to make measurements until you tell it to Stop. You should let it take at least a few dozen data points, and perhaps many more on your repeated runs.

You can change the horizontal and vertical axis of the graph while the program is running. Simply click on the lowest or highest values of either scale, and edit its value. You may want to do this for the x direction simply to keep from running off the graph. Expanding the vertical scale to include only a small range of values near your data points will allow you to check for consistency and possible long term drift.

3.2.2   Dependence of the Period on the Initial Angle

An advanced analysis of the simple pendulum tells us that the period T is

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta_0}{2} + \frac{9}{64}\sin^4\frac{\theta_0}{2} + ...\right), \tag{3.7}$$

where θ_0 is the maximum angle to the vertical during the motion. For "small enough" launch angles θ_0 the terms $\sin^2(\theta_0/2)$, and $\sin^4(\theta_0/2)$ are very small so that we obtain eq. (3.1). For example, with $\theta_0 = 10°$ the period would be increased by about 0.2% above the value predicted by eq. (3.1), and for $\theta_0 = 2°$ the increase is only 0.008%.

Can such small effects be detected in your experiment?

3.2.3   Measurement of g

Measure the period of the pendulum using a fairly long length l and the steel ball. Estimate the uncertainties of l and T. From equation 3.1, you can calculate g as:

$$g = 4\pi^2\frac{l}{T^2}. \tag{3.8}$$

Using calculus (Appendix B.3), one can show that:

$$\left(\frac{\sigma_g}{g}\right)^2 = \left(\frac{\sigma_l}{l}\right)^2 + \left(2\frac{\sigma_T}{T}\right)^2 \tag{3.9}$$

Using these formulae, estimate the measured value of g and the experimental uncertainty σ_g.

Report your measured value of g to your AI. Measured values of g will be collected from all groups of lab partners for statistical analysis of precision and uncertainties. Appendix C reports the results of such an analysis based on data collected in Ph101 Lab 3 during a previous year. You may wish to read Appendix B to appreciate better this error analysis procedure.

A precise measurement of g, using a different technique, was made by J. E. Faller in Palmer Hall (now the Frist Student Center) room 130 for his 1962 Ph. D. Thesis. He found

that:

$$g = 980.1604 \pm 0.0007 \, \text{cm/s}^2. \tag{3.10}$$

How does your measured value compare with this? How does your uncertainty compare with the one obtained by J. E. Faller? If you repeated the experiment 100 times, what would be the uncertainty on your answer? In principle, could the uncertainty reach 10^{-6} m/s^2?

3.2.4   Dependence of the Period on Length

Measure the pendulum period with 5 different lengths l around 50, 75, 100, 125 and 150 cm. In each case, set up the pendulum, and then find the value of l, measuring from the pivot point at the top of the pendulum to the center of the ball. For each length, run Pendulum Timer for at least ten pendulum periods, record the average period T, and analyze your data for g and σ_g as in the previous part.

Enter your measurements of l and T into Excel, and use logarithmic plotting (described in section 3.1.2) to find the exponent α in:

$$T \propto l^{\alpha}. \tag{3.11}$$

Does your measurement of the exponent α agree with the expected value of 0.5 to within the error reported by Excel?

3.2.5   Dependence of the Period on Mass

Prior to the insight of Galileo that all objects fall at the same rate if air resistance can be neglected, one might have supposed that the period depended on the mass m of the hanging weight. Test this by measuring the period of the pendulum with a steel ball, an aluminum ball, and a wood ball, always using the same length of string. Are the periods you measure the same within the measurement precision?[1] *Note that if you don't use the same length of string for the different masses, you might confuse the dependence of the period on length (sec. 3.2.3) with possible dependence of the period on mass.*

3.3   The Conical Pendulum (not for credit)

A conical pendulum is one in which the mass swings around at constant speed in a (horizontal) circle, as shown in Fig. 3.2, rather than swinging back and forth in a vertical plane as in Fig. 3.1.

Using what you have learned in class, derive a formula the period T of a conical pendulum of length l at angle θ from the vertical. (Hint: first find a formula for the speed of the

---

[1]Pendulum motion is a variant on falling motion, which doesn't depend on the mass of the falling object if air resistance can be neglected. As Galileo argued, if heavier objects fell faster, then two masses tied together should fall faster than either object by itself. But, if the lighter mass of the two fell more slowly, it would "pull back" the heavier object, and the tied pair would fall more slowly than heavier mass alone.
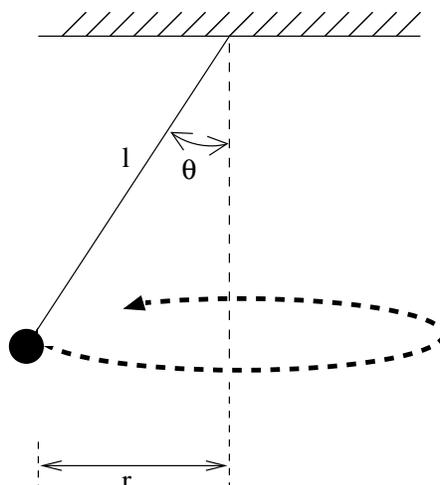
Figure 3.2: A sketch of a conical pendulum. The pendulum swings around in a horizontal circle.

pendulum, then use that to find the time it takes to go around the circle). Your result should be the same as the simple pendulum formula (3.1).

We will use the same setup as before, though we will not use the photogates. This will also be a low-precision experiment, and you need not undertake uncertainty analysis. First, set up the pendulum as before, with a length of around 50 cm. Now unclamp the pendulum *and its short horizontal supporting rod* from the experimental setup. The string should remain clamped into the rod, so that the length l of the pendulum does not change.

Notice that if you move the rod in the proper manner you can get the pendulum moving in a more-or-less circular motion. Test your formula for the pendulum period by measuring the period of circular motion for at least three angles θ.

To measure the period, use the stopwatch provided. It is difficult to measure θ directly, but you can find it by measuring the diameter of the circular motion ($2r$ in figure 3.2) by operating the pendulum near a meter stick. This isn't very accurate, but you should be able to at least qualitatively verify the period formula. In particular, your formula should predict that either the period increases with θ or it decreases with θ. Does this prediction hold true?

3.4   Appendix: Derivation of the Pendulum Formula

A simple pendulum is sketched in Fig. 3.1, and a free-body diagram of its mass is given in Fig. 3.3. Note that x is measured along the arc of motion of the mass; at the instant pictured, the x axis is at angle θ above the horizontal. The only force in the x direction is gravity:
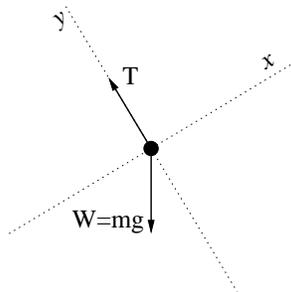
$$F_x = -mg\sin\theta. \tag{3.12}$$

Figure 3.3: Free-body diagram of the pendulum mass.

For small angles, and θ in radians, we can approximate $\sin \theta \simeq \theta$. Further noting that $\theta = x/l$, eq. (3.12) becomes:

$$F_x = -mg\frac{x}{l} . \tag{3.13}$$

Newton's second law, combined with the definition of acceleration, says:

$$F_x = ma_x = m\frac{d^2x}{dt^2} . \tag{3.14}$$

Using eq. (3.13) for F_x and canceling m on both sides of the resulting equation gives:

$$\frac{d^2x}{dt^2} = -\frac{g}{l}x. \tag{3.15}$$

This differential equation has solutions like:

$$x(t) = x_0 \sin\left(\sqrt{\frac{g}{l}}t\right). \tag{3.16}$$

Note that this goes through one cycle when the argument to the sine function is 2π, that is, when:

$$t = 2\pi\sqrt{\frac{l}{g}} \tag{3.17}$$

so this is the period of the pendulum.

Lab 4

Collisions in Two Dimensions

In this laboratory you will study two-dimensional collisions between two objects (pucks) on a horizontal, low-friction surface (an air hockey table), as sketched in Fig. 4.1.[1] Use the CCD camera to take a series of images before, during, and after the collisions, and then analyze the images to test conservation of momentum and energy and to evaluate the motion of the center of mass of the objects.
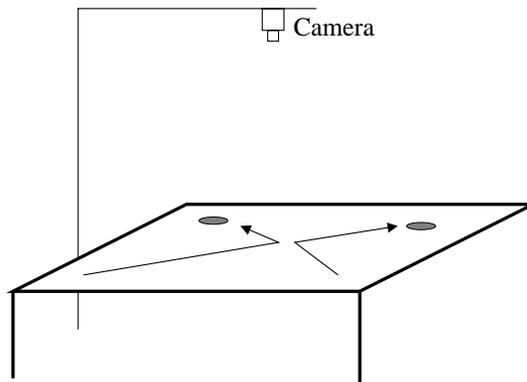
Figure 4.1: A sketch of the experimental apparatus.

4.1   Background: Two-Body Collisions

A collision between two bodies is a brief interaction between them during which the forces are very strong (but equal and opposite according to Newton's third law).

This lab involves collisions only between pucks of equal mass, which substantially simplifies the analysis.

4.1.1   Momentum Conservation

The total momentum of two masses m_1 and m_2 with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\mathbf{p}_{\text{total}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$. During a brief collision any external forces are small compared to the forces between the two masses, and the latter forces sum to zero. Newton's second law tell us that during the

---

[1]Please don't actually play air hockey with the lab equipment! The pucks break easily and are difficult to replace.

collision the rate of change of total momentum is equal to the total force, which is negligible. Hence, the total momentum is the same before and after the collision:

$$m_1\mathbf{v}_{0,1} + m_2\mathbf{v}_{0,2} = m_1\mathbf{v}_{f,1} + m_2\mathbf{v}_{f,2}, \qquad (4.1)$$

and we say that the total momentum is "conserved".

Since momentum is a vector, each component is independently conserved. Write down the equations for the momentum conservation in the x and y (horizontal) directions $m_1 = m_2$. In this lab, the vertical velocities of the pucks are always zero, so momentum is "obviously" conserved along the vertical direction.

4.1.2   Center of Mass Motion

A concept related to the total momentum of a system is its center of mass velocity, $\mathbf{v}_{\text{cm}}$, defined by

$$\mathbf{v}_{\text{cm}} = \frac{\mathbf{P}_{\text{total}}}{m_{\text{total}}} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}, \qquad (4.2)$$

for a two-body system.

Since total momentum is conserved in a collision, the center of mass velocity is unchanged during a collision.

If in addition, the total external force on the system is zero, then the total momentum and the center of mass velocity are constant at all times.

A constant center of mass velocity implies that some aspect of the system moves along a straight line at constant speed. That special property is the center of mass position vector, $\mathbf{r}_{\text{cm}}$, defined by

$$\mathbf{r}_{\text{cm}} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}, \qquad (4.3)$$

for a two-body system.

Write down the equations for horizontal coordinates x_{cm} and y_{cm} of the center of mass for a system of two equal-mass pucks. Where is the center of mass located, between or outside the pucks? What are the distances from the pucks to the center of mass in terms of the distance between the pucks?

4.1.3   Elastic and Inelastic Collisions

Mechanical energy may or may not be conserved during a collision (in contrast to the total momentum which is always conserved during collisions).

If mechanical energy is conserved, we say that the collision is "elastic" (meaning that if the pucks compress slightly during the collision, they rebound to their original shape after the collision).

The mechanical energy of a two-body system is the sum to the kinetic energies of the two masses. Hence, in an elastic collision,

$$\frac{1}{2}m_1 v_{0,1}^2 + \frac{1}{2}m_2 v_{0,2}^2 = \frac{1}{2}m_1 v_{f,1}^2 + \frac{1}{2}m_2 v_{f,2}^2. \tag{4.4}$$

Write down the simplified expression for the case of equal masses.

If mechanical energy is not conserved during a collision, we call this an "inelastic" collision. In particular, if the two pucks stitck together after the collision (sec. 4.2.5), this collision is called "totally inelastic", and

$$v_{\text{final}} = v_{\text{cm}} = \frac{m_1 v_{0,1} + m_2 v_{0,2}}{m_1 + m_2}. \tag{4.5}$$

An extreme form of inelastic collisions occurs when the objects stick together after the collision. In general, the objects then spin about their (moving) center of mass position. Nonetheless, momentum is still conserved in such inelastic collisions.[2]

4.2   Specific Instructions

The camera and software are the same as you used in Lab 2 (the bouncing ball). If needed, see that writeup for camera and image analysis instructions.

