Princeton University
Physics Department

Physics 103/105
Lab Manual

Fall 2009

Physics 103/105 labs start Monday September 21, 2008. It's important that you go to the lab section that you signed up for. We will be expecting you!

You should have a lab book and a scientific calculator when you come to your first lab. (See details in the Orientation section following.)

Each week, before you come to lab:

Read the procedure for that week's lab, and any additional reading required.

The **Prelab problems** are **optional**, but please work them if it appears that they will be of help to you.

Also, for the first week:

Read the "Orientation to Physics 103/105 Lab" and "Error Analysis – Guidance and Reference Text" sections of this packet, and the assigned sections in Taylor.

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PHYSICS 103/105 LAB MANUAL
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LAB SCHEDULE

Remember: Always read the writeup and any reference material before coming to lab.

Date	Topics	Experimentation	Equipment
Sept 21-24	Experiment # 1 Encountering the Equipment; Motion in Two Dimensions	Learn to use camera and software. Take data on bouncing ball. Calibrate m/pixel. Analyze results using both VideoPoint and Excel.	PC & CCD camera. Ball & launcher, backboard.
Sept 28-Oct 1	Experiment # 2 Describing Measurement Variability	Characterizing deviations in repeated "measurements"	Tokens and shaker box. Special Excel program.
Oct 5-8	Experiment #3 Free Fall, Terminal Velocity, etc.	Digitizing camera images again. More use of Excel.	PC & CCD camera. Ball & launcher, backboard. Very light objects.
Oct 12-15	Experiment # 4 Collisions and Conservations	Look at collisions in two dimensions	Air table, with pucks and other objects. PC & CCD camera
Oct 19-22	Experiment # 5 Inclined Planes and Energy Conservation	Look closely at objects moving under "constant acceleration." Isolate and quantify effects of friction.	PC & CCD camera. Tilted Ramps Carts Cafeteria of "stuff."
Oct 26-29	Experiment #6 Two Experiments in Rotational Motion	Loop-the-Loop and rolling off a log.	Special apparatus.
Nov 2-5	NO LAB	--- MIDTERM BREAK ---	
Nov 9-12	Experiment # 7 Fluids	Forces in fluids and fluid flow	

Nov 16-19	Experiment # 8 Coupled Pendulums and Normal Modes	Interacting pendulums having slightly different frequencies.	
Nov 23-26	NO LAB	--- THANKSGIVING ---	
Nov 30-Dec 3	Experiment # 9 Precision Measurement of g	Use a precision pendulum to get a very accurate measurement of g.	Special apparatus.
Dec 7-10	Experiment # 10 The Speed of Sound and Specific Heats	Measure the speed of sound, density of gasses.	Gas column, frequency generator, vacuum pump, scale.

Orientation to Physics 103/105 Lab

WELCOME TO PHYSICS 103/105 LAB!

Physics 103/105 labs start the first full week of classes. It's important that you go to the lab section that you signed up for. We will be expecting you!

You should have a lab book and a scientific calculator when you come to your first lab. (See details in the Orientation section following.)

Each week, before you come to lab:

Read the procedure for that week's lab, and any additional reading required.

The **Prelab problems** are **optional**, but please work them if it appears that they will be of help to you.

Also, for the first week:

Read the "Orientation to Physics 103/105 Lab" material below.

I. What Physics 103/105 labs Are Like

You will soon find that Physics 103/105 Lab is not like any lab you've had before. You will be expected to think and be creative, not just follow instructions. The lab manuals for each experiment will step you through many of the new techniques that you will need, but you will not generally be given recipes to follow for each experiment. The manual for each week will typically describe a basic experiment that all students will be expected to complete. Beyond that, the manual will suggest a variety of additional experiments and extensions that you may find interesting and challenging. You can pick one or more of these to work on, or you can invent a new experiment on your own. The lab employs versatile equipment that will allow you to do the "standard" experiment quickly and then branch out to new challenges that match your interests and expertise.

In Physics 103/105 Lab, you will generally work in teams of two or three students. (Larger groups become unwieldy.) Your TA will randomly assign teams. Students will work in the same groups each week but, there is always some flux as team members change sections, or as some experiments require larger or smaller groups. Always be flexible about letting others work with you, or splitting up your existing team if it makes the groups more even. Your TA may mix things up and switch groups in

I.	What Physics 103 labs are like
II.	Notebooks
III.	Prelabs
IV.	Grading
V.	Attendance
VI.	Feedback

the middle of the term to give people a chance to work with other individuals.

You will find that learning how to plan and work together will be crucial to the effectiveness of your team. Among your responsibilities in physics 103/105 is to be sure that all of the people in your team get a chance to contribute to your success. Take turns at the computer so that every team member gets a chance to drive. If one or more of your team members is lost, take a few minutes to get them up to speed—you'll need their input later when things get tougher! You will also find that you learn a lot by discussing problems with others, *even when you think you're the one doing the teaching*. Most scientists and engineers today work in teams, some large and some small, depending on the size of the task.

Periodically, your TA will come by your lab bench and ask you questions about what's going on. Of course, you can ask your TA questions about what's going on too. You will find that your TA is a great resource. On the other hand, you shouldn't be surprised if your TA doesn't answer all of your questions directly. Your TA's job is to help you learn, and sometimes the best way to do that is by making you struggle on your own for a while.

Physics labs won't always follow the lecture step for step. Sometimes, you will learn things in lab before you see them in lecture or precept. In other cases, you will see topics in lab that won't be covered in the lecture at all. While physics labs are good for helping you solidify what you learn in lecture (either before or later), they are also great for going beyond the lecture, showing you additional techniques and phenomena that you won't see in other parts of your physics course.

Among the important techniques you will learn in lab that won't be covered in lecture are data analysis and error analysis, or analyzing the uncertainty in your measurements. These are critically important skills that you will need in your professional lives as scientists and engineers, and things you may not learn any place else. In some cases, analyzing your data and estimating your uncertainty will be the most difficult parts of a lab. This actually mirrors the professional lives of real scientists and engineers, who may spend very little time actually taking data, and a lot of time analyzing their data and planning what to do next.

The process you will go through in this lab closely approximates the working experience of professional scientists and engineers. It goes something like this: your team decides on an interesting investigation, makes a plan, organizes to do the work, gets and analyzes data, thinks about the results, and repeats or improves the experiment until you are satisfied with your accomplishments. The most valuable tool that you have in the lab (or anywhere else) is your common sense. A good scientist or engineer is always thinking: Do we have a clear idea of what are we trying to do? Do our results so far make sense? Is there a better way to do it? Practicing scientists almost never get their final results the first time they do an experiment. Don't just go through the motions: plan, think, and understand. Try to get to the result quickly, think about it, and then do a better experiment or try a variation to test a "what if?" idea. Along the way you will

learn a lot of physics, especially if you are thinking and discussing (or arguing about) ideas with your teammates.

II. Notebooks

For physics 103/105 labs, you will not turn in any formal laboratory reports. Instead, you will be required to keep a notebook. You can get one at the U-Store. The spiral bound 5X5 QUADRILLE RULED (80 Sheets 11" X 8.5" Green tint) is recommended (cheap and adequate). If those are not available, try to get something close; quadrille paper is particularly important.

Start each week's lab on a new page; write the date at the top, and who your other team members are. From that point on, your notebook should be a comprehensive record of everything you do and think in 103/105 lab that day; sort of like a diary or a journal.

Your lab notebook is not the same as a "lab report." It contains similar information, but in much less polished form. DO NOT waste time making your notebook look pretty. It should be neat enough to be easily understandable, BUT NOT NEATER. Feel free to cross things out, draw arrows, make freehand sketches, and cuss. Use scissors and tape to include printouts in your notebook whenever you use a computer to make a diagram or a graph. Also, include all of the things you tried that didn't work out how you planned; they're an important part of what you are learning too. Just be sure that your notebook is easily readable, and always take some time to write some understandable sentences explaining what you are about to do, what you've just done, and what you have learned from it.

Suppose, for instance, that you have just measured the speed of a car that has rolled down a ramp, and it didn't come out like you expected it to. You might write:

> "The value we came up with for the speed of the car was 4.2 m/sec; that's about 15% lower than we expected. Maybe friction (while rolling) is more important than we thought? Jen suggested we try raising the ramp: that way the speed of the car should be greater, and the effect of friction should be (relatively) smaller."

You could then try the experiment again, including the new data and a new graph in your notebook, along with your conclusions:

> "Dang! The speed increased to 6.7 m/sec, but now that's 25% lower than we expected it. The problem didn't get better, it got worse! So much for that idea: we can conclude that friction while rolling down the ramp probably isn't the problem...."

Notice that although the problem isn't solved yet, the student was able to draw a partial conclusion: that the discrepancy in speed is not due to rolling friction. These conclusions are vital for your notebook. Without them, your notebook is just a bunch of meaningless numbers. Of course, a final paper based on this experiment might never include this degree of detail, especially about this part of the experiment that seems to be producing inconsistent results. But solving these difficulties is an important part of your

experience in lab, and your description of your experience is what your notebook is all about.

Students are often unsure about how much detail to put in the notebook. <u>What we most want to see in your notebooks is evidence of thinking</u>, and not just "correct" results. A reasonable rule of thumb is to imagine that your notebook will be read by a friend of yours who is taking a similar physics course, perhaps at another school. Your friend should be able to pick up your notebook, perhaps along with your lab manual, and understand what you have been doing. If you have made a mistake in your experiment, your friend should be able to look back a few pages in your notebook and figure out where you might have gone wrong.

At the end of your lab period, leave your notebook on the shelf in your lab room for your TA to grade. <u>DO NOT REMOVE your notebook from the lab room</u>. This rule is for your own benefit. When students take their notebooks home, it starts a sort of arms race for who has the prettiest colored graphs and the best spelling. Don't waste time beautifying your notebook; it's a record, not an artwork.

Professional researchers rely heavily on their notebooks for reference when they write the papers or reports describing their work. Notebooks are the means to recall and check, sometimes years later, what went on in the lab. Many researchers have their notebooks regularly notarized, in case they are needed for patent verification. It's important to learn how to keep a good lab notebook. Someday your job may depend on it.

III. Prelab work

<u>You must read (and think about) the lab handouts and the assigned reading in Taylor's book on Error Analysis before coming to lab</u>. Otherwise you will waste a lot of time in getting up to speed. If your lab partners aren't reading the write-ups ahead of time, ask them to do so. Many of the writeups refer you to Appendix materials which you will be expected to read carefully.

<u>You can also complete the optional "prelab problems" in the back of the manual for each laboratory before coming to lab</u>. The prelab problems are not too hard or time consuming, but they can be helpful in getting you prepared for each week's material.

IV. Grading

The main intent of Physics 103/105 lab is to offer you a chance to be scientifically and technically creative. We want you to get and pursue ideas. Our evaluation of your work will depend a lot on your creativity and understanding, and very little on "getting the right answer." Experimentation is hard, and even good experiments don't always turn out how you plan them too

> **Golden rule of 103/105 lab grading:** *In general, we won't hold it against you if an experiment doesn't produce the answer you think it should or doesn't quite get finished, as long as you worked hard and were intellectually and creatively engaged for your entire lab period.* The flip side of the pact is this: *In general, you are required to be intellectually and creatively engaged for your entire lab period, even if your experiment goes well. If you finish one experiment, then think of another.*

Whatever you do, don't just sit around doing nothing. Your grade will suffer, and you will attract unwanted attention.

For Physics 103 Lab grade is worth 15% of your course grade in the class (Physics 105 will announce its grading scheme in class). But for both classes, your lab grade is calculated as follows:

> 75% Notebooks: Graded after each lab for completeness, thoughtfulness, and readability.
> 25% Participation: Graded for each lab for creativity and industry.

V. Attendance

Attendance each week is vital in Physics 103/105 Lab. If you must miss your regular lab section due to illness, University-related travel, or religious reasons, contact the Ph103 Lab Manager: Prof. Kirk McDonald (kirkmcd@princeton.edu) by email at to arrange to go to another lab section.

Missing one lab without notice or without a valid excuse will lower your **PHY 103/105 course grade** by **one full letter grade**. *Missing two labs* will result in **automatic fail of the class.**

If you realize that you have missed a lab because you have slept through it or have forgotten what day it was, then you can make it up later that week in another lab section. If you realize you have missed your lab, try to contact Prof. McDonald. However, if you have only one time slot left that you can make up the laboratory, don't wait to hear from the department office; just show up, explain the situation to the TA, and we'll sort the paperwork out later.

Makeup labs will offered, by appointment with Prof. McDonald, during the last full week of class, Dec. 14-17. However, it is strongly recommended that in case of schedule difficulties some week, you attend some lab session that week, and work with other students on the lab.

VI. Feedback

Physics 103/105 Labs are always changing. As always, we think we've nailed it, but chances are we've missed some things. To help us do better in future weeks and future years, we want to hear feedback from you.

If you have any questions, comments, or concerns about Physics 103/105 lab, please bring them up with your TA, the lab manager, or the course head. Thanks!

**Physics 103/105 Labs start on Monday, September 21, 2009.
See you soon!**

Error Analysis – Guidance and Reference Text

All of science is based on observation and measurement, and virtually all measurements are imprecise. Similarly, quantitative work in engineering, medicine, sociology, and many other fields involves consideration of imprecisely known inputs or results. Thus we come to the topic of "Error Analysis," which should really be called "Estimation of Uncertainties."

In all of your lab activities, you need to be concerned with the precision of your measurements, and with their accuracy and repeatability. And when multiple measurements are combined to yield a computed result, you need to understand how the uncertainties in the measurements affect your final conclusions.

We won't teach Error Analysis as a separate subject. Rather, you will be asked to develop intuitive and conceptual strengths as we go. That way, we hope that the formal mathematics and equations will become natural expressions of your understanding of what you are doing.

The major resource we expect you to use is <u>An Introduction to Error Analysis</u>, by John R. Taylor. That's the paperback book with the picture of a train wreck on its cover. It is a classic in the field, and should be kept as a permanent item in your professional library no matter what field of science or technology you go into (and also if you want to read and critique such things as newspaper articles on health and environment).

Most weeks, you will be asked to read a few sections in Taylor before you come to lab. Make sure that you do so. The purpose is to get you to understand the concepts and to be ready to apply them in the upcoming lab. We have assigned only the basic sections of the text. You may want to look at some extension material if you are interested in special topics, or in the background mathematics. You may also want to look at some of Taylor's problems (especially the odd-numbered ones whose solutions appear in the back of the book) if you are uncertain of your understanding.

The subject of Error Analysis can be painful, especially if you approach it as an application of abstruse formulas. But the intellectual content is really very basic, and quite intuitive. Feel free to discuss with your lab instructor how you should be doing things. We hope that by the end of the course you will feel comfortable with our introduction to the subject, and prepared (perhaps starting with the unassigned portions of Taylor) to go as much further in the subject as your interests may take you.

LAB #1: Encountering the Equipment; Motion in Two Dimensions

BEFORE YOU COME TO LAB:

1. Read the Orientation to Physics 103/105 lab, above.
2. Read the Introduction and Physics Background sections, below.
3. Read Taylor's Sections 1.5-1.6 and 2.1-2.2. (You should have read Sections 1.1-1.3 last week.) If you are unfamiliar with spread sheet programs, read Appendix A – Data Analysis with Excel
4. If you find it helpful, work the optional PreLab problem set attached at the end; You can discuss this with your TA at the beginning of the lab.

This lab is about <u>motion</u>, and how to describe it. The intuitive precision that a baseball player shows in moving to exactly the right location to catch a fly ball immediately after it leaves the bat is amazing. But in the technical realm it is useful to make a mathematical description. Describing motion requires measuring position and time. In Galileo's era, a measurement-based approach was in its infancy. But it is fundamental to all science.

Much of your time in this lab will be spent mastering a video-based method of measuring position and time. But don't lose sight of what is really important – characterizing motion itself. In what sense does motion sustain itself, and how do external agents affect it? Can two-dimensional motion be analyzed in terms of two separate descriptions, each involving only one dimension? Newton's simple equation, $F = m\,a$ (with F and a being vectors), underlies a formal answer to such questions. But formalism has to be related to measurement and intuition. That is what this lab is really about.

A. Introduction

In this lab you will use a TV camera to make videos of moving objects, and a computer to analyze the motion. It is important that every member of your group learn how to use the computer and software. Be sure that you all take turns and that, when you are the operator, your partners understand what you are doing.

First, open the Physics 103 folder, located on your computer's Desktop. You will then see a screen with icons for the programs that we will use in this lab. These include:

- VideoPoint Capture: We use this to record a video.

- VideoPoint: the program in which you will analyze your videos, by measuring positions in the video frames, and graphing and analyzing the results.