4.2.1   Setup

Turn on the two blowers to supply air to the table. Make a few trial runs at colliding two simple pucks (no magnets, no Velcro bands). *For your measurements, the collisions should not be "head-on" (1-dimensional). But while practicing, try to arrange a head-on collision between a moving puck and one at rest. What is the simple result of such a collision?* Weigh each of the pucks and record the results in your notebooks.

Now start playing with VideoPoint Capture to get a good video of a collision between your two pucks when both have nonzero initial velocity. Include a meter stick near the edge of the camera's field of view. The pucks must be visible for 4-6 frames before and after the collision. *It is better if none of the puck velocity vectors, nor* $\mathbf{v}_{\text{cm}}$, *is parallel to a coordinate axis.* (Use the Preview Screen to see if you need to try again. If so, use the <<Record button at the left of the screen.) When you get a good video, name it and store it in a file. Then open it in VideoPoint.

In the initial screen in VideoPoint, choose "2" as the number of objects whose positions you want to track. Note that the Coordinate System screen has two lines, labeled Point S1 and Point S2. These labels identify your two pucks.

You may want to make the movie screen as large as possible before making your measurements. It can be maximized by clicking on the middle button at the top right of the screen.

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[2]There is another conservation law that applies to collisions, namely conservation of angular momentum. This law can be helpful in understanding the details of inelastic collisions, but we do not pursue this here.

When you are ready to digitize the puck's positions, move the cursor to the center of the first puck in the first video frame, and click on it. Note that the cursor's label changes from Point S1 to Point S2. Click on the center of the second puck in this frame. Note that the frame advances automatically, and the label changes back to S1. Continue clicking alternately on the two pucks, and advance through the frames to the end.

Some time during your clicking, stop and turn on the Edit/Leave Trails option. You will have to do this two times (once when the cursor shows S1, and once when it shows S2). The trails for one puck will be red, and the others will be green. If you have gotten mixed up in your clicking, you can start over again by clicking on Edit/Clear All/OK.
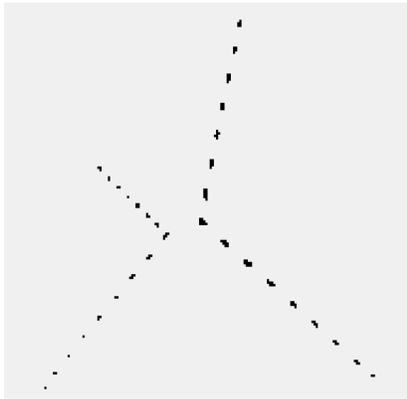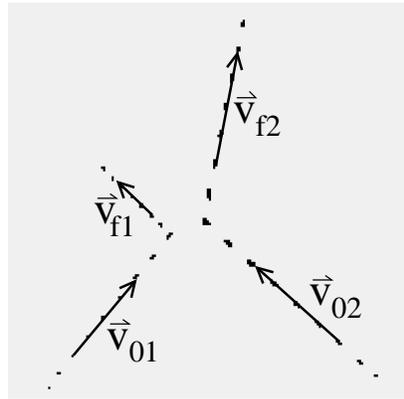

Figure 4.2a: Image of a collision.
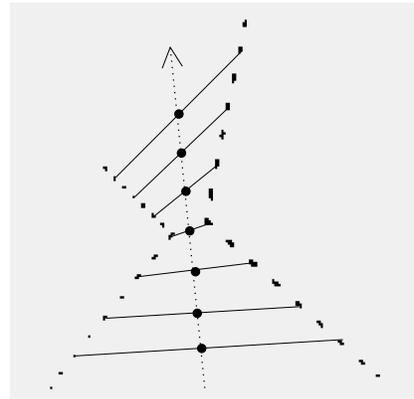
Figure 4.2b: Calculating initial and final velocities.

Figure 4.2c: The center of mass at a given time is highlighted by a dot.

4.2.2   Center of Mass Motion

When you have finished digitizing your video, the movie window will show the trails of both of the pucks, in all of the frames. If you haven't already done so, maximize the size of the movie screen. Then print out a copy of the video image. Figures 4.2a, b and c suggest how you could use the image to carry an analaysis, using a ruler to find the positions of the center of mass of the system of two equal-mass pucks in each frame.

However, it is more accurate to transfer your data as to the (x, y) positions of the pucks at the various times of the frames into an Excel worksheet. Calculate the corresponding coordinates $(x_{\text{cm}}, y_{\text{cm}})$ of the center of mass using Excel to implement eq. (4.3) (See Appendix A.3). Print plots of x_{cm} *vs.* y_{cm}, x_{cm} *vs.* t and y_{cm} *vs.* t.

If there are no external forces (and there shouldn't be in this experiment), the center of mass will travel in a straight line at constant velocity. Do the positions of your center of mass follow a straight line, and does its speed appear constant?

4.2.3   Is Momentum Conserved?

Test conservation of momentum by checking that the equations you derived relating the total momentum before and after the collision hold true. To do this, you will need to find the x and y velocities of the pucks before and after the collision. You can do this using the data you have already stored in an Excel worksheet, or go back to VideoPoint and se View/New Graph to make four time plots of the components of the velocities of both pucks. Go back to the movie window and use Movie/Scale Movie to change your data and graphs to physical units.

From your graphs, determine what the velocity components of each of the pucks were, just before and just after the collision. Then use the masses of the pucks to determine if momentum was conserved. You need not do detailed uncertainty analyses, but do check that the sum of the momenta before and after the collision agree, component by component, to within a few percent. Otherwise, look for a gross error somewhere.

Is the speed (magnitude of the velocity) of each puck the same, before and after the collision?

4.2.4   Is Energy Conserved?

Check energy conservation using the velocities you derived. Calculate the squares of the speeds, multiply by half the masses, and see if the total kinetic energy of the system is the same before and after the collision. If not, what fraction of the energy was lost? Into what other forms of energy might the lost kinetic energy have been converted?

4.2.5   Velcro Pucks and Totally Inelastic Collisions

Put Velcro straps around a pair of (non-magnetic) pucks. Now when the pucks collide they will stick together. Make a video of a collision for which the pucks rotate slowly after colliding. (See Fig. 4.3b.)

Even though the individual pucks aren't moving in straight lines after the collision, the center of mass should move in a straight line. Print out the image and use a ruler to find the center of mass in each frame (as in sec. 4.2.2). Do the points line up? Is momentum conserved, *i.e.*, is $\mathbf{v}_{cm}$ thet same before and after the collision? Is translational kinetic energy conserved, *i.e.*, does $m(v_{0,1}^2 + v_{0,2}^2)/2 = mv_f^2$?
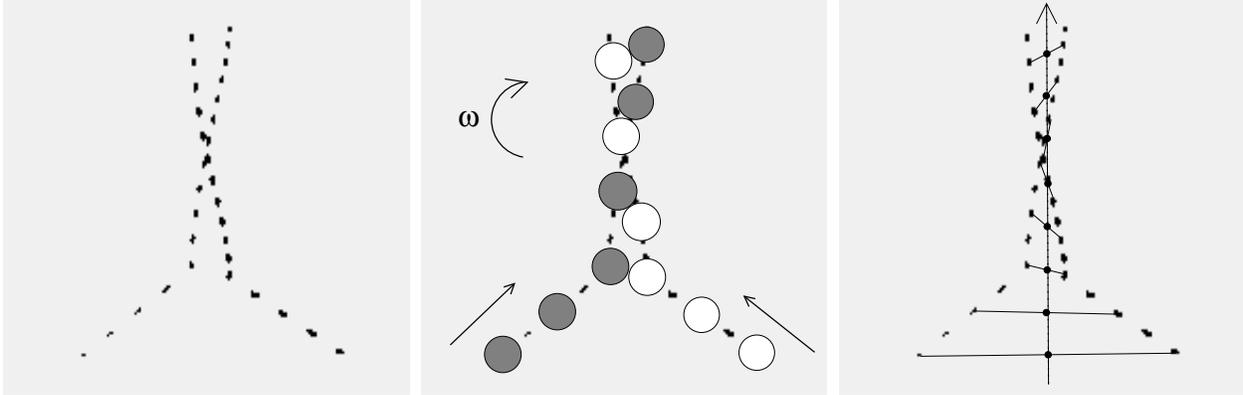
Figure 4.3a: Colliding Velcro pucks.

Figure 4.3b: Diagram of puck motion. After the collision, the pucks stick together (and slowly rotate in a clockwise direction in this example).

Figure 4.3c: Center of mass of the Velcro pucks.

4.2.6   Optional Study: Magnetic Pucks

Experience in this lab has been that a substantial fraction of energy is usually lost in collisions in which the pucks actually hit each other. This energy loss can be reduced if the pucks interact without contact. This can be done using magnetic pucks, which can repel each other without actually touching. *Note: the magnet pucks are fragile, and can be damaged if dropped or harshly handled.*

Practice launching the magnetic pucks so that they come close to each other, and their paths are changed, but they do not touch. You may need to use relatively low puck velocities, and to decrease the frame rate when taking your videos.

Once you've gotten a good video using the magnetic pucks, open it in VideoPoint and redo the conservation of momentum and conservation of energy calculations above. Is kinetic energy conserved better than before? Is momentum conserved?

Lab 5

Rotational Motion

5.1 Experimental Apparatus

In this experiment you study rotational motion and its connection with translational motion. The experimental apparatus is sketched in Fig. 5.1. A large disk (radius r_1) and a smaller disk (radius r_2) rotate together on an axle. A string is wound around the smaller disk, and a mass m is attached. As the mass falls under gravity, the pulley undergoes angular acceleration.
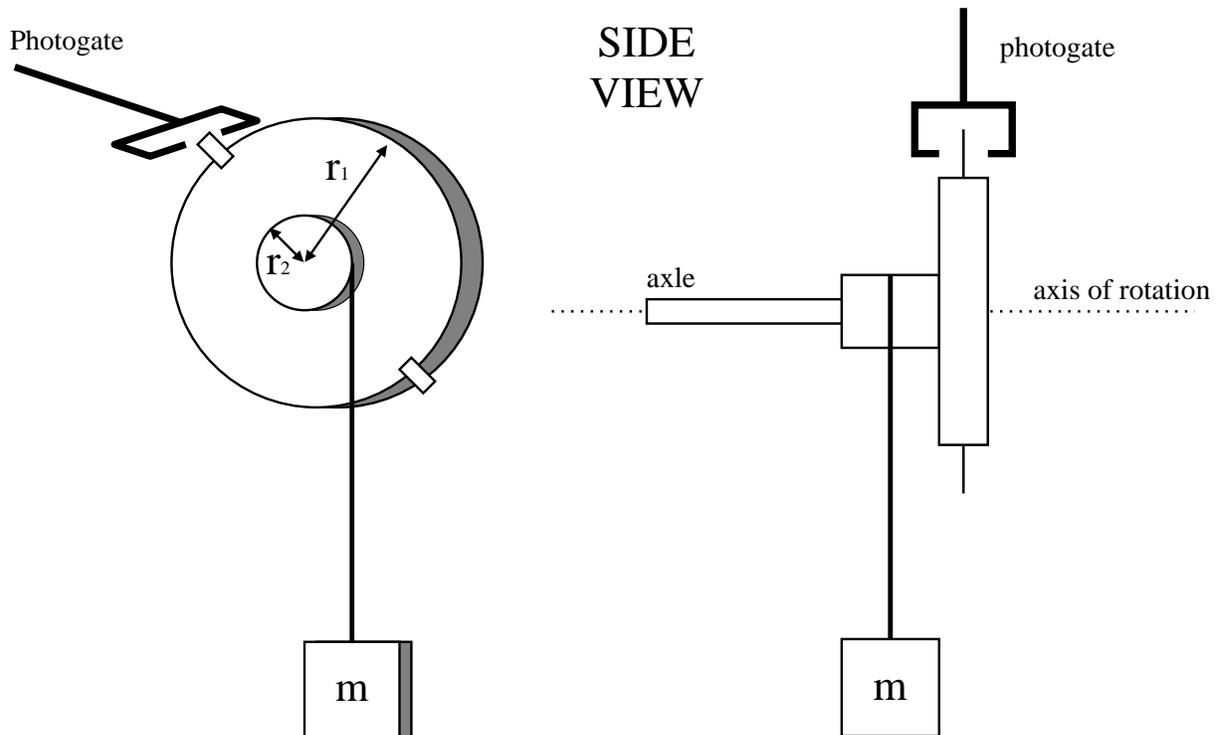

Figure 5.1: A sketch of the experimental apparatus.

In this type of apparatus, sometimes called Atwood's machine, the vertical acceleration a of mass m is much less than g, so the behavior of accelerated motion is easier for the human eye to discern.