- Excel with WPTools: an extension of the common Excel spreadsheet program, which allows more detailed calculations and graphing. The WP (Workshop Physics) extensions make it easy to use Excel's sophisticated graphing functions.

- WordPad: a utility word processor, useful in printing out an image of any computer screens that you need to discuss in your write-ups.

- Student Data: a folder with space for you to put your files.

Capturing a Single Still Image

Your first task is to capture a still picture of yourself to give to your lab instructor to help him or her remember your name and face.

1. Open VideoPoint Capture.

2. A Preview Screen appears, with a picture of whatever your camera is viewing at the moment. Wave a hand in front of the camera. Check that the camera is focused on it. If not, rotate the distance-setting ring on the camera lens to get a sharp image. If the picture is too dark or too light, rotate the aperture ring on the camera lens to change the amount of light level reaching the camera's sensor. (The lens admits light through a circular opening, or aperture, whose diameter is controlled by rotating the ring. The larger the diameter, the more the light and the brighter the picture.)

3. Position yourselves (re-aiming the camera if necessary) so that your faces fill the screen. Then reach over, hold down the ALT key, and hit the Print Screen button near the right end of the top row of keys on your keyboard. (This actually makes a copy of the screen, rather than printing anything.)

4. Now, open WordPad. Then choose Edit→Paste to insert the image that was copied using the Print Screen command. (Please do NOT change any of the computer or display settings in an attempt to make the image larger. Such changes may interfere with your analysis program.)

5. Click on the printer icon in the WordPad menu to make a hardcopy of your picture. ***Write your names on a printout of your picture and give it to your instructor***. (Extra copies to take home, or to send your grandmother, are allowed.)

Stop and Think – What is a digital image?

Before trying to capture a video in the computer, let's be sure that we understand what the moving image on a television screen actually consists of. It is a series of individual stationary, or still, pictures, appearing at a rate of 30 pictures, or *frames*, a second. Standard movie films consist of strips of such still pictures, recorded photographically at a slightly lower frame rate.

Digital computers, and hence our electronic imaging systems, work only in quantized units. Your camera divides the area that it sees into 76,800 elements, arranged into a grid 320 cells wide by 240 cells high. You will see later that VideoPoint measures x- and y-positions with twice this resolution, dividing the x-direction into 640 integer steps and the y-direction into 480 steps. In either case, a 1-step by 1-step cell is called a *pixel*, for "picture element."

To determine the coordinates of an object in an image frame using VideoPoint, we just position the cursor over the object, and the system will count the pixel rows and columns to get to the location of the cursor, starting at the lower left of the image on the screen. Then, if we know how many pixels correspond to a meter long object, we can calculate real distances from the pixel counts. This is called "scaling the image." We will return to this shortly.

With real distances known from scaling the images, and with the time interval between two chosen frames also known (a multiple of 1/10 sec, say), we can easily calculate such things as velocities and accelerations of moving objects in our videos.

Note that the system reports locations rounded to the nearest integer pixel count. As a result, no position measurement can be known more accurately than within ± 0.5 pixel spacing, or to within ± 1/2 of whatever distance is equivalent to one pixel in real units (mm, meters, *etc.*).

B. Physics Background

In this lab you will analyze the trajectory of a bouncing ball. The ball will be accelerated by gravity in the –y direction, but not in the x direction. Assuming that only gravitational forces are significant, the ball's motion should obey the equations

$$x = x_0 + v_{0x}t \,, \tag{1}$$

$$y = y_0 + v_{0y}t - 1/2\, gt^2 \,. \tag{2}$$

In this experiment, a golf ball rolls down a fixed rail (the "launcher"). After it leaves the launcher it falls freely, bouncing at least twice on the table top. The camera takes a series of pictures at fixed time intervals between the first and second bounces, when the ball is moving freely. By measuring the position of the ball in each picture, you can test equations (1) and (2).
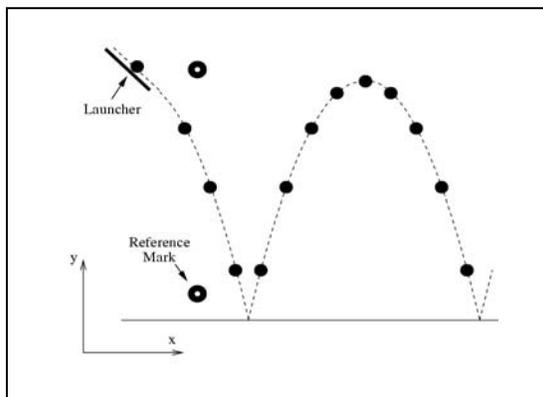
C. Acquiring the Data

To make your first video,

 Open VideoPoint Capture and use the Preview Screen to

1. Aim the camera, and check the lens focus and aperture setting

2. Include a meter stick in the picture, for scaling the video

3. Make a few trial runs, adjusting the launcher as necessary for the camera to catch one full bounce

4. Set the frame capture rate to 30 frames per second, using the button at the lower left of the image.

Make an Actual Movie of the Bouncing Ball:

1. Start recording by clicking on the Record button

2. Start the ball rolling down the launcher, and let it complete one full bounce

3. Hit the Escape button on your keyboard to stop recording. (The Stop button on the screen doesn't always work.)

4. Note: Although you don't need to rush things, don't record too much "dead air" before and after the actual experiment – it just ties up computer memory.

Viewing Your Video:

After VideoPoint capture takes a few seconds to organize your video, it stops to display the first frame of your movie in an Editing window. Note that you can

1. Play the movie by clicking on the little triangle at the lower left. Watch the progress as indicated by the moving "progress button" below the image. (Clicking on the triangle while the movie is playing will stop it at whatever frame it has reached.)
2. Jump to any frame by manually moving the progress button with your cursor.
3. Single-step from one frame to the next by clicking on the left and right arrow buttons below the image.

Each of you should spend some time playing with the Editing screen controls, to understand their functions and to get a "feel" for the fact that a video or movie is really only a sequence of still pictures.

Editing Your Video:

In general, your original video will contain more frames than you will want to use for analysis. Today, we want to analyze the ball's motion between two consecutive bounces off the table.

To get rid of the unnecessary frames at the beginning and end of your video,
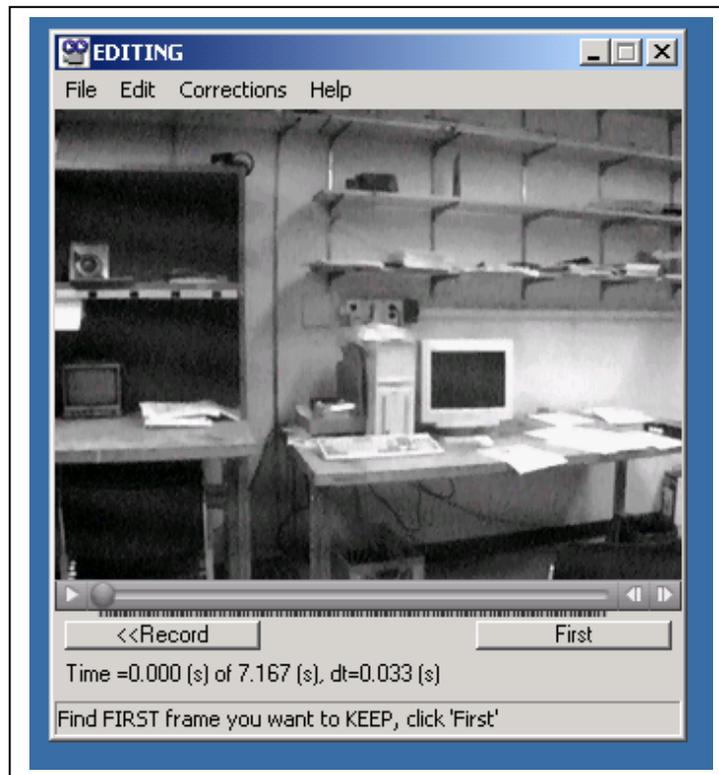
1. Use the progress and single-step buttons to choose the first frame after the ball has bounced from the table.
2. Click on the First button. All of the earlier frames are eliminated.
3. Move the video back to the last frame before the ball bounces again. Then click on Last. All of the later frames are eliminated, and your video contains just one bounce.

4. Press **Keep** to save the file, or **Record** to discard the data and record a new movie.

<u>Naming and Storing Your VideoPoint Capture Files; Opening VideoPoint to Start Your Analysis:</u>

We have provided a folder identified by your lab group's meeting time in which you should store all of your data and analysis files. This folder is found in the top-level **Student Data** located on the hard drive labeled **Students (D:)**. It should be accessible from the **Physics 103** folder on your desktop.

To save your video,

1. Click on the **Save** button on the editing screen.

2. On the screen which appears, titled **Save Movie File as:**, find your section's folder.

3. Provide a name for your file which will help you find it later. You may want to insert words before the suggested name (which indicates the month and the day by four digits, followed by two additional digits which simply cycle from 01 to 999).   The resulting file name might be **Bouncing Ball 091501**.

4. After choosing the correct folder, and editing the file name, click on the **Save** button.

5. Click on the **Open in VideoPoint** button to exit from VideoPoint **Capture** and start an analysis of your movie.
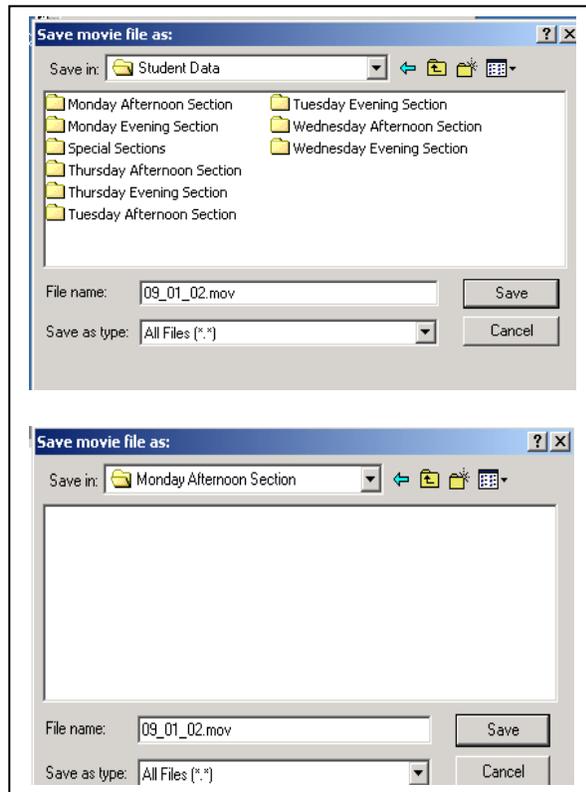
# D. Data Analysis Using **VIDEOPOINT**

With your movie now opened in VideoPoin*t*, you are ready to do some physics. In the **Number of Points** screen which first appears, click **OK** to **1**, since there is only one object that we want to analyze (the bouncing ball). If you see an information screen titled **About VideoPoint**, just click on the little **x** at the top right to make it disappear.

"Picking Off" Coordinates

Three windows appear when VideoPoint is first opened. The *Movie* window shows the first frame of your edited video, with your file name appearing in its title bar. It looks very much like the editing screen in VideoPoint Capture. (Confirm that you can play the movie, and use the progress and single-step buttons as before. The *Movie* window shows you, at the upper right hand corner, which frame you are viewing, and the total number of frames in your video.)
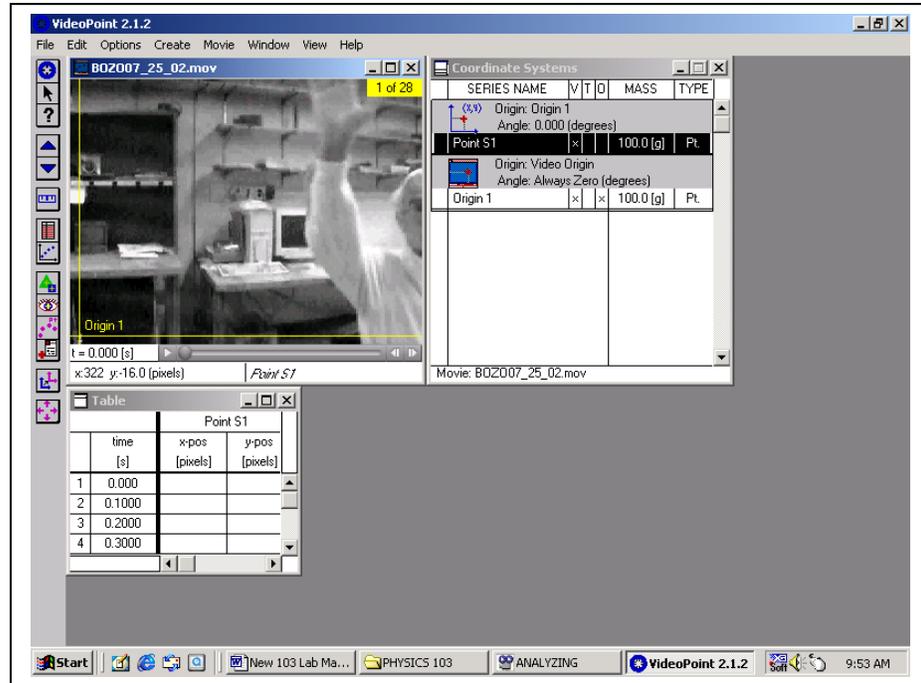
The *Movie* window is where you will "pick off" the coordinates of the golf ball's position in each frame. Note the yellow coordinate axes on the screen, and note that the cursor changes to a cross-hair pointing device when you move it into the image area. Finally, note that both the time of the frame and the *x*- and *y*-coordinates of the cursor are shown at the lower left of the *movie* window.

To enter your positional data,

1. Go to the first frame of your edited movie.

2. Position the cursor over the image of the golf ball, and click your left mouse button. (You may want to maximize the size of the *Movie* window, for increase accuracy.)

3. Note that the movie advances automatically to the next frame. (If it doesn't, go the Options menu and put a check mark next to the Auto Frame Advance option.)

4. Click on the image for several more frames. Stop and turn the Edit / Leave/Hide Trails option on and off, and note that the ball's history is shown superimposed on whatever frame you stop at.

Now stop and look at VideoPoint's Table window. It shows the time of each frame of your movie, and records the x- and y-pixel locations of each position that you identified by clicking on a frame. The three data columns are similar to those in a spread sheet program, but don't allow any manipulation of

> **Summary of Options When Picking Off Coordinates:**
>
> Note that the *Movie* cursor looks like a crosshair if no coordinate has been chosen for a given frame, and like an arrow otherwise.
> Use Edit/Leave or Hide Trails to turn trails on or off.
> Use the Coordinate System window to Select a point series by highlighting it
> Move a *selected* point in a given frame, by positioning the tip of the arrow on the point and holding down the left button.

their contents. (Later on, we will copy and paste the data to an Excel spread sheet and do some calculations.)

1. Finish your data entry by going back and picking off the coordinates of the golf ball in each of the frames of your video.

2. If you make a mistake in positioning your mouse in a particular frame, you may want to correct the position of the point you chose. You can move the old point to a new location. With the item selected in the Coordinate Systems window, and two concentric circles showing around the previously chosen location, first place the arrow-shaped cursor at the center of the circles. Then, while holding down the left button on the mouse, move the location circle to the new, corrected, position. Releasing the mouse button completes the change, and enters the corrected coordinates into the data table.

Graphing Your Data

To use VideoPoint to make graphs, first click on the View → New Graph menu item. For each graph, you must tell the system which variable you want plotted on the x-axis, and which on the y-axis. Normally, you will want either time or x-position to run horizontally, and x- or y-position, velocity, or acceleration to run vertically.

Let's make and print out some graphs. Start with an XY graph. Choose Point S1 / x / Position for the Horizontal Axis, and S1 / y / Position for the vertical axis. Note that, if you go back to the *Movie* window and advance through the frames, a circle moves on the graph to indicate which of the points plotted corresponds to the movie frame currently visible.

Plot Series
Horizontal Axis: time
time, Point S1, Point S2, Point S3
Vertical Axis: Point S1 / x
Position, Velocity, Acceleration, Momentum, Net Force [=ma], Kinetic Energy (KE) magnitude, Potential Energy (= mgy *), Total Energy (KE + PE)
* y-axis must align with g-vector   Cancel Add OK

(Your dialog box will not show *Point S2* and *Point S3*, since you have made measurements on only one object.)

To print a graph, first make its screen active by clicking anywhere on it. (The title bar of the graph will be colored blue after your click.) Then click on the File → Print menu bar option, and click on Print and OK

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when you are asked to confirm your request. If you maximize the size of the graph window before printing, your printed graph will be larger.

You can also make a printout of any one of the frames of your video, simply by printing when the movie window is selected.