5.2   Moment of Inertia of the Pulley

5.2.1   Background

When a body of mass m rotates about an axis, its motion can be analyzed using quantities analogous to those found useful for linear motion. Namely,

$$
\begin{aligned}
\text{position } x &\rightarrow \text{ angle } \theta, \\
\text{velocity } v &\rightarrow \text{ angular velocity } \omega, \\
\text{acceleration } a &\rightarrow \text{ angular acceleration } \alpha, \\
\text{mass } m &\rightarrow \text{ moment of inertia } I, \\
\text{force } f &\rightarrow \text{ torque } \tau, \\
F = ma &\rightarrow \tau = I\alpha.
\end{aligned}
$$

The moment of inertia I of a uniform disk is

$$I = \frac{1}{2}Mr^2 = \frac{\pi \rho r^4 h}{2}, \tag{5.1}$$

where $M = \pi \rho r^2 h$ is the mass of the disk of mass nedisty ρ, radius r and thickness h (see Cutnell and Johnson Table 9.1). According to eq. (B.13) of Appendix B, the relative uncertainty σ_I/I in the moment of inertia I is given by

$$\left(\frac{\sigma_I}{I}\right)^2 = \left(\frac{\sigma_M}{M}\right)^2 + \left(2\frac{\sigma_r}{r}\right)^2 = \left(\frac{\sigma_\rho}{\rho}\right)^2 + \left(\frac{4\sigma_r}{r}\right)^2 + \left(\frac{\sigma_h}{h}\right)^2. \tag{5.2}$$

5.2.2   Specific Instructions

The actual pulley (Fig. 5.1) consists of two disks of masses M_1 and M_2 with radii r_1 and r_2, respectively. The total moment of inertial I of this pulley is

$$I = I_1 + I_2 = \frac{1}{2}M_1 r_1^2 + \frac{1}{2}M_2 r_2^2 = \frac{\pi \rho}{2}(r_1^4 h_1 + r_2^4 h_2). \tag{5.3}$$

Dismount the (aluminum) pulley and measure its outer radius, r_1, and thickness h_1. Also measure the radius, r_2, and thickness, h_2, of the smaller disk, and then calculate the moment of inertial I using eq. (5.3), taking the density of aluminum to be

$$\rho = 2.70 \pm 0.01 \text{ g/cm}^3. \tag{5.4}$$

Estimate the uncertainty σ_I using eq. (5.2).

Does the moment of inertia of the smaller disk make a significant contribution to the total moment of inertia I of the pulley?

5.3 Measuring I from Rotational Dynamics

5.3.1 Background

An alternative method of measuring the moment of inertia I of the pulley uses its acceleration by a falling mass. The mass is attached to a string, which is wrapped around the smaller part of the pulley (radius r_2), as shown in Fig. 5.1.

Analysis of the system is straightforward. First consider the pulley. The tension T of the string exerts a torque[1]

$$\tau = Tr_2 \tag{5.5}$$

about the axle of the pulley. This torque leads to an angular acceleration α of the pulley, related to torque by

$$\tau = I\alpha. \tag{5.6}$$

Combining these gives

$$I = \frac{Tr_2}{\alpha}. \tag{5.7}$$

Next consider the falling mass m. The forces on mass m are tension and gravity, so Newton's second law says:

$$mg - T = ma, \tag{5.8}$$

where g is the gravitational acceleration constant and a is the acceleration of the mass. Since the string is wrapped around the pulley at radius r_2:

$$a = \alpha r_2. \tag{5.9}$$

Combining these gives an expression for tension:

$$T = m(g - \alpha r_2). \tag{5.10}$$

Substituting this into eq. (5.7) yields:

$$I = \frac{mgr_2}{\alpha} - mr_2^2. \tag{5.11}$$

For an estimate of the uncertainty σ_I on the moment of inertia I, note that eq. (5.11) can be rewritten using eq. (5.9) as

$$I = mr_2^2 \left(\frac{g}{a} - 1\right) \approx \frac{mgr^2}{a} = \frac{mgr_2}{\alpha}, \tag{5.12}$$

where the approximation holds since $a \ll g$. Then, using eq. (B.13) we have

$$\left(\frac{\sigma_I}{I}\right)^2 \approx \left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_\alpha}{\alpha}\right)^2 + \left(\frac{\sigma_{r_2}}{r_2}\right)^2. \tag{5.13}$$

---

[1]Clockwise torques and downward translational motion are taken to be positive.

5.3.2 Specific Instructions

Use a falling mass of $m \approx 200$ g to measure the moment of inertia of the pulley. Measure the value of m with a scale. Wrap the string around the smaller disk in a single layer, avoiding the groove. For precision results, measure the diameter of the string with the Vernier caliper,[2] and increase your previous measurement of the radius r_2 of the smaller disk by the radius of the string to obtain the lever arm through which the torque acts.

Use a photogate and the program Princeton Timer A to measure the angular acceleration α of the pulley. Two tabs are attached to the larger pulley; they will cause the photogate to be blocked at intervals of π radians. Make sure that both tabs block the timer gate.

The system will record the times when the photogate is blocked. Thus, each time reported by the computer is the time it takes the pulley to turn an integer multiple of π radians.

The string should be wound in a single layer on the small pulley. Start the Timer program before releasing the mass (recall sec. 1.2.2), and stop the Timer before the mass hits the ground. Leave the Timer on long enough to record at least 5 time intervals.

The angular motion of the pulley due to the constant angular acceleration α is described by

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2. \tag{5.14}$$

You can find both α and σ_α by fitting a curve to a plot of θ versus t. Use Excel and WPTools with a Polynomial of Order 3. The fit coefficient a_3 should be 0 to within 1-2 times the reported error SE(a_3). Then, $\alpha = 2a_2$ and $\sigma_\alpha = 2$ SE(a_2).

Use your measurement of α and eq. (5.11) to calculate the moment of inertia I of the pulley. Calculate the uncertainty σ_I using eq. (5.13).

Compare this value of I with the value (5.3) you calculated from the dimensions of the pulley. Do they agree within the larger of the uncertainties σ_I obtained from eqs. (5.2) and (5.13)?

5.4 The Block and Tackle

In this part of the lab, you analyze the block-and-tackle system sketched in Fig. 5.2. First examine the system in equilibrium, then analyze the kinetic and potential energies of the system in motion.

Before setting up the block-and-tackle system, wind the string used in the first part of this lab around the pulley and secure it with a piece of tape. Please don't remove it.

Construct the system as shown in the figure. The "fixed" pulley is the one used in the first part of this lab. The "moving" pulley should have about 100 g as a load. Measure the mass m_2 of the moving pulley (including its load) before assembling the system. Attach a string to the axle of the fixed pulley (or better, to a nearby piece of the supporting structure),

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[2]Lengths can be measured to 0.1 mm using the Vernier caliper. Consult with your AI if you are unfamiliar with this technique.
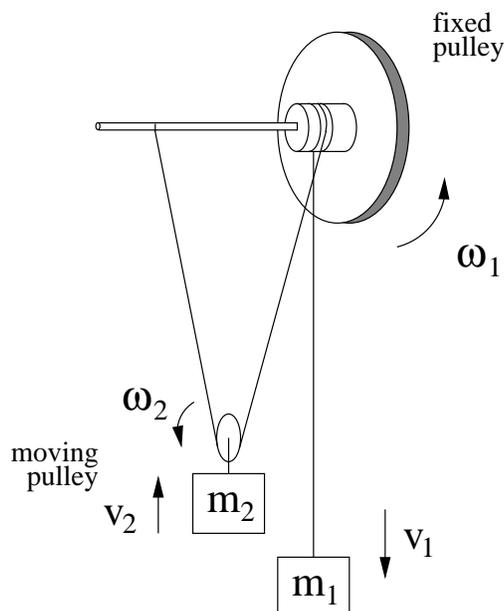
Figure 5.2: A sketch of the black-and-tackle system

pass it around the moving pulley, back up over the fixed pulley, and down to the hanging mass. The string should be long enough that when the hanging mass (m_1) is near the fixed pulley the moving pulley (m_2) is close to the floor. The string should loop around the groove in the hub of the fixed pulley, and it should not slip when the pulley and the masses move. Loop it around twice if necessary to avoid slippage. Also be careful that the string freely winds and unwinds without tangling.

5.4.1   Static Equilibrium of the Block and Tackle

Adjust the mass of the hanging weight m_1 until the system is in static equilibrium. Keep adding weights until the hanging mass would fall if only 1–2 g more were added. Remove the hanging mass and measure m_1. What relation do you expect between m_1, m_2, and the tension in the string? Do your measured values of m_1 and m_2 agree with this expectation?

5.4.2   Dynamics and Conservation of Energy of Block and Tackle

Background

Consider what happens if m_1 is increased, so the system is no longer in equilibrium. Mass m_1 will fall downwards, causing the moving pulley to move upwards and both pulleys to rotate. The motion can be analyzed in terms of forces and torques, but it turns out that an energy analysis is simpler.

The kinetic energy KE of the system at any instant is:

$$\text{KE} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2. \tag{5.15}$$

The moment of inertia of the moving pulley, I_2, is sufficiently small that the last term may be ignored. The other terms can be simplified by noting that:

$$v_1 = \omega_1 r_2 \quad \text{and} \quad v_2 = \frac{1}{2}\omega_1 r_2, \tag{5.16}$$

where r_2 is the radius of the fixed pulley at the point the string is wrapped, including the radius of the string as in sec. 5.3.2. (Why is there a factor of $1/2$ in expression for v_2?) Inserting these expressions into eq. (5.15) and simplifying gives:

$$\text{KE} = \frac{1}{2}\left[\left(m_1 + \frac{1}{4}m_2\right)r_2^2 + I\right]\omega^2 \tag{5.17}$$

Now consider the change in potential energy between two times. This is just the change in mgh for both masses, but here we will measure θ instead h. If the fixed pulley rotates by an angle of $\Delta\theta$, mass 1 falls by height $h_1 = r_2\Delta\theta$, while mass 2 rises by height $h_2 = r_2\Delta\theta/2$ (do you see why?). The change in potential energy is then:

$$\Delta\text{PE} = m_1 g h_1 - m_2 g h_2 = \left(m_1 - \frac{1}{2}m_2\right)g\, r_2\Delta\theta. \tag{5.18}$$

If friction in the system can be ignored, then the kinetic energy (5.17) at any moment should be equal to the change in potential energy (5.18) since the beginning of the motion.

Specific Instructions

Add 150–200 g to the hanging mass and re-attach it to the system. Release the mass and record the time intervals of successive half-turns of the fixed pulley, using the same procedure as in the first part of the lab. Be sure that you start the timer after the mass has started falling, and stop the timer before the mass hits the floor.

To calculate the initial and final kinetic energies using eq. (5.17), you need to know the initial and final angular velocities of the pulley. The measurements reported are the times it takes the pulley to turn by successive multiples of π radians. Thus the average angular velocity in any interval is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi}{\Delta t}. \tag{5.19}$$

Use the time measurement for the first time interval reported by Princeton Timer to calculate ω_0, the average angular velocity during this interval, which is equal to the angular velocity at the midpoint of this time interval. Then use eq. (5.17), along with your measurements of m_1, m_2, r, and I from the first part of the lab to find the kinetic energy of the system at the middle of the first time interval. Similarly, use Δt from the last time interval to calculate ω_f and the kinetic energy at the midpoint of this time interval.

If the data set contains N numbers, then the pulley rotated a total of $\Delta\theta = (N-1)\pi$ radians between the midpoints of the first and last time intervals. Use this in eq. (5.18) to calculate the change ΔPE in potential energy between the midpoints of these time intervals.

Does your measurement of ΔPE agree with the difference between initial and final kinetic energy? You need not perform a detailed uncertainty analysis, but if the numbers don't agree within 5–10%, look for a gross error somewhere.

5.5   Angular Momentum Toys

Sometime during the lab period, go to the back of the room where several angular momentum toys are set up. Take a ride on the rotating stool, pulling the dumbbells in and out to change your moment of inertia. Also try twisting the spinning bicycle wheel while sitting on the stool.

Try pouring water into the funnel attached to a copper tube gadget. What is the resulting motion of the system? Is angular momentum conserved in this device?

Lab 6

Springs and Simple Harmonic Motion

6.1   Background

Springs provide a useful model for many physical systems that exhibit oscillatory behavior. The key characteristic of springs is that, when stretched or compressed by an amount Δx from their relaxed position, they exert a force proportional to Δx:

$$F = -k\Delta x, \tag{6.1}$$

where k is the spring constant. If a mass, m, is attached to one end of the spring, it will oscillate with angular frequency:

$$\omega = \sqrt{\frac{k}{m}}. \tag{6.2}$$

The corresponding period is:

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{6.3}$$

In the first part of the experiment you will verify these equations. In later parts you will analyze systems with more than one spring.