If you need a printout of your data table, first be sure to first maximize the table window by clicking on the middle one of the three boxes at the right end of the Table title bar. Then issue the print commands. When you want to make the table small again, just click on the middle box, as before. (If your table is very long, you may have to print more than one time, scrolling vertically to change which data lines are presented each time.)

The Initial Physics Discussion

Print out and think about the following graphs. Each of you should have your own copy of the group's printouts, and **note your own conclusions on your own plots**.

1. Do the plots of *x*-position, velocity, and acceleration all confirm that the golf ball is subject to no horizontal force?

2. Does a plot of *y*-position *vs.* time look like you expected? (What is that?)

3. What does a plot of *y*-velocity *vs.* time look like? What does that show?

4. What about a plot of *y*-acceleration *vs.* time?

You should know that we expect to see a constant, negative, acceleration of the golf ball in the *y*-direction. Numerically, we expect that the acceleration of the golf ball will be *g*, which is approximately equal to 9.80 m/sec^2. But we have yet to scale our movie, in order to find the conversion factor between pixels and meters.

Before we scale the movie, let's look at our results in terms of pixels, and take a look at how accurate those results seem to be.

E.   Final Analysis – Fitting Your Data; Looking at Data "Jitter"

Now for some more quantitative analysis.   First, let's fit a quadratic curve to *y* as a function of time. The computer can show us the quadratic function that "comes closest" to passing through all of the data points on our *y vs. t* plot.

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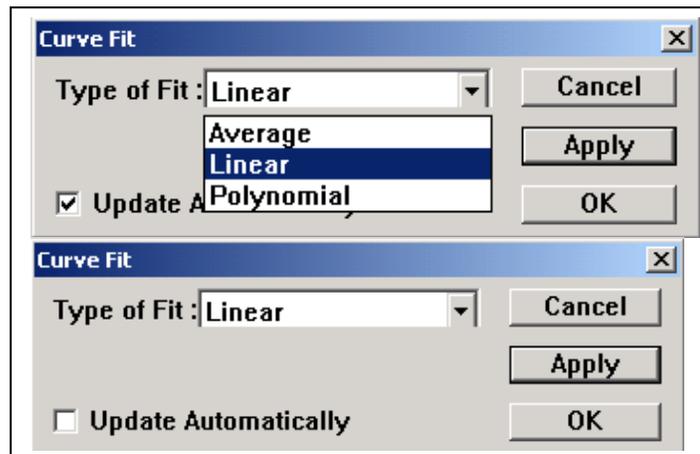
To do this, go to your graph of y-position *vs.* time, and then click on the red F (for Fit) button near the upper right hand corner. A dialog box will appear. Select the Polynomial option, and then choose Order 2, and click on Apply. VideoPoint's best-fit line should appear on the graph.

At the top of the graph is the algebraic function resulting from the plotted fit. (Here, x refers to whatever is the horizontal variable on the graph, not to our x coordinate. Also, you should ignore the computer's R^2 (R-squared) parameter. It is simply one of several statistical measure of the goodness of the fit, and VideoPoint often calculates it incorrectly.)

The number multiplying the squared term in the fit corresponds to the "*1/2 g*" factor in equation (2). If we multiply our number by 2, that gives our experimental value for *g*, in units of pixels/sec^2.

Before moving on to scale the movie, and get *g* in m/sec^2, let's look at how closely our data follows the computer's "best fit" curve.

Zooming in VideoPoint; Looking at Data "Jitter"

Maximize your plot of *y vs. t*. Probably all of the points lie close to the curve, but how close?

"Magnify" a region of your VideoPoint plot, by "zooming in" on it with the following steps:

5. While holding down the Ctrl (Control) key of your keyboard, hold down on the left mouse button and use the cursor to draw a rectangular box around a region of interest containing one of your data points. The graph then shows only that region, in a magnified view.

6. If you want to magnify some more, do it again. (But if you magnify too much, the fit line may disappear, and you'll have to start over by returning to the unmagnified graph.)

7. When you want to return to your original unmagnified view, just hold down on the Ctrl key and double-click with the left button of your mouse.

Use the zooming feature to look at several points of your graph, and to judge how far the points are vertically away from the best-fit curve. **Record your results, and calculate the average of the magnitudes of the deviations that you found**. [WARNING – after a "zoom," the scales on VideoPoint's graphs may not be quite what they seem. Often, there is an additional decimal place which is not shown on the screen's axis labels. To correct this, you can double-click on the axis, and delete the extra digit to cure the problem. For now, it is easier to use the cursor readout at the lower left corner of

Graph Scale			
Axis:	Vertical	▼	Cancel
Min:	-4.000	Max: 1.000	Apply
☑ AutoScale		Ticks: 5	OK
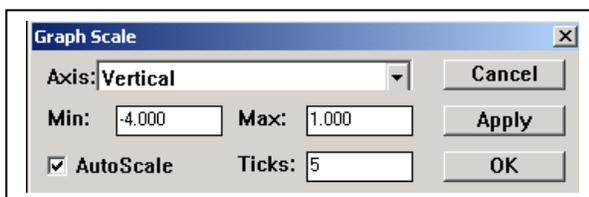
the graph to estimate the differences between your data point and the nearby curve.]

The average deviation which you have calculated is an estimate for how much an individual measurement varies from the "true" value indicated by the curve. It includes the random ± 0.5 pixel "quantization error," plus any inaccuracies you introduced in picking off your points.

F. Using **Excel** for Further Analysis

Spreadsheet programs are powerful tools for data analysis, in finance and other fields as well as in science and engineering. Our version, **Excel with WP Tools**, includes extensions generated by an academic project, **Workshop Physics** (WP), to allow easier generation of graphs from spread sheet data. It also provides more information than **VideoPoint** does when fitting functions to graphs. Although we will use **Excel** more generally later this semester, we would like today to export your data into **Excel**, generate graphs, and do fits to your **YT** data much as you did using **VideoPoint**.

To export your data into **Excel**:

1. Maximize the data table window in **VideoPoint**, and click on the heading box for **time**. Then hold down the *Ctrl* key and click on the **y-pos** heading box. Those data columns should then be highlighted.

2. Use the **Edit / Copy** command to copy the highlighted data.

3. Open the **Excel with WP Tools** program in the **Physics 103** folder. When the program starts up, click **No** when it asks if you want to reopen **WPTools**, and then use **File / New Workbook / OK** to start up a new spread sheet. Then click on an open data cell and click on *Edit / Paste* to enter the data you copied from **VideoPoint**.

To make a graph in **Excel with WPTools**, you have to identify what you want plotted:

1. First, "swipe" your cursor across the data you want to plot on the horizontal axis. (In this case, swipe across the **time** values.)

2. Then, while holding down the **Ctrl** key, swipe across the corresponding data you want for the vertical axis (the **y-pos** values).

3. Click on **WPTools / Scatter Plot** to generate the plot.

You should check that your **YT** plot is equivalent to the one you had in **VideoPoint**.

Now, let's do a quadratic fit in **Excel**:

1. Click on one of the data points of your graph, to select and highlight the points plotted.

2. Click on **WPTools / Polynomial Fit** and choose **Order 2** to create the fitted function. Several boxes describing the fit should appear on your graph window.

[If not, go to WPTools / Preferences and make sure that the options for displaying fit equations and statistics on the plot are both checked. If not, check them, and redo the plot.]

3. Note that the parameters of the fit are

    a_0 = the constant term in the quadratic equation

    a_1 = the multiplier for t in the equation

    a_2 = the multiplier for t^2 .

    These Excel values should agree closely with your VideoPoint fit results. [As before, ignore the R^2 parameter value.]

4. Note the value for σ (lower-case Greek letter sigma). This is the computer's estimate for how far a typical data point deviates from the fitted curve.

    Does the value of σ seem reasonable, given your own estimates of typical deviations of your data points from the VideoPoint curve? **You should comment on the magnitude of the typical deviations, and on possible causes, in your notebook.**

5. Note also the values for $SE(a_0)$, $SE(a_1)$, and $SE(a_2)$. These are values of what are called the "standard errors" in the values which the computer calculated for a_0, a_1, and a_2. These are the computer's estimates for how precisely your data determine the fit parameters, given the "jitter" of the data about the curve.

6. You can consider Excel's value for a_2 and $SE(a_2)$ to be your physics result for the value of $1/2$ g and the estimated error in that value. (Still in units of pixels/sec^2, of course, since we still haven't converted pixels to meters.) Record your result for g in your lab book, and calculate the relative precision, $SE(a_2)$ / a_2.