6.2   Specific Instructions

The setup for the first part of the lab is shown in Fig. 6.1. A spring connects a glider of mass M to one end of an air track. A hanging mass, m, is attached to the other end of the glider via a string (actually a piece of magnetic tape).
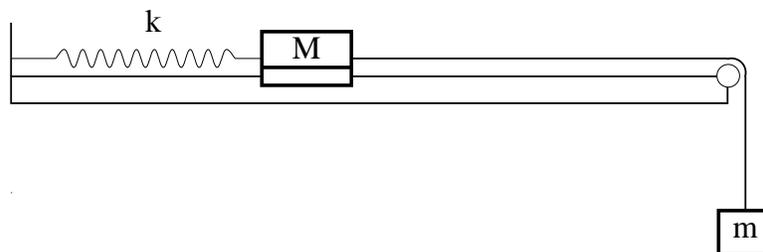

Figure 6.1: Equipment setup for the first part of the experiment.

6.2.1   Equilibrium

Analyze the system in equilibrium and find the equation which relates the mass, the spring constant and the displacement.

Measure the equilibrium positions, x, of the glider with an empty hanger and also with at least four different weights. Measure the masses of the hanging weights on a balance or digital scale for precision results.

Fit a line to a plot of m versus x. Use the equation you previously derived to extrapolate k and its uncertainty, σ_k, from the fit parameters. Use $g = 9.8\,\mathrm{m\,s^{-2}}$ where needed.

6.2.2   Oscillations of a Single Spring

If you displace the glider and release it, it will oscillate back and forth on the air track. Set up a photogate so that it is blocked twice per cycle (once for each direction of motion). Now you can use Pendulum timer to measure the oscillation period. For each measurement you make, find the period for several cycles of the motion, and use the average period. Just use the Stat (for Statistics) button located above the graph.

The period of oscillation of the glider should be:

$$T = 2\pi\sqrt{\frac{m_{\text{total}}}{k}} = 2\pi\sqrt{\frac{m + M}{k}}, \tag{6.4}$$

independent of the amplitude of the oscillation (just as the period of a pendulum was independent of amplitude). Test this by recording the period of oscillation for glider displacements of around 4, 6, and 8 cm. Use larger displacements if necessary to avoid the effects of damping. Use the same hanging mass m for each of these trials. Do you measure the same period within the experimental uncertainty?

Next test the dependence of T on mass $m + M$. Measure the period of oscillation with the hanger empty, then with 20 g of additional mass, and finally with 40 g of additional mass.

Compare these measured periods with periods calculated using eq. (6.4).

Another useful comparison can be made by rewriting eq. (6.4) as

$$k = 4\pi^2 \frac{m + M}{T^2}. \tag{6.5}$$

Use this to calculate three values of the spring constant k from your three measurements of period T as a function of mass M. For a simple statistical analysis, take your best estimate of k to be the average of the three values, and the uncertainty to be $\sigma_k = |\mathrm{Max}(k) - \mathrm{Min}(k)|/2$. Compare these results with those you obtained in part 6.2.1.

6.2.3   Two Springs

Set up the glider with two identical springs as shown in Fig 6.2. What is the effective spring constant of the combination of springs? What is the expected period?
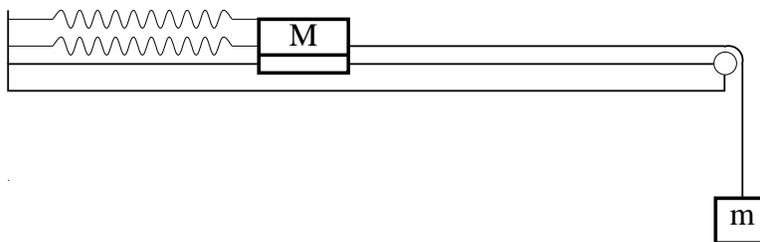
Figure 6.2: A glider connected to the air track via two springs.

Measure the period of the system. Does it agree with what you expected? You need not do a detailed uncertainty analysis, but the numbers should be in the same ball park. If not, repeat sec. 6.2.1 to verify the spring constants of the springs that you used.

6.2.4   Three Springs, Two Masses

Set up two gliders and three identical springs as shown in Fig. 6.3. Displace one of the gliders from its equilibrium position while holding the other fixed. Then let them both go at the same time. The resulting motion can be quite confusing!
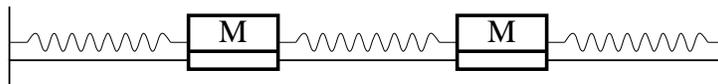

Figure 6.3: Two gliders and three springs.

You need not analyze the motion of the gliders in its full complexity. Instead, you will study two special cases of motion called 'normal modes, in which all movement takes place it a single frequency. It turns out that any one-dimensional system of springs that includes N masses has N such modes. Thus the two-glider system has two normal modes.

The first normal mode occurs when the gliders move but the distance between them does not change. Displace both gliders by the same amount and in the same direction, and release them. The middle spring should keep a constant length, and the gliders should oscillate with the same period. Measure the period of oscillation of this mode (T_1) using the photogate aligned on one of the gliders.

The other normal mode occurs when the gliders move in opposite directions with equal amplitudes. Displace one glider to the right and the other to the left by the same amount, and release them. Again the gliders should oscillate with the same period. Measure this period (T_2). Does the center of mass of the gliders combination move?

Compare your measured periods with those deduced in sec. 6.2.5 below. You don't need to do a quantitative analysis of the uncertainty, but your measured periods should be close to those calculated in the analysis.

If you start the system by holding one glider fixed at its equilibrium position and displacing the other glider, you will create a motion that is a combination of the two modes studied above. With time, one or the other of the two modes may be more apparent, in an alternating pattern. However, if you are not attuned to the presence of the two modes, the motion may appear to be chaotic (a technical term meaning behavior that appears to be complex but which nonetheless obeys simple laws of physics).

6.2.5   Analysis of Normal Modes for 3 Springs and 2 Masses

During the first normal mode the force from the middle spring never changes since its length never changes. Therefore, the changing force on, say, the left glider is only due to the change in length of the leftmost spring. If k is the spring constant of each of the springs, then the force on the left glider is:

$$F_L = -k\Delta x_L, \tag{6.6}$$

where Δx_L is the displacement of the left glider. Thus the period of oscillation in the first normal mode is:

$$T_1 = 2\pi\sqrt{\frac{M}{k}}. \tag{6.7}$$

During the second normal mode the left glider experiences changing forces from both the left and middle springs. After a displacement Δx_L the leftmost spring exerts force $-k\Delta x_L$ on the left glider. Meanwhile the middle spring exerts force $-k(2\Delta x_L)$ on the left glider since the middle spring is compressed (or stretched) twice as much as the leftmost spring is stretched (or compressed). Note that the direction of the force on the leftmost glider is the same for both springs, so the net force on the left glider is:

$$F_L = -3k\Delta x_L. \tag{6.8}$$

Thus the "effective" spring constant is $3k$ and the period of this mode is:

$$T_2 = 2\pi\sqrt{\frac{M}{3k}}. \tag{6.9}$$

Lab 7

Fluids

7.1 Flow of a Viscous Fluid in a Circular Pipe

7.1.1 Background

It is a remarkable fact that fluid immediately adjacent to an immobile surface, such as the wall of a pipe, always has zero velocity. For fluid some distance y from the surface to flow at velocity v, a force must be applied:

$$F = \frac{\eta A v}{y} \tag{7.1}$$

where A is the area of the surface (or, equivalently, the area of the layer of fluid), and η is the coefficient of viscosity.

Fluid flow through a circular pipe is slightly more complicated. The flow rate, Q, (= volume of fluid passing any cross section of the pipe each second) through a circular pipe of radius R and length L is given by Poiseuille's law:

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L} \tag{7.2}$$

where ΔP is the difference in pressure at the ends of the pipe. The R^4 dependence is dramatic: a small reduction in R leads to a large reduction in the flow rate in the pipe (a blood vein, for example).

If v is the flow velocity at the center of the pipe, then the flow rate is $Q = \pi R^2 v/4$, where the factor of 4 arises due to the variation of velocity with distance from the wall of the pipe. From eq. (7.1), $v = FR/\eta A$, where the force F is the pressure difference times the cross-sectional area, $F = \pi R^2 \Delta P$, and the area A of the surface of a length L of the pipe is $A = 2\pi RL$. Combining these facts, we arrive at Poiseuille's law (7.2).

7.1.2 Specific Instructions

Use the apparatus shown in Fig. 7.1 to test Poiseuille's law and to measure the viscosity of a fluid. The fluid is a heavy machine oil, which fills the large vertical cylinder. Its weight produces the pressure at the bottom of the cylinder and, therefore, at one end of the small horizontal tube. The other end of the horizontal tube is at atmospheric pressure. Thus the pressure difference across the length of the small tube is $\Delta P = \rho g h$, where h is the height of the fluid above the outlet of the (possibly curved) tube.
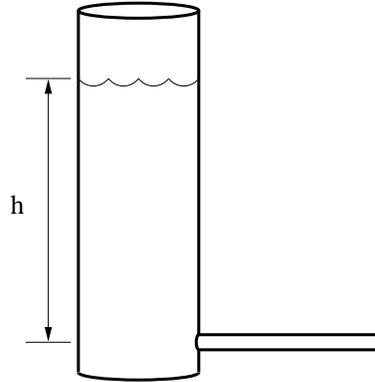
Figure 7.1: Apparatus for the study of Poiseuille's law. The vertical cylinder is partly filled with oil. It is open to the atmosphere at the top. The height h is to be measured between the liquid level and the outlet of the tube.

Measure the mass of an empty graduated cylinder

Measure the flow rate in each of the three available tubes (radii 0.370, 0.307 and 0.242 cm), using a stopwatch and a graduated cylinder. *Let the oil drop directly to the bottom of the cylinder so the height of the oil surface gives an accurate measure of the volume of the oil.*

Measure the mass of the cylinder plus oil, and read the volume of the oil in the cylinder to determine the density ρ of the oil.

Hints: Keep the small tube horizontal to minimize the effect of gravity on the flow. Measure the height of the fluid in the vertical cylinder before and after the oil flows out, and use the average value. From which point should the height be measured?

7.1.3   Analysis

First use your data to test the assertion that Q is proportional to R^4. Although it isn't strictly true, you may assume that each tube has the same length L. Then you can write eq. (7.2) as:

$$Q = \text{Constant} \times R^\alpha \tag{7.3}$$

You want to check that the exponent α is 4. Do this by analyzing your measurements of Q using logarithmic plotting, as discussed in sec. 3.1.2.

Next, determine the value of the viscosity η of the oil. For this part of the analysis, assume that the exponent $\alpha = 4$. Rework the Poiseuille's equation to extract the value of the coefficient of viscosity, and use your three measurements of Q to calculate three values of η. Are the values close to each other? Formal uncertainty analysis isn't required, but look for a procedural error if the measurements are grossly different.

7.2   Terminal Velocity

7.2.1   Background

An object falling through a viscous fluid feels three forces.

Gravity pulls the object downward:

$$F_{\text{grav}} = mg = \rho V g, \qquad (7.4)$$

where ρ and V are the density and volume of the object, respectively, and g is gravitational acceleration.

The buoyant force pushes the object upward:

$$F_{\text{buoy}} = \rho_{\text{f}} V g, \qquad (7.5)$$

where ρ_{f} is the density of the fluid. Finally, there is a drag force which acts to oppose the motion of the object.

The drag force on a spherical object of radius R moving with velocity v is given by Stokes' law:

$$F_{\text{drag}} = 6\pi \eta R v, \qquad (7.6)$$

where R is the radius of the sphere.

Stokes' law (7.6) is a variant on the basic rule (7.1). The area of the sphere is $A = 4\pi R^2$, its volume is $V = 4\pi r^3/3$, and the $y = 2R/3$ is the distance from the surface of the sphere at which the surrounding fluid is hardly perturbed by the motion of the sphere.

When these three forces balance, no net force acts on the sphere, so it falls with constant velocity, called terminal velocity. Combine eqs. (7.4)-(7.6) for the three forces acting on the sphere to derive an expression for the terminal velocity.

7.2.2   Specific Instructions

Test the equation you just derived by measuring the terminal velocity of small lead spheres falling through the oil you analyzed in the first part of the experiment. The density of lead is $\rho = 11.4 \,\text{g}\,\text{cm}^{-3}$.