It is actually better to use a fit of order one higher than you think is needed to represent your data, and check that the value, a_n, of the highest-order parameter is consistent with zero to within the standard error $SE(a_n)$. In the present case, try a fit using Order 3.

Before we convert pixels to meters and get our final physical result for the value of g and its uncertainty, let's review some of the concepts we have used.

1. Things are accelerated by gravity in the vertical direction, but not horizontally.

2. Constant acceleration leads to a quadratic variation in position.

3. An electronic camera can measure positions in pixels, which have later to be converted to meters. The integer-pixel position measurement leads to an uncertainty of $\pm$ 0.5 pixel in our knowledge of the true position.

4. Every individual measurement is inexact, and we need to understand what a typical uncertainty in a measured quantity might be.

5. Our result for the value of g (in pixels/sec^2) is inexact, because the measurements are inexact.

Our focus on the deviations of measured points from the fitted line assumes that the fit itself is our best measure of the true function, and that the deviations are due to random jitter in the individual measurements. Our manual study of the typical deviations, and the computer's calculation of σ, are attempts to characterize the range of random variations (including pixel quantization, mouse vibration, *etc.*).   These are examples of <u>random errors</u>, which are presumed to exist independently at each measurement.   Other errors affect all of your measurements at once. They are called <u>systematic errors</u>. An example occurs when you convert pixels to meters in your video. The conversion factor clearly affects all of your points at once.

G. Scaling the Movie

We have now to determine how many pixels in our picture correspond to one meter in the real world. We will do this by measuring how long a meter stick was in our video, as measured in pixels. The resulting value for our conversion factor (pixels per meter, or the inverse, meters per pixel) will allow us to convert any measurement from pixels to meters (or pixels/secs to meters/sec^2).

Our conversion factor will have some error, which we can estimate based on the pixel quantization error, plus a "guesstimate" as to how reproducibly we can position our cursor on the ends of the meter stick. Once we know the conversion factor, and its estimated error, we can use it to convert all of our measured *x*- and *y*-positions to meters from the origin, rather than pixels. But the conversions will all involve the factor, which has some error associated. This is a systematic error, since it affects all measurements equally.

First, choose any frame in which the meter stick is clearly visible. Move the cursor manually to either end of the meter stick, and note the pixel coordinates at the two positions, from the live display at the lower left corner of the movie screen. Calculate the distance between the two points, in pixels. Using the distance in pixels, calculate your movie's scale factor, *F*, in units of pixels per meter. Use this to convert your value for the acceleration of the golf ball from pixels/sec^2 to m/sec^2.   Note the result in your lab book.

Is your answer close to the expected value of g? What effects might <u>systematically</u> bias your answer away from 9.8 m/s^2 ?

Now let's consider what is the accuracy with which your measurements determine a value for *g*. If the uncertainty in g is called Δg, then the <u>fractional, or relative, uncertainty</u> in *g* is defined as Δg/*g*.

When we want to estimate the uncertainty in our determination of *g*, we will have to use both the computer's estimate of the effects of random error, $SE(a_2)$, and our estimate of the uncertainty in the conversion factor *F* from pixels to meters. That is,

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\text{SE} \ (a_2)}{a_2}\right)^2 + \left(\frac{\Delta F}{F}\right)^2},$$

where F is your scale factor. <u>It is up to you to estimate a value for ΔF, perhaps by repeated trials.</u> Taking responsibility for estimating the accuracy with which you make a measurement is not easy, but you have to do it. Take time to think it through!

<u>Record your final result for *g* and for Δ*g*</u>. Note any comments you might have on the reliability of your conclusion, and on a comparison of your result with the accepted value for *g*, namely 9.80 m/sec².

H. Storing Your VideoPoint File

You will want to store your VideoPoint analysis file. As before, put it in your lab section's folder.

First, click on File → Save As in the VideoPoint menu bar. As before, use the up arrow and double-clicking to cause your section's folder identification to appear in the Save in: box at the top of the dialog box which appears. It will be convenient to use the same name as the one you gave the video file which you saved at the end of the VideoPoint Capture activities. This name should be visible at the top of the movie screen, if you have forgotten it.

If you use Windows Explorer to look at the contents of your section's data folder, you will see that there are actually three files associated with any movie that you have analyzed. A large file with the extension .mov contains the actual movie. A small file, having the same name with the extension .mov.#res added comes along with the movie file. The VideoPoint file .vpt is also a small one. It has whatever name you gave it, but can be used only if the movie file is also available on your computer.

PRELAB Problems for Lab#1: Encountering the Equipment; Motion in Two Dimensions

1. Assume that in a digital image there are 600 lines and 800 columns (typical for some computer display screens). Each point in the image lying at the intersection of a line and a column is called a picture element, or "pixel." Each pixel has its own brightness and color. The intensity of each of the three primary colors at each pixel may be described by an integer number, lying between 0 and 255. Each number takes 8 digital bits, or 1 "byte," to define.

   (a) How many pixels are there in the image?

   (b) How many bytes of computer memory are required to record the image?

   (c) If a video or movie consists of a series of 30 such images every second, how many bytes of information must be transmitted every second in order to display the movie? How many bits per second? (This is what sets the electrical engineer's design specification for a television system.)

2. Isaac Newton is said to have considered the motion of an apple falling from the tree in thinking about gravitational forces. If we look at the apple's vertical motion, starting from the instant when it is released from rest, the distance it falls increases with time according to the formula for uniform acceleration,

$$s = 1/2 \, g \, t^2 \, .$$

Galileo, who died the year Newton was born, studied motion under uniform acceleration, but did not have Newton's mathematics. He concluded that

> *Starting from rest, distances traveled in successive equal increments of time are in the proportions 1 : 3 : 5 : 7 : ....*

   (a) Using the formula $s = 1/2 \, g \, t^2$, calculate the vertical distances traveled by a ball falling from rest after 0.1, 0.2, 0.3, 0.4, and 0.5 seconds. (To honor Newton's heritage, let's use English units and take $g = 32$ ft / sec^2.)

   (b) Show that your results are consistent with Galileo's description.

If you are intrigued by this result, you might want to prove that Newton's result produces Galileo's rule, for any set of equal time increments. Hint: think about the quantity $[(N + 1) \, \Delta T)]^2 - [\, N \, \Delta T)]^2$.

LAB #2: Describing Measurement Variability

BEFORE YOU COME TO LAB:

1. **Review the "Data Analysis with Excel" material (Appendix A).**
2. **Read Taylor's Sections 4.1-4.4.**
3. **Carefully read at least sections A through C, below, and scan the remainder of the lab writeup.**
4. **Consider the optional the PreLab problem set attached.**

A. Introduction

Physics, and indeed all science, is based on observation and measurement. Laws of science are formulated based on certain observations and tested by further measurements. One of the goals of introductory physics labs is to provide you with experience in handling measurements – gathering your own data, summarizing and describing it, taking responsibility for working with it, and reaching defensible conclusions based on it.

Physical measurements are generally susceptible to unknown and/or uncontrollable variations. For example, a wooden meter stick may change length based on changes in the humidity in the air. The current in an electrical circuit is subject to random fluctuations based on the quantization of the charge on sub-atomic particles, and on their thermal motion. Or we simply may not know how accurately a meter is calibrated.

Taking responsibility for measurements implies taking responsibility also for estimating the uncertainty in raw data and in any calculated results which follow from the measurements. Otherwise, how could one ever say whether a new result is, or is not, consistent, with an old law? Or, how could you know that two mechanical parts will actually fit together, since neither could have been manufactured to <u>exactly</u> the specified dimensions? These questions lead scientists and engineers to define <u>tolerances</u> for dimensional accuracy in manufacturing parts which must fit together and to define <u>units of measurement</u> very carefully. No longer is our unit of length based on some king's foot, or on scratches made on an iridium bar. Nowadays, we use wavelengths of light to define the meter.

The very assumption that "Laws of Nature" exist implies that experimental results are reproducible. But the inevitable existence of variations in measurements implies that real results are not <u>exactly</u> reproducible. And, of course, we are neglecting the effects of quantum mechanics, in which a single measurement cannot be made or predicted with arbitrary accuracy, but only the statistical properties of nominally-identical measurement trials. At all levels, variability in measurement is a reality.

Very often, a scientist's response to variations in his or her measurements is to make repeated trials, and note how irreproducible a measurement turns out to be. As you might presume intuitively, the best use of a set of not-quite-reproducible, nominally equivalent, measured values involves taking their average, and assuming that the result is better at approximating the true value than is any single measurement. But we need to be able to characterize the range of variation seen in any set of data, and to estimate the accuracy with which we think we have determined our best value. This, in turn, requires developing a language with which to communicate our results, their variations, and an estimate of the true value of the quantity which we have measured.