Measure the diameter of one of the spheres, taking an average of several measurements if it isn't really spherical. Measure the velocity of the sphere falling through the oil using a stopwatch. Repeat the experiment for at least three different spheres. Are the measured values close to the values predicted by your equation?

Solve for the viscosity η and compare with your result from sec. 7.1.

7.3   Buoyant Force

7.3.1   Background

The density of gas in a helium balloon is less than the density of the surrounding air ($\rho_{\mathrm{He}} = 0.165P\,\mathrm{kg\,m^{-3}}$ for pressure P in atmospheres, $\rho_{\mathrm{air}} = 1.29\,\mathrm{kg\,m^{-3}}$ at 1 atm pressure), so the balloon feels a net upward force. The buoyant force (7.5) can be balanced by hanging a mass below the balloon, as shown in Fig. 7.2.
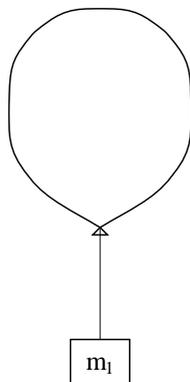

Figure 7.2: Apparatus for the study of buoyant force.

The total weight is:

$$W_{\mathrm{total}} = \left(m_1 + m_{\mathrm{string}} + m_{\mathrm{balloon}} + m_{\mathrm{He}} \right) g \tag{7.7}$$

where m_1 is the mass hanging below the balloon, m_{string} is the mass of the string, m_{balloon} is the mass of the (empty) balloon, and m_{He} is the mass of the helium within the balloon.

The masses of the balloon, string, and hanging weight can be measured on a precision scale, and $m_{\mathrm{He}} = \rho_{\mathrm{He}} V$ where V is the volume of the balloon.

7.3.2   Specific Instructions

Measure the mass of the empty balloon. Fill it with helium, and after stopping the flow of helium, measure the pressure within the balloon before tieing off the end of the balloon. *You may need the following conversion factors: 1 atm = 15 psi = 1.013×10^5 Pa. Note that the gas regulator measures* gauge *pressure $P_g = P - P_{\mathrm{atm}}$ rather than the absolute pressure P.*

Next measure the volume of the balloon. One way of doing this is to put it on a table, hold a meter stick vertically next to it, and use a wooden board to help measure its size on the meter stick (see Fig. 7.3). If d_1 is the size of the balloon in its longest dimension and d_2 is the size from side to side, its volume is approximately:

$$V = \frac{4}{3}\pi \left(\frac{d_1}{2} \right) \left(\frac{d_2}{2} \right)^2. \tag{7.8}$$
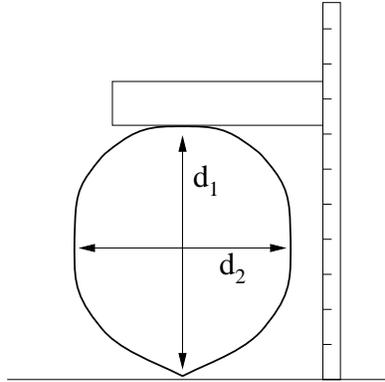
Figure 7.3: Measuring the dimensions of a balloon.

Cut a piece of string 1-2 feet long, measure its mass and tie it to the bottom of the balloon. Finally, tie a 5 g hanger to the string and keep adding weights to the hanger until the balloon is in equilibrium. To fine-tune the hanging weight, you may want to use small paper clips (about 0.3 g each) or pieces of tape. After you have achieved equilibrium, detach the hanger and its weights, and measure their total mass on a scale.

Now you have all data you need to test the buoyancy formula. Calculate the buoyant force and the weight. Are they close to each other?

Lab 8

Speed of Sound and Specific Heats

8.1 Safety

Tanks of various gases will be available in the lab. Choose just one of these for your measurement of the speed of sound and, of course, use the same gas for measurement of the density. The tanks contain gas at high pressure:

Do NOT try to move the tanks; if it is absolutely needed, ask the assistance of an instructor.

Every tank has three valves: the main valve on the throat of the bottle, which allows gas to enter a large reduction valve in the regulator, which bleeds the gas slowly out of a small needle valve into the rubber tubing.

You need to use only the needle valve. The shutoff valve and the regulator valve have already been adjusted: do NOT tamper with them.

The pressure has been set high enough to give a flow that will fill your flask in a reasonable time.

If someone should increase the pressure it could cause a flask to blow up! Do NOT touch the regulator valve.

As you open the needle valve check for a gentle breeze of gas flowing out of the rubber tube when held near your lips (the "lip test"). Keep the needle valve closed except when you need some gas flow.

8.2 Velocity of Sound

In this lab you will measure the velocity of sound in a gas, and you will weigh the gas in order to find its density. These measurements will lead to a determination of C_P/C_V, the ratio of specific heats at constant pressure and constant volume, and thus allow you to infer the molecular structure of the gas. Measurements of macroscopic lengths, masses, and forces provide insights on molecules of size $\approx 10^{-10}$ meters and mass $\approx 10^{-23}$ grams!

The speed v of longitudinal sound waves in gases is given by:

$$v = \sqrt{\frac{\gamma P}{\rho}}. \tag{8.1}$$

where ρ is the gas density in kg/m^3, P is the gas pressure in Pascals, and γ is the ratio of the specific heat capacity at constant pressure C_P to the specific heat capacity at constant

volume C_V:

$$\gamma = \frac{C_P}{C_V}. \tag{8.2}$$

The kinetic theory of gases predicts that $\gamma = \frac{5}{3}$ for an ideal monoatomic gas and $\gamma = \frac{7}{5}$ for an ideal diatomic gas. At standard temperature and pressure, $\gamma_{\text{Ar}} = 1.66$, $\gamma_{\text{CO}_2} = 1.29$ and $\gamma_{N_2} = 1.40$.

Using the ideal gas law,

$$PV = nRT, \tag{8.3}$$

where the gas constant is $R = 8.31$ J/mole-°K, we can express the gas density as

$$\rho = \frac{m}{V} = \frac{Mn}{V} = \frac{MP}{RT}, \tag{8.4}$$

where M is the molar mass. What density does the ideal gas law predict for gases Ar, CO_2 and N_2 at standard temperature and pressure? What is the velocity of sound for these gases according to eq. (8.1)?

If the sound wave has a pure frequency f and wavelength λ, then the velocity of sound is related by

$$v = f\lambda. \tag{8.5}$$

8.2.1   Resonance

The apparatus for measuring the speed of sound is a vertical cylinder of gas (see Fig. 8.1). The length of the column of gas is adjusted by changing the height of a metal reservoir of oil. The gas volume is interconnected to the top of the oil reservoir so that the gas volume does not change when the reservoir is raised or lowered.

A loudspeaker is attached to the top of the cylinder, and it generates sound from an electrical audio oscillator. The loudspeaker also responds to sound vibrations (like a microphone) and thus the sound waves set up in the cylinder of gas affect it also. This action is detected by a meter. When the cylinder is in "resonance" you will get a large reading on the meter (and you may be able the hear that the output of the loudspeaker is louder).

A resonant condition occurs when a pattern of longitudinal standing waves (sound waves) is established in the gas column. This will happen only for a certain set of the lengths of the gas column when the loudspeaker is driven at a pure frequency f. At the resonance, the vertical vibration of the gas molecules is maximal (an antinode) at the tube end where the loudspeaker is located. Of course, the vibration of the gas molecules is zero (a node) at the liquid surface in the tube.

Deduce an expression which relates the height h of the gas column to the wavelength λ of the sound at the resonances. Note that the resonating wave within the column has a node at the surface of the oil and an antinode at upper, open end of the column. What is the spacing Δh between neighboring resonances? Use eq. (8.5) to relate the speed v of sound to the spacing Δh and the frequency f at the resonances.
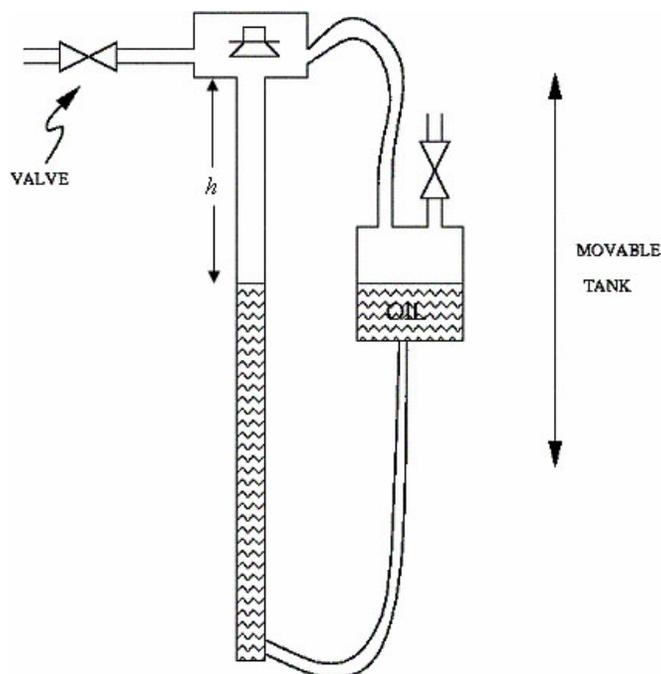
Figure 8.1: Apparatus for measuring the speed of sound.

8.2.2   Specific Instructions

As a preliminary step, turn on the oscillator at a convenient frequency ≈ 800 Hz. Lower the oil reservoir top down about 60 cm while watching the meter and listening to the sound. You should be able to find several positions of the oil level at which resonance occurs.

The meter reading should be near the maximum of its scale at resonance – if it goes off scale reduce the output level of the oscillator. You might find that a different frequency (10-20%) will give a stronger indication of resonance. Explore a bit.

You are now ready to fill the cylinder with gas from one of the tanks. Note that the gas can be fed into the apparatus in a way to flush out the gas previously there. Raise the oil as high as possible during flushing to facilitate this. Be sure to open and close the valves in the right order so that the pressure in the tube is not raised above atmospheric pressure. Lower the oil level from the top and note the approximate location of the first three resonances. Then raise the oil slowly through each resonance. *Upward motion makes is easier to determine the level of oil as you will discover on trying it both ways.* Find the position of resonance with the maximum possible accuracy.

Use the relation you previously derived to the speed of sound from the spacing Δh between resonances and the frequency f of the audio oscillator.

Tip: a good way find a resonance position is for one person to slowly raise the oil level while another records the meter readings. The maximum can be found by graphing the meter reading versus position.

Also explore the resonances keeping the height h of the oil constant while varying the

frequency f fo the oscillator. What is the lowest resonant frequency for the height that you have chosen?

8.3   Density of the Gas

Find the gas density ρ weighing a known volume of the gas at atmospheric pressure. Two flasks of equal volume are provided, one coated with plastic and the other clear.

Determine the useful volume of the coated flask by filling the clear flask with water to the level of the rubber stopper in the coated flask. Measure the volume of water.

Evacuate the coated flask with the vacuum pump. You can tell when this is achieved when a sharp clacking sound of the pump replaces the initial slurping sound.

Weigh the evacuated flask. *Be careful not to drop it. The plastic coating is to minimize hazard of breakage.*

Then fill the coated flask with gas at one atmosphere pressure, using the procedure described below, and reweigh.

- To fill the flask, open the needle valve, repeat the lip test (sec. 8.1), and when a gentle flow is found attach the hose to your flask and open the stopcock. The flask is full to over one atmosphere pressure when the hissing noise at the needle valve stops. Shut the stopcock. Shut the needle valve and remove the tubing. Open the stopcock slightly and gas will hiss out allowing the pressure to drop to atmospheric. *If you leave the stopcock open too long, ordinary air will mix with the gas in the flask, thereby reducing its purity.*

- Weigh the filled flask. After a minute, open the stopcock again briefly and reclose. Reweigh. Repeat after another minute. Reweigh. If the weight has not changed again you are finished. If it has decreased, repeat again. Why does the weight change at all?

- You must measure the change in weight (a few grams) between the evacuated and filled flask, which can be done by moving the rider on the balance. *Be sure that the weights in the balance pan are not changed while you fill the flask with gas.*

8.4   What Gas Did You Use?

The tanks are labeled so that they can be distinguished, but the labels indicating what gases they contain are hidden. Sometime into the lab your AI will provide you with a list of gases in the tanks. Can you determine which gas you used only on the basis of your measurement of the gas density, or only on the basis of your measurement of the velocity of sound?

Use eq. (8.1) and your measurements of ρ and v to determine $\gamma = C_P/C_V$ for your gas. Was your measurement accurate enough to distinguish between monatomic, diatomic, and polyatomic gases?

Appendix A

Data Analysis with Excel

Computers are used for data analysis in any modern physics laboratory, and the Physics 101 lab is no exception. We have built our data analysis system around the program Excel, which is widely used on and off campus. We've added some Workshop Physics (WP) tools to make graphing data easier, and to let you do regression calculations with uncertainties, but otherwise we are using the standard, off-the-shelf software.[1]

If you are already familiar with Excel, great! If not, we'll give brief instructions here. Like any software, it can be confusing at first, so don't hesitate to ask your instructors and your fellow students for help. Play around with the program a bit to get comfortable with it.

A.1   Starting things up

- If the computer isn't already on, turn it on and wait for it to boot up.

- If the Physics 101 window isn't already open, double click on the Physics 101 icon to open it

- Double click on Excel with WPtools (look for the X logo) to get the program running.

A.2   Entering Data: a Simple Example[1]

When Excel is started up, you need to open a spreadsheet to work in. If you are asked if you want to reopen WPtools, click No. Then go to File→New and click on OK to open a new Workbook. (If you wanted to open a pre-existing spreadsheet, you would use the File→Open menu command; if you want to save the new spreadsheet, use the File→Save menu command. Since we'll be working with fairly small datasets, neither of these is really necessary for your lab work.)

Suppose you wish to record some (x, y) data pairs in two columns of a spreadsheet. Go to Excel, and start entering the data in the upper left most cell (called A1). To do this, move the cursor to this cell and click on it with the left mouse button. Enter the first x data value

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[1]Some online documentation for WPtools is at
http://physics.dickinson.edu/~wp_web/wp_resources/Documentation.html
[1]Note: Menu commands are described as follows: File→Open means move the cursor to the word File on the line near the top of the screen, press *and hold* the **left** mouse button, drag the cursor down to the word Open, and release the button.

here, pressing Return when you are done. The cell below it (called A2) will automatically be selected next; enter the second x data value here. Work down the first data column in this way. If you need to go back and correct any of the numbers, simply move the cursor with the mouse, click on the relevant cell, and re-enter the number.

Once you've entered the first column of numbers, move the cursor to the top cell of the second column (cell B1) and click the left button to select it. Enter the first y data value. Press Return/Enter, and enter subsequent y data in the rest of the cells.

A.3   Calculations in Excel

Now that your two columns of data are in the computer, select them. Do this by moving the cursor to the top left cell, pressing *and holding* the **left** mouse button, dragging the cursor to the lowest filled cell in the second column, and then releasing the mouse button. The block of numbers you entered will now be "selected", indicated by a blue-grey color.

Go to the WPtools pull-down menu and select Linear Fit. Immediately Excel will display a plot of your data, along with the values and uncertainties of the best-fit line. Print out a copy of the results if desired.[2]

Sometimes you will want to transform your raw data in some way before plotting it. For example, you may have entered two columns of data as above, but you want to convert the y values from inches to meters. This is where a spreadsheet program becomes really handy. Select a blank cell somewhere on the sheet (cell C1 would be a good place). Instead of entering a number, enter the formula '=0.0254*B1, and press Return. Excel will display the expected numerical value in cell C1, and it will also remember the formula. This is useful for two reasons, first, if you change the value in B1, the number in C1 will be automatically updated. Second, you can copy the formula in C1 to other cells, transforming the rest of column B using the same formula. To do this, first select cell C1. The cell becomes outlined, and note that there is a little square in the lower right corner of the outline. Move the cursor to this square, push and hold the **left** mouse button, drag the cursor down several cells, and release the mouse button. Voila! Excel will use the same formula to multiply all the cells in column B by 0.0254.

Excel can do much more complicated arithmetic. For example, you could use the formula =sqrt(A1) * B1 to take the square root of the values of cells in column A, multiply them by the values in column B, and put the result in some other column.

You might also want to take differences between the successive items in your data list. If you type into cell C2 the formula =B2-B1, and then use the little square to fill this formula into the cells B3, B4, *etc.*, then you will obtain the differences in column C.

If you do a transformation like this, and then you want to do a plot or a curve fit, the columns of data you want to plot may not be adjacent to each other. No problem. Say you want to plot cells A1-A10 on the horizontal axis and cells C1-C10 on the vertical axis. First

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[2]You must first "grab" the plot by left-clicking on an open area inside it. If the plot legends are obscuring the graph, drag them aside with the mouse. Your can add labels to your plot using the Edit labels option on the WPtools menu bar.

select A1-A10. (Go to cell A1, hold down the left button, drag the cursor to A10, then release the mouse button). Then *hold down the* Ctrl *key* and select C1-C10. Now both cell groups A1-A10 and C1-C10 will be selected, but not B1-B10. Run the WPTools→LinearFit routine, and you will get the plot you want.

A.4 Accumulating Values via Excel Tricks

There will be times in the Physics 101 lab when you want to accumulate sums of a series of values. For example, you might have measured a series of time intervals,

$\Delta t_1 =$ interval between event 1 and event 2,
$\Delta t_2 =$ interval between event 2 and event 3,
$\Delta t_3 =$ interval between event 3 and event 4,
etc.

You may wish to convert these into a continuous time scale. In other words, you may want to declare that $t = 0$ at the time of event 1, and then find

time of event 2 $= \Delta t_1$,
time of event 3 $= \Delta t_1 + \Delta t_2$,
time of event 4 $= \Delta t_1 + \Delta t_2 + \Delta t_3$,
etc.

This is easy to do. Say that Δt_1, Δt_2, *etc.*, are in cells A1, A2, *etc.*, and you want to put the accumulated times in column B. First put a 0 in cell B1 (since $t = 0$ for the first event). Then go to cell B2 and enter the formula =SUM($A$1:A1). The SUM function simply adds up the cells in the range specified.

The usefulness of the $ notation becomes apparent when you want to calculate the rest of the times. Select B2, move the cursor to the square in the lower righthand corner of the cell border, press and hold the left mouse button, drag the cursor down several cells, and release the button. The cells in column B are now filled with SUM functions, but in a special way: The $A$1 in the SUM function call remains the same in all the cells (because of the $), but the second part of the function call changes from A1 to A2 to A3, ... In other words, cell B3 now reads =SUM($A$1:A2), cell B4 reads =SUM($A$1:A3), and so on. These are exactly the formulae we want for the event time calculations, so column B is now filled with calculated values of t.

A.5 Further Notes about Workshop Physics Routines

- Use the WPtools→Polynomial Fit menu command, and set Order=2 to fit lines of the form $y = c_0 + c_1 x + c_2 x^2$.

- If you enter non-numerical text in the cell above each column of data, it will be used to label the horizontal and vertical axes on the plot.

- The fitting routines always use the first selected column for the horizontal (x) points, and the second selected column for the vertical (y) points.

- Empty rows are usually ignored (but partially-empty rows may corrupt the fit).

- To delete a plot, select it (move cursor to it and click once), then press the Delete key. To delete a column of the sheet, select the entire column (by clicking on the letter at the top) and use Edit→Delete.

- If data are modified after running a fit, the associated plot will be automatically updated, but the fit parameters will not be re-calculated. Usually it is best to delete both the old plot and fit parameters after updating data.

Appendix B

Estimation of Errors

While the subject of **error analysis** can become quite elaborate, we first emphasize a basic but quite useful strategy, discussed in secs. B.1-2. Then, we distinguish between **random** (or **statistical** uncertainties and **systematic** uncertainties in sec. B.3. Random uncertainties follow the famous bell curve, as sketched in secs. B.4-5. The important distinction between the uncertainty on a single measurement, and the uncertainty on the average of many repeated measurements is reviewed in secs. B.7-7. The subject of **propagation of errors** on measured quantities to the error on a function of those quantities is discussed in sec. B.8.

B.1   67% Confidence

Whenever we make a measurement of some value v, we would also like to be able to say that with 2/3 probability the value lies in the interval $[v - \sigma, v + \sigma]$. We will call σ the **uncertainty** or **error** on the measurement. That is, if we repeated the measurement a very large number of times, in about two thirds of those measurements the value v would be in the interval stated.

B.2   A Simple Approach

Repeat any measurement three times, obtaining a set of values $\{v_i\}$, $i = 1, 2, 3$. Report the average (mean),

$$\bar{v} = \frac{1}{N} \sum_{i=1}^{N} v_i \qquad \text{(for } N = 3\text{)}, \tag{B.1}$$

as the best estimate of the true value of v, and the uncertainty σ as

$$\sigma = \frac{v_{\max} - v_{\min}}{2}. \tag{B.2}$$

If you take more than three measurements, you can still implement this procedure with the aid of a **histogram**. Divide the range of observed values of v into 5-10 equal intervals (called **bins**). Located the bin that contains each measurement, and draw a box one unit high above that bin. Stack the boxes on top of one another if more than one measurement falls in a bin. To estimate the error, determine the interval in v that contains the central 2/3 of the measurements, *i.e.*, the central 2/3 of the boxes you just drew, and report the error as 1/2 the length of this interval.

B.3   Random and Systematic Uncertainties

The uncertainty in a measurement of a physical quantity can be due to intrinsic random uncertainty (colloquially: error) as well as to systematic uncertainty.

Random uncertainties lead to difference in the values obtained on repetition of measurements. Systematic uncertainties cause the measurement to differ from its ideal value by the same amount for all repetitions of the measurement.

Random uncertainties can arise from vibrations of the components of a set-up driven by random thermal fluctuations, random noise in the electronics, and/or many other small but uncontrolled effects including quantum fluctuations.

In principle, the effect of random uncertainties can be made as small as desired by repetition of the measurements, such that the dominant uncertainty is due to systematic effects (which can only be reduced by designing a better measurement apparatus).

B.4   The Bell Curve

In many cases when a measurement is repeated a large number of times the distribution of values follows the bell curve, or Gaussian distribution:

$$P(v) = \frac{e^{-(v-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\sigma}, \tag{B.3}$$

where $P(v)dv$ is the probability that a measurement is made in the interval $[v, v + dv]$, μ is true value of the variable v, and σ is the standard deviation or uncertainty in a single measurement of v. See Figure B.1.
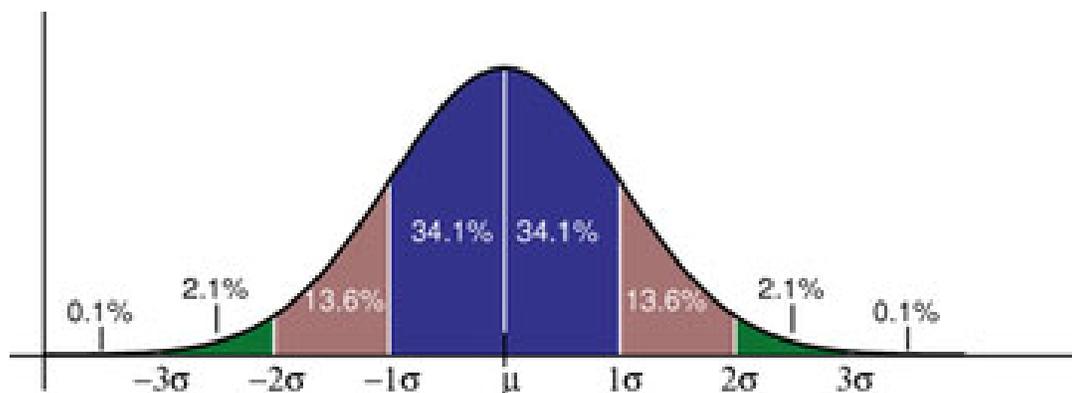

Figure B.1: The probability distribution measurements of a quantity with true value μ and Gaussian uncertainty σ of a singe measurement. About 68% of the measurements would fall in the interval between $\mu - \sigma$ and $\mu + \sigma$, and 95% would fall in the interval $\mu \pm 2\sigma$.

The Table lists the confidence that a single measurement from a Gaussian distribution falls within various intervals about the mean. If the 100 students in Ph101 each make 100

Table B.1: The probability (or confidence) that a measurement of a Gaussian-distributed quantity falls in a specified interval about the mean.

Interval	Confidence
$\pm\sigma$	68%
$\pm2\sigma$	95%
$\pm3\sigma$	99.7%
$\pm4\sigma$	99.994%

measurements during these lab sessions, then 10,000 measurements will be taken in all. The Table tells us that if those measurements have purely Gaussian 'errors', then we expect one of those measurements to be more than 4σ from the mean.