In this lab, you will make repeated measurements similar to a "coin tossing" experiment, where the experimental question is, "How many coins will come up "heads" when N coins are tossed in the air and allowed to fall back down?" Simply by symmetry, we expect that, on average, 50% would come up heads, and 50% tails.

In our case, we have provided you with "not quite cubes" to toss. These started as symmetric cubes, but we machined off two opposite faces so that the length of one of the sides is less than that of the other two. We have colored the two machined faces red, for easy identification, and the question will be, <u>If we toss N "not quite cubes" in the air, how many red faces will be "up" after they fall back down?</u> If we had a true cube, symmetry would tell us that the answer would be $N/3$. In our case, you won't know the answer until you take some data. And then there will be the question of variability in individual trials, and of characterizing that variation.

We will address these questions experimentally. We will first focus on the <u>average</u>, or <u>mean</u>, of our measurements. Then we will define and apply a measure of variability, the <u>RMS (root mean square) deviation from the mean</u>, also known as the <u>Standard Deviation</u>, which is used throughout science, engineering, and in any other field which relies on statistical analysis of data (economics, epidemiology, testing of pharmaceuticals, *etc*.).

The statistical language which we will develop in this lab will be of use to you throughout your professional lives, and will also be important in such areas as understanding newspaper accounts of "cancer clusters" or other statistics-dependent topics. You will, of course, be expected to apply the language to your later activities in this course.

B. Conceptual Outline

It is important not to get lost in the jargon of uncertainty analysis. In this lab, we will work through the following:

1. Physical measurements are often irreproducible, for many reasons.

2. In response, we often take repeated measurements of the same thing.

3. Having made a set of repeated measurements, we have to decide how to handle the slightly-varying results.

4. Theory confirms the intuitive feeling that the best estimate for the "true" result, given a set of N nominally-equivalent results, is gotten by <u>averaging the results</u> (that is, adding them up and dividing the total by N). NOTE – "average" and "mean" are the same thing.

5. We need some way of describing the range of variation in our results, relative to their mean. We need also to be able to estimate the accuracy with which the average determines the true value.

6. The <u>Root Mean Square of the Deviation from the Mean</u>, = the <u>Standard Deviation (SD)</u> are useful ways to characterize the deviations of a set of measurements from their mean. The result is often labeled with the Greek letter sigma (σ_x, if we are measuring x).

7. The accuracy with which a set of N measurements determines their own mean, or the final best estimate of the measured quantity, is given by the <u>Standard Deviation of the Mean (SDOM)</u>, calculated by dividing the Standard Deviation of the measurements from their mean by the square root of the number of measurements.

C. Analysis Tools

We have provided an Excel spreadsheet for use in analyzing your "coin toss" (actually, "cube toss") data. The following figure shows what you will see when you open Princeton Coin Toss Program in your Physics 103 window on your computer desktop.

The spread sheet is set up, in its Primary Data Cells column, to record the numbers of "hits" (*i.e.,* of "Red Faces Up" cubes) which are found in each of up to 30 sequential tosses of some fixed number of cubes, N.

Every time a number is entered in one of the Primary Data Cells, it shows up as a dot on the Scatter Plot. This graph plots the Number of Hits observed on the horizontal axis, and can accept up to 30 points before it runs out of vertical space.

The frequency chart, or Histogram, is a little more complicated. After you have entered some number of results for hits observed, it counts how many times you entered 1, how

many times you entered 2, and so on, up to a maximum number of 30 hits observed. The horizontal axis, as in the Scatter Plot, shows the number of hits. The vertical axis shows the number of times you got a given value for the number of hits.

In preparing for lab, you should check a few entries in the Data Cells, and confirm that the corresponding dots show up on the Scatter Plot where you would expect. You should also check that the histogram shows the correct number of occurrences for two or three of the possible hit results. *Do you really understand the data table, and both plots?*

D. The Experiment - Things to Do (Phase 1)

Now let's take some data of our own.

First, delete any existing entries in the entire data section of the spreadsheet. (Just "swipe" across the 5 columns and 30 rows to highlight the data region, and hit Delete.)

- Next, **put 20 "cubes" in the large box, toss it around vigorously for a while, and then count the number of hits (red faces up)**. Enter the number in the top cell of the Primary Data column. That's the first trial of your experiment. *Did your point show up on the two graphs in the way you would expect?*

- **Repeat the experiment for a total of 30 trials**, entering your result each time in the next cell in the Primary Data column of the spread sheet. Check that the two graphs are behaving properly.

Print out your data sheet and graphs to look at and use in the discussions of the following section. (If you want to print out only a graph, just highlight it before issuing the print command.)

You have now recorded the results for 30 independent measurements of the same quantity. Their variability, although quite large, represents the variability in any physical measurement. Getting different results in tossing coins is not, in principle, different from getting different results in repeatedly measuring the length of your foot. In either case, the question becomes, What is the "best value" of the measurement? And then there is the issue of how we describe the variability in the measurements. These questions are the subject of the next Section.

File   Edit   View   Insert   Format   Tools   Data   Window   Help

L15

	Trial Number	Data Cells	
		Primary	Secondary
1	15		
2	17		
3	14		
4	15		
5	18		
6	18		
7	16		
8	16		
9	17		
10	18		
11	14		
12	14		
13	19		
14	14		
15	21		
16	18		
17	17		
18	16		
19	15		
20	13		
21	14		
22	15		
23	14		
24	12		
25	13		
26	11		
27	15		
28	15		
29	18		
30	16		

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Scatter Plot (Primary Cells Only)

Trial Number vs. Number of Hits

Frequency Chart (Histogram of Results)

Number of Occurences vs. Number of Hits

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E. Looking at Your Data – Averages and Deviations

If our cubes were truly symmetric, we would expect, on the average, a red face would land "up" 1/3 of the time. (Two out of the six faces of the cube are red.) On the other hand, if we had shaved down the faces so far that the "cube" would have looked like a thin, square, "coin," we would expect that a red face would show up almost 100% of the time. (There is little chance that a real coin will wind up standing on its edge.) On this basis, we expect that the average number of hits that we observe might be greater than 7 (~1/3 of 20) and, of course, less than 20. *How does your data look?*

- **Using your graphs, record in your lab book an initial "guesstimate" of the average number of hits.**

Your first estimate of the "true" number of hits to be expected can be based on an "eyeball" judgment based on the clustering of the points along the *x*-axis of your Scatter Plot, or of the peak or center of the Histogram. But that's not a very precise way of describing either the average or the variability in your results.

- **To more precisely characterize your data, calculate the following**. (You can do this manually or, ideally, take the opportunity to do it in Excel, using the blank spreadsheet cells in the middle of the page.):

  a) The **<u>average</u>** of the hit results for your 30 trials. (Add the numbers up and divide by 30.)

  b) For <u>each</u> trial, the **<u>deviation</u>** of that result from the average just calculated.

  c) For the 30 trials, the **<u>average of the deviations</u>** just found in each trial. (If this doesn't turn out to be zero, there's something wrong!)

The average is, as is discussed in the Error Interludes, your best estimate for the "true" number of hits to be expected. But, since the average deviation is always zero, <u>we still have to find a useful and quantitative way to describe how far the number of hits in any single experimental trial is likely to deviate from the average number</u>. (This is, in some sense, a measure of the width of the Histogram.)

The standard way of quantifying the variations in a set of nominally-equivalent measurements is to use what in Engineering is called the RMS deviation (from the mean), and in other fields may be called the Standard Deviation (from the mean). (We ignore for now a slightly different definition of the Standard Deviation that is sometimes used.)

This way of characterizing fluctuating measurements, or the data sets resulting, is so important that it is worth putting in a box:

We define the **Root-Mean-Square (RMS) Deviation from the Mean** *of any set of data by the following operation*:

Take the Square Root ....

of the Mean (or Average) ....

of the Square ....

of the Deviations of each of the data values ....

from the Mean of those values.

You will note that taking the square root of the square ensures that the RMS Deviation associated with any physical quantity will be positive, and will have the same units (if any) as the original quantity (length, time interval, counts, ...).

The RMS Deviation, or the Standard Deviation, is an estimate of how far a typical **single** measurement will deviate from the mean of a large number of measurements. It should be equal to roughly half the width of the peak graphed in your histogram.