B.5   Estimating Uncertainties When Large Numbers of Measurements Are Made

One can make better estimates of uncertainties if the measurements are repeated a larger number of times. If N measurements are made of some quantity resulting in values v_i, $i = 1, ...N$ then the mean is, of course,

$$\bar{v} = \frac{1}{N} \sum_{i=1}^{N} v_i, \tag{B.4}$$

and the standard deviation of the measurements is

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (v_i - \bar{v})^2}. \tag{B.5}$$

Calculus experts will recognize that the operation $(1/N)\sum_{i=1}^{N}$ becomes $\int P(v)\, dv$ in the limit of large N. Then, using the Gaussian probability distribution (B.3) one verifies that

$$\bar{v} = \langle v \rangle = \int_{-\infty}^{\infty} v P(v)\, dv, \qquad \text{and} \qquad \sigma^2 = \left\langle (v - \bar{v})^2 \right\rangle = \int_{-\infty}^{\infty} (v - \bar{v})^2 P(v)\, dv. \tag{B.6}$$

B.6   The Uncertainty on Mean of a Uniformly Distributed Quantity

Not all measurable quantities follow the Gaussian distribution. A simple example is a quantity with a uniform distribution, say with values v equally probable over the interval $[a, b]$. It is clear that the average measurement would be $(a + b)/2$, but what is the uncertainty of the measurement? If we adopt the simple prescription advocated in secs. B.2 we would report the uncertainty as $(b - a)/3$ since 2/3 of the time the measurement would fall in an

interval $2(b-a)/3$ long. If instead we use the calculus prescription for σ given in eq. (B.6) we find that

$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{b-a}{3.46}, \tag{B.7}$$

which result is often used by experts.

B.7   The Uncertainty in the Mean

Thus far we have considered only the uncertainty or spread in measured values of some quantity v. A related but different question is: what is the uncertainty on our best estimate of v (which is just the mean value of our measurements, $\bar{v} = (1/N)\sum v_i$)?

The uncertainty on the mean $\bar{v}$ is surely less that the uncertainty, σ, on each measurement v_i. Indeed, the uncertainty on the mean is given by

$$\sigma_{\bar{v}} = \frac{\sigma}{\sqrt{N}}, \tag{B.8}$$

where σ is our estimate of the measurement error obtained from one of the methods sketched previously.

Appendix C illustrates eq. (B.8) using measurements of g from past Ph101 labs.

B.8   The Uncertainty on a Function of Several Variables (Propagation of Error)

In many cases we are interested in estimating the uncertainty on a quantity f that is a function of measured quantities a, b, ... c. If we know the functional form $f = f(a, b, ...c)$ we can estimate the uncertainty σ_f using some calculus. As a result of our measurements and the corresponding 'error analysis' we know the mean values of a, b, ... c and the error estimates σ_a, σ_b, ... σ_c of these means. Our best estimate of f is surely just $f(a, b, ...c)$ using the mean values.

To estimate the uncertainty on f we note that the change in f due to small changes in a, b, ... c is given by

$$\Delta f = \frac{\partial f}{\partial a}\Delta a + \frac{\partial f}{\partial b}\Delta b + ... + \frac{\partial f}{\partial c}\Delta c. \tag{B.9}$$

If we just averaged this expression we would get zero, since the 'errors' Δa, ... Δc are sometimes positive, sometimes negative, and average to zero. Rather, we square the expression for Δf, and then average.

$$\Delta f^2 = \left(\frac{\partial f}{\partial a}\right)^2 \Delta a^2 + ... + \left(\frac{\partial f}{\partial c}\right)^2 \Delta c^2 + ... + 2\frac{\partial f}{\partial a}\frac{\partial f}{\partial c}\Delta a\Delta c + ... \tag{B.10}$$

On average the terms with factors like $\Delta a \Delta c$ average to zero (under the important assumption that parameters a, b, ... c are independent). We identify the average of the squares of

the changes relative to the mean values as the squares of the errors: $\langle \Delta a^2 \rangle = \sigma_a^2$, *etc.* This leads to the prescription

$$\sigma_f^2 = \left(\frac{\partial f}{\partial a} \right)^2 \sigma_a^2 + ... + \left(\frac{\partial f}{\partial c} \right)^2 \sigma_c^2 + ... \tag{B.11}$$

Some useful examples are

$$f = a \pm b \pm ... \pm c \quad \Rightarrow \quad \sigma_f = \sqrt{\sigma_a^2 + \sigma_b^2 + ... + \sigma_c^2}, \tag{B.12}$$

and

$$f = a^l b^m ... c^n \quad \Rightarrow \quad \frac{\sigma_f}{f} = \sqrt{l^2 \left(\frac{\sigma_a}{a} \right)^2 + m^2 \left(\frac{\sigma_b}{b} \right)^2 + ... + n^2 \left(\frac{\sigma_c}{c} \right)^2}, \tag{B.13}$$

where l, m and n are constants that may be negative.

For more detailed and rigorous analyses one can consult, for example:

- P.R. Bevington and D.K. Robinson, *Data Reduction and Error Analysis for the Physical Science*, 2nd ed. (McGraw-Hill, New York, 1992).

- J.R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (University Science Books, 1997).

Appendix C

Standard Deviation of the Mean of g

Suppose you make N repeated measurements of a quantity g, such as the acceleration due to gravity. How well is the value of g determined by these measurements?

For example, during the 2006 sessions of Ph101 Lab 3 a total of 37 different measurements of g were made, as shown in the histogram Fig. C.1.
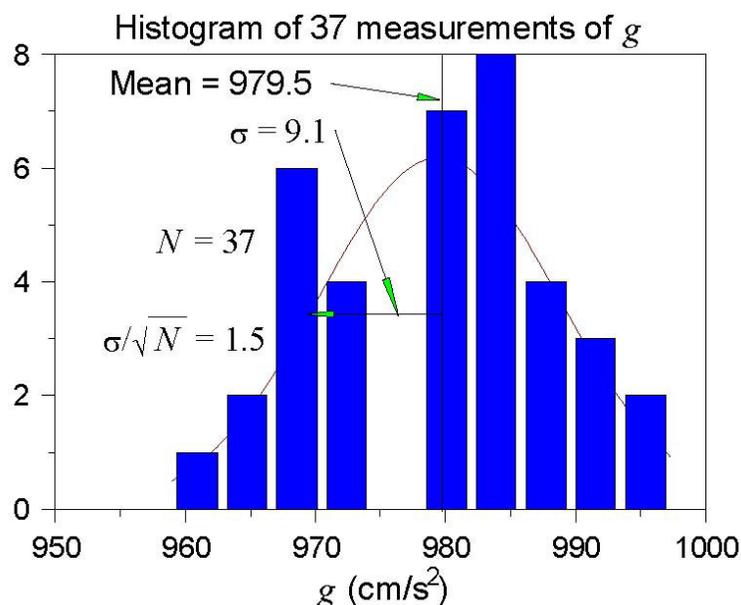

Figure C.1: Histogram of the values of g measured in the 2006 Ph101 Lab 3. The horizontal axis is g, and the vertical axis is the number of times a value of g was reported to lie with the range of g corresponding to the width of a vertical bar.

A histogram is a graph containing M vertical bars in which the height of a bar indicates the number of data points whose value falls within the corresponding "bin", i.e., within the interval $[g_j - \Delta/2, g_j + \Delta/2]$, where $g_j, j = 1, M$ and the centers of the M bins and Δ is the bin width. One can make a histogram of a data set $\{g_i\}$ using Excel/Tools/Data Analysis/Histogram. *Enter the data $\{g_i\}$ in one column of an* Excel *spreadsheet. Click on the* Input Range: *box of the* Histogram *window; then click and hold the left mouse button on the first data point, and drag the mouse to the last data point to enter the cell addresses of the data. Click on* Chart Output *and then* OK *to create a basic histogram. If the number/spacing of "bins" chosen by* Excel *is awkward, fill a new column with a linear series of 5-10 steps that begins near the lowest g_i and ends near the highest; create a new histogram with the* Excel *addresses of the first and last elements of the bin list in the box* Bin Range:.

The mean value $\bar{g}$ is calculated according to

$$\bar{g} = \frac{\sum_{i=1}^{N} g_i}{N} \, , \tag{C.1}$$

and was found to be $\bar{g} = 939.5$ cm/s^2 for the data shown in Fig. C.1.

The distribution of the value of g is approximately Gaussian, and the standard deviation of this distribution is calculated according to

$$\sigma_g = \sqrt{\frac{\sum_{i=1}^{N}(g_i - \bar{g})^2}{N-1}} \, , \tag{C.2}$$

with the result that $\sigma_g = 9.1$ cm/s^2.

The standard deviation σ_g is a good estimate of the uncertainty on a single measurement of g. However, after 37 measurements of g, the uncertainty on the mean value $\bar{g}$ is much smaller than σ_g.

An important result of statistical analysis is that the standard deviation (*i.e.*, the uncertainty) of the mean of the N measurements is related to the standard deviation of the distribution of those measurements by,

$$\sigma_{\bar{g}} = \frac{\sigma_g}{\sqrt{N}} \, . \tag{C.3}$$

For the data shown in Fig. C.1, where $N = 37$, we obtain

$$\sigma_{\bar{g}} = \frac{9.1}{\sqrt{37}} = 1.5 \text{ cm/s}^2. \tag{C.4}$$

That is, we can report the result of all 37 measurements of g as

$$g = 979.5 \pm 1.5 \text{ cm/s}^2. \tag{C.5}$$

As a check that eq. (C.3) is valid, we can analyze the data another way. Namely, we can first calculate the means $\bar{g}_i$ for the 5 different sessions of Ph101 Lab 3. Then, we can make a histogram of these 5 values, as shown in Fig. C.2.

The mean of the 5 means is 979.6 cm/s^2, which is essentially identical to the mean of the 37 individual measurements of g. The standard deviation of the 5 means shown in Fig. C.2 is calculated to be 1.6 cm/s^2, which is essentially identical to the previous calculation (C.4) of the standard deviation of the mean.

Concluding Remarks: If N were much larger than what we have here, the histogram C.1 would approach the Gaussian distribution (the bell-curve) shown in Appendix B. The peak in the histogram would be very close to the mean value $\bar{g}$ of the measurements, which represents the best estimate of g from the data. The standard deviation $\sigma_g \approx$ width/2 is a measure of the uncertainty of a single measurement,[1] while $\sigma_g/\sqrt{N}$ is the uncertainty on the best estimate $\bar{g}$.

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[1]Strictly speaking, the full width at half maximum of a Gaussian distribution is $2.35\sigma_g$.
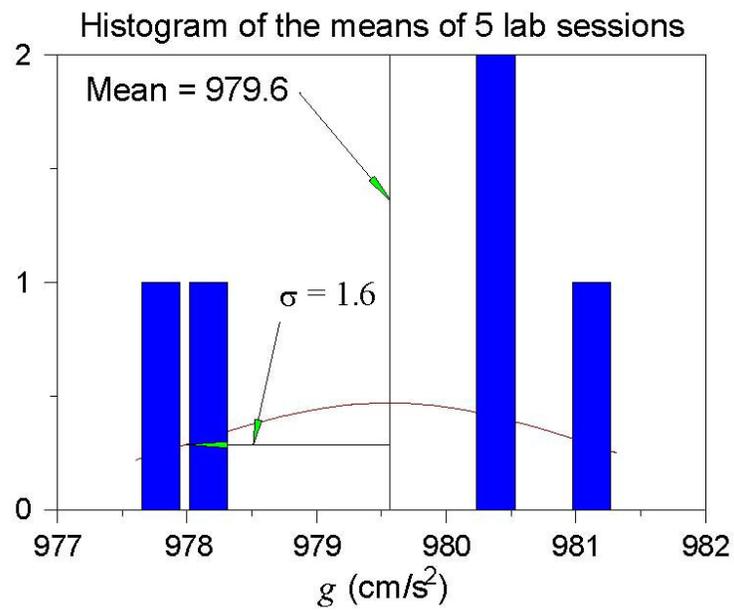
Figure C.2: Histogram of the mean values of g measured in the 5 sessions of Ph101 Lab 3 in 2006.

Appendix D

Polynomial Fits in WPtools

D.1 Polynomial Regression

In this technical Appendix we sketch the formalism used in the polynomial regression method for fitting data. This is a generalization of the method of linear regression.

We start with a set of data (x_j, y_j), $j = 1, ...m$, and we wish to fit these data to the nth-order polynomial

$$y(x) = \sum_{i=0}^{n} a_i x^i. \tag{D.1}$$

In general each measurement y_j has a corresponding uncertainty σ_j. That is, if the measurements were repeated many times at coordinate x_j the values of y_j would follow a gaussian distribution of standard deviation σ_j. We indicate in sec. D.2 how the program WPtools proceeds in the absence of input data as to the σ_j.

Because of the uncertainties in the measurements y_j we cannot expect to find the ideal values of the coefficients a_i, but only a set of best estimates we will call $\hat{a}_i$. However, we will also obtain estimates of the uncertainties in these best-fit parameters which we will label as $\sigma_{\hat{a}_i}$.

The best-fit polynomial is then

$$\hat{y}(x) = \sum_{i=0}^{n} \hat{a}_i x^i. \tag{D.2}$$

The method to find the $\hat{a}_i$ is called least-squares fitting as well as polynomial regression because we minimize the square of the deviations. We introduce the famous chi square:

$$\chi^2 = \sum_{j=1}^{m} \frac{[y_j - \hat{y}(x_j)]^2}{\sigma_j^2} = \sum_{j=1}^{m} \frac{\left(y_j - \sum_{i=0}^{n} \hat{a}_i x_j^i\right)^2}{\sigma_j^2}. \tag{D.3}$$

Fact: $\exp(-\chi^2/2)$ is the (un-normalized) probability distribution for observing a set of variables $\{y_j(x_j)\}$ supposing the true relation of y to x is given by eq. (D.2).

A great insight is that $\exp(-\chi^2/2)$ can be thought of another way. It is also the (un-normalized) probability distribution that the polynomial coefficients have values a_i when their best-fit values are $\hat{a}_i$ with uncertainties due to the measurements $\{y_j\}$. Expressing this in symbols,

$$\exp(-\chi^2/2) = \text{const} \times \exp\left(-\sum_{k=0}^{n}\sum_{l=0}^{n} \frac{(a_k - \hat{a}_k)(a_l - \hat{a}_l)}{2\sigma_{kl}^2}\right), \tag{D.4}$$

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or equivalently

$$\chi^2/2 = \text{const} + \sum_{k=0}^{n} \sum_{l=0}^{n} \frac{(a_k - \hat{a}_k)(a_l - \hat{a}_l)}{2\sigma_{kl}^2}. \tag{D.5}$$

The uncertainty on $\hat{a}_k$ is σ_{kk} in this notation. In eqs. (D.4) and (D.5) we have introduced the important concept that the uncertainties in the coefficients $\hat{a}_k$ are correlated. That is, the quantity σ_{kl}^2 is a measure of the probability that the values of a_k and a_l both have positive fluctuations at the same time. In fact, σ_{kl}^2 can be negative indicating that when a_k has a positive fluctuation then a_l has a correlated negative one.

One way to see the merit of minimizing the χ^2 is as follows. According to eq. (D.5) the derivative of χ^2 with respect to a_k is

$$\frac{\partial \chi^2/2}{\partial a_k} = \sum_{l=0}^{n} \frac{a_l - \hat{a}_l}{\sigma_{kl}^2}, \tag{D.6}$$

so that all first derivatives of χ^2 vanish when all $a_l = \hat{a}_l$. That is, χ^2 is a minimum when the coefficients take on their best-fit values $\hat{a}_i$. A further benefit is obtained from the second derivatives:

$$\frac{\partial^2 \chi^2/2}{\partial a_k \partial a_l} = \frac{1}{\sigma_{kl}^2}. \tag{D.7}$$

In practice we evaluate the χ^2 according to eq. (D.3) based on the measured data. Taking derivatives we find

$$\frac{\partial \chi^2/2}{\partial \hat{a}_k} = \sum_{j=1}^{m} \frac{\left(y_j - \sum_{i=0}^{n} \hat{a}_i x_j^i\right)\left(-x_j^k\right)}{\sigma_j^2} = \sum_{i=0}^{n} \sum_{j=1}^{m} \frac{\hat{a}_i x_j^i x_j^k}{\sigma_j^2} - \sum_{j=1}^{m} \frac{y_j x_j^k}{\sigma_j^2}, \tag{D.8}$$

and

$$\frac{\partial^2 \chi^2/2}{\partial \hat{a}_k \partial \hat{a}_l} = \sum_{j=1}^{m} \frac{x_j^k x_j^l}{\sigma_j^2} \equiv M_{kl}. \tag{D.9}$$

To find the minimum χ^2 we set all derivatives (D.8) to zero, leading to

$$\sum_{i=0}^{n} \sum_{j=1}^{m} \frac{x_j^i x_j^k}{\sigma_j^2} \hat{a}_i = \sum_{j=1}^{m} \frac{y_j x_j^i}{\sigma_j^2} \equiv V_k. \tag{D.10}$$

Using the matrix M_{kl} introduced in eq. (D.9) this can be written as

$$\sum_{i=0}^{n} M_{ik} \hat{a}_i = V_k. \tag{D.11}$$

We then calculate the inverse matrix M^{-1} and apply it to find the desired coefficients:

$$\hat{a}_k = \sum_{l=0}^{n} M_{kl}^{-1} V_l. \tag{D.12}$$

Comparing eqs. (D.7) and (D.9) we have

$$\frac{1}{\sigma_{kl}^2} = M_{kl}. \tag{D.13}$$

The uncertainty in best-fit coefficient $\hat{a}_i$ is then reported as

$$\sigma_{\hat{a}_i} = \sigma_{ii} = \frac{1}{\sqrt{M_{ii}}}. \tag{D.14}$$

D.2 Procedure When the σ_j Are Not Known

This method can still be used even if the uncertainties σ_j on the measurements y_j are not known. When the functional form (D.1) correctly describes the data we claim that on average the minimum χ^2 has value $m - n - 1$.[1] To take advantage of this remarkable result we suppose that all uncertainties σ_j have a common value, σ. Then

$$\chi^2 = \sum_{j=1}^{m} \frac{[y_j - \hat{y}(x_j)]^2}{\sigma^2} \approx m - n - 1, \tag{D.15}$$

so that

$$\sigma_j = \sigma = \sqrt{\frac{\sum_{j=1}^{m}[y_j - \sum_{i=0}^{n} \hat{a}_i x_j^i]^2}{m - n - 1}}. \tag{D.16}$$

In practice it appears that the error estimates from this procedure are more realistic if a fit is made using a polynomial with one order higher than needed for a 'good' fit to the data.

Using eq. (D.16) as the estimate of the uncertainty σ on each of the measurements y_j, the matrix M_{kl} of eq. (D.9) becomes

$$M_{kl} = \frac{m - n - 1}{\sum_{j'=1}^{m}[y_{j'} - \sum_{i'=0}^{n} \hat{a}_{i'} x_{j'}^{i'}]^2} \sum_{j=1}^{m} x_j^k x_j^l. \tag{D.17}$$

The estimate (D.14) of the uncertainty on the fit coefficient $\hat{a}_i$ is now given by

$$\sigma_{\hat{a}_i} = \frac{1}{\sqrt{M_{ii}}} = \sqrt{\frac{\sum_{j'=1}^{m}[y_{j'} - \sum_{i'=0}^{n} \hat{a}_{i'} x_{j'}^{i'}]^2}{(m - n - 1)\sum_{j=1}^{m} x_j^{2i}}} \tag{D.18}$$

When WPtools performs a polynomial regression it generates a plot of the data points and the best-fit curve, along with numerical values of various parameters associated with the fit. Figure D.1 gives an example of a fit to a set of 8 data points of the form $y = x^2$. The fit is to the form $y = a_0 + a_1 x + a_2 x^2$. The fit coefficients are $a_0 = -0.4107$, $a_1 = -0.3274$ and $a_2 = 1.1964$. The uncertainties (standard errors) on the fit coefficients are reported as $SE(a_0) = 4.0070$, $SE(a_1) = 2.0429$ and $SE(a_2) = 0.2216$, as calculated according to eq. (D.18). *Note that the uncertainties on coefficients a_1 and a_1 are larger than the coefficients themselves, which tells us that these coefficients are indistinguishable from zero.*

Also indicated on the plot are the values $R^2 = 0.9915$ and $\sigma = 2.8721$. The latter is the uncertainty in the data points $\{y_j\}$, calculated according to eq. (D.16) with $m = 8$ and $n = 2$. The quantity R^2 is defined by

$$R^2 = \frac{\sum_{j=1}^{m}[\hat{y}(x_j) - \overline{y}]^2}{\sum_{j=1}^{m}[y(x_j) - \overline{y}]^2}, \tag{D.19}$$

where the average $\overline{y} = \sum_{j=1}^{m} y(x_j)/m$. This is a measure of the "goodness of fit". If the fit is perfect then $\hat{y}_j = y_j$ for all j and $R^2 = 1$. It is not obvious, but $R^2 \leq 1$ always. The extreme case of $R^2 = 0$ occurs when the fit has the trivial form $\hat{y}(x) = \overline{y}$ for all x, which in general is a bad fit. The qualitative conclusion is that if R^2 is not close to 1, the fit results are to be regarded with suspicion.

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[1] The whole fitting procedure does not make sense unless there are more data points (m) than parameters ($n + 1$) being fitted.

Series1(X)=a_0+Σa_n(X)^n

R^2=0.9915
σ =2.8721

a_0=-0.4107
a_1=-0.3274
a_2=1.1964

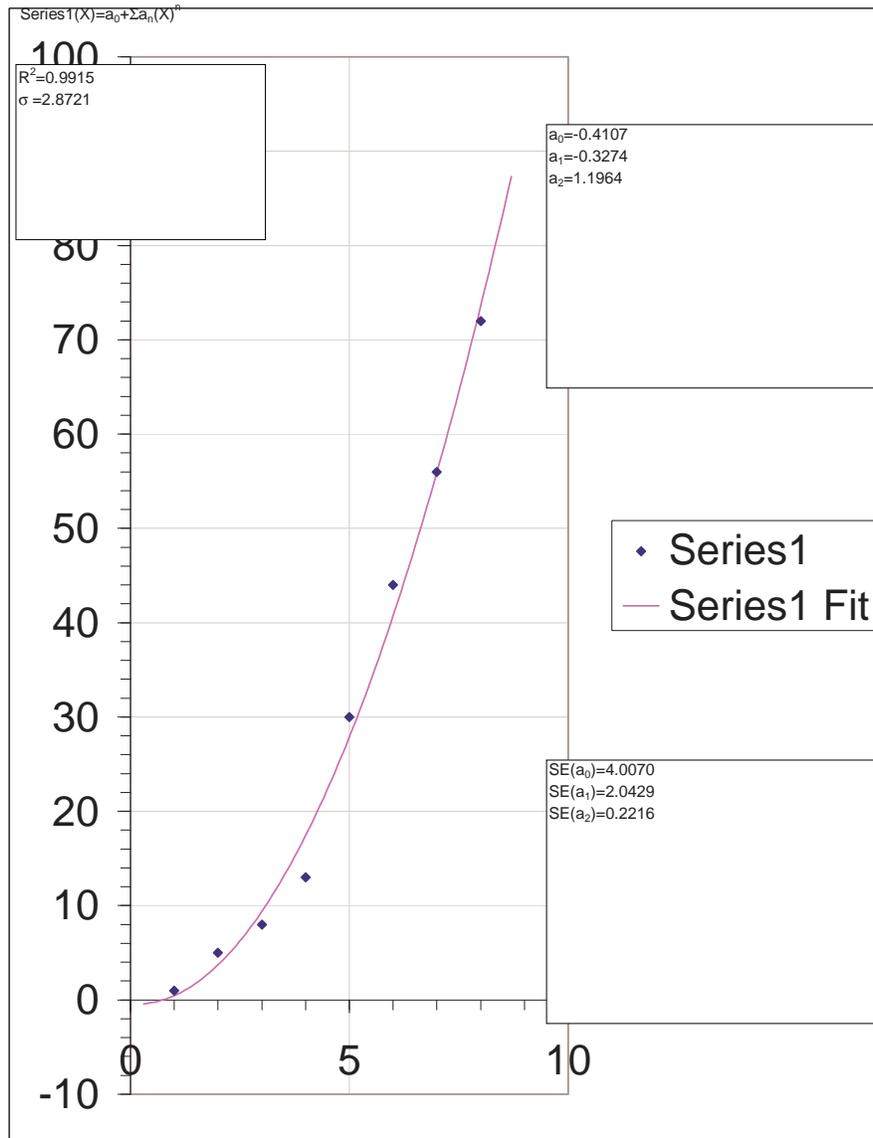SE(a_0)=4.0070
SE(a_1)=2.0429
SE(a_2)=0.2216

Series1
Series1 Fit

Figure D.1: Sample plot from WPtools Polynomial Fitting.