You should be able to judge the average value of the measurements which were used to create your histogram to considerably better accuracy than the half-width of the peak. *Can you?* The accuracy with which you can make this judgment is the final accuracy of your experimental results. You are using multiple measurements to improve the precision in your conclusion. This requires a standard way of expressing that precision. This is the final result in our mathematical description of variability.

The **Standard Deviation of the Mean** of a **set** of measurements is an estimate of how accurately we can judge our "best value," or how precisely we can judge the center of the frequency chart, or histogram. It is calculated by dividing the Standard Deviation of the measurements themselves by the square root of the number of those measurements.

F. Formal Summary of Means and Standard Deviations

(a) The best value for a set of measurements is determined by taking their **average**, or **mean**:

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_N}{N}$$

or

$$\bar{x} = \frac{\sum x_i}{N}$$

(b) The **RMS Deviation from the Mean**, or the **Standard Deviation** (SD), is defined as

$$\sigma_x = \sqrt{\frac{\sum (\bar{x} - x_i)^2}{N}}$$

> NOTE 1: If our measurement variations follow what is called the "Normal Distribution," = "bell curve" = "Gaussian statistics", we expect that <u>approximately 2/3 (more accurately, 68%) of any set of measurements will fall within ± 1 σ of their true mean value</u>. Engineers, especially, will find it useful also to know that roughly 95% of all such measurements are expected to fall within a ± 2 σ range, and 99.7% within a ± 3 σ range. But, be wary – not all measurements follow the Normal distribution.

> NOTE 2: An alternative definition of the Standard Deviation, which matters only for small values of N, replaces the N in the denominator by (N-1). The difference will not concern us here, but is OK to use a calculator that has a "Standard Deviation" function which uses the alternative definition.

(c) The **uncertainty in our estimate for the best value** for a set of N measurements having standard deviation σ_x is obtained by dividing their standard deviation by $\sqrt{N}$. This quantity is called the ***standard deviation of the mean (SDOM)***:

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

Intuitively, we see that our uncertainty decreases as we average more measurements.

H. Thing to Do (Phase 2)

Now let's do the calculations for the results of your 30 tosses of the 20 cubes:

If you haven't already done it, use some of the blank cells to the right of your data array in Excel to calculate

- **the average, or mean, or your 30 measurements**
- **the deviations of each of your results from their own mean**
- **the squares of those deviations**
- **the mean of the squares of the deviations**

Now, **take the square root to get the RMS measure of the variability in your results.**

- **Does your result look reasonable, in terms of your histogram? Indicate the average, and the range of ± 1 σ about the average, on a printout of your graph. What fraction of your data points fall within this range? Is it close to 2/3 (or 68%)?**

Finally, **divide the RMS deviation of the measurements by the square root of 30 (the number of measurements you took) to get the Standard Deviation of the Mean**. Record the final result in your notebook as

N_{red} = your average value ± your Standard Deviation of the Mean .

We might ask **<u>What is the probability that any single cube will land with a red face up?</u>** As you would expect, this is gotten just by dividing N_{red} by 20, to calculate the average value. **Call this probability *p*.** *What is your result?*

How precisely have we determined *p*? (Just divide the standard deviation of the mean of N_{red} by 20.) **Record your final result for *p* (including its uncertainty) in your notebook.**

Probability theory predicts the relationship between *p* and the variability in our results for N_{red}. The only assumption required is that all our "cubes" are identical, and they all fall independently of each other. The predicted standard deviation for N_{red} depends on *p* and on the total number of cubes (N_{total} = 20) which were tossed each trial.

$$\sigma_{N_{red}}(\text{predicted}) = \sqrt{(1-p)\,p\,N_{total}}$$

How does your measured standard deviation compare to this predicted value?

NOTE: Since p times N_{total} is simply the number of red faces that we expect when N_{total} cubes are tossed, p times N_{total} is just equal to the average value for N_{red}. So the above equation becomes

$$\sigma_{N_{red}}(\text{predicted}) = \sqrt{(1-p)} \quad \bullet \sqrt{N_{red}(\text{average})}$$

That is, the predicted standard deviation describing the variability in N_{red} is proportional to the square root of the value of N_{red} itself. In our case, you have probably determined that p is roughly 0.5. Then, the factor relating the standard deviation to the average expected value for N_{red} turns out to be roughly equal to the square root of 0.5, or 0.707.

IMPORTANT GENERALIZATION FOR SCIENCE LITERACY

In more realistic physics situations (involving such things as radioactive decay, occurrences of disease in human populations, *etc.,* rather than coin tossing), it is very common to have the theoretically expected variation in a counting measurement be equal to the square root of the number measured, (rather than .707 times the square root). This is a very important result, applicable to such things as the number of Merit Scholars graduating from a high school, the number of cancer cases occurring near power lines, and similar societally-important issues

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I. Things to Do (Phase 3)

Making Things Easy in Excel Calculations

There are several Excel functions that you could have used to speed up your calculations earlier in this lab. These include SUM, AVERAGE, and STDEVP.

In each case, these functions operate on a range of values to give a result:

> SUM (...) calculates the total of the values in the range of cells indicated by ...
>
> AVERAGE (...) calculates the average of the values in a range
>
> STDEVP (...) calculates the Standard Deviation of the values in a range of cells from their average (using N in the denominator)

To use the SUM function, choose the cell in which you want the answer to appear. Click on that cell, and type = SUM (. Then either enter the range of cells manually, or just swipe the cursor from top left to bottom right of the region of the spread sheet that contains the values you want to sum. Close the parenthesis, and hit Return, after which the sum appears.

You may want to take time to repeat your earlier calculations, if you did them "manually" in Excel. Just check that you get the same answer using the neat functions above.

Using Excel's Random Number Generator to "Do the Experiment"

If you have time, you might want to play with the same experiment, done in the computer. Excel can generate data for as many "tosses" as you like, following the mathematical rule for coin-tossing type experiments (Binomial Statistics). It can fill both the Primary and Secondary Cells in your spread sheet, so that you can see the prettier results that come from repeating the experiment many more times.

To use Excel's random number generator, refer to the following figure. The Random Number generator screen appears when you click on Tools / Data Analysis / Random Number Generation in Excel's menu bar. Then choose values for the parameters indicated, and hit Return. The numbers should be entered into your data range, and the graphs should update themselves to reflect the results.

You might set up your spread sheet to calculate averages, standard deviations, and standard deviations of the mean for the combined primary and secondary data cells. Then, filling all these cells repeatedly using the random number generator, you can observe how much these quantities vary from run to run. *Do the variations fall in the expected range?*

PRELAB Problems for Lab#2: Describing Measurement Variability

In this PreLab, the only mathematics that you need is the concept of average, or mean value. That's easy, of course, but you need to apply it in the context of the statistical concepts which are the focus of the lab.

1. Review the full-page Excel spread sheet shown early in the writeup, so we can do some "guesstimating."

   a) What is the largest data value shown? What is the smallest? Can you see how each of these appears on the graphs?

   b) Simply from looking at the frequency chart, estimate the average value of the counts recorded in the 30 trials.

   c) Also just from looking at the graphs, estimate the range of values, centered on your estimated average, which contain 2/3 of the data points.

2. Let's do some more quantitative work:

   a) Make a "data column" of any 8 integers lying between 10 and 15. Some will be listed more than once, of course, but don't make them all the same number.

   b) Sketch the frequency chart for your data entries, and record the "guesstimates" for average and the range of variation, as in Problem 1.

   c) At the bottom of the data column, calculate the average of your numbers.

   d) To the right of the data, make a second column of numbers recording the deviations of each of the data entries from their average. At the bottom of this column record the average of the deviations. (It better be zero!)

   e) Make an additional column recording the squares of your deviations, and calculate the average of these. Then calculate the square root of that average.

   f) Compare your quantitative results to your "guesstimates."

LAB #3: FREE FALL, TERMINAL VELOCITY

BEFORE YOU COME TO LAB

- **Read Taylor's Section 2.7 (on fractional uncertainties) and Section 3.3 (on provisional rules for handling sums, differences, products, and quotients in propagating uncertainties). You should continue to use the concepts of error analysis in recording your lab notes. Important elements include the confidence level associated with an estimate of uncertainty, proper use of absolute and relative error descriptions, and avoiding misuse of significant figures.**
- **Read the writeup for this lab, and plan how you will approach it intellectually.**
- **Consider the optional PreLab problem set attached.**

A. Introduction

The emphasis in this lab is once again on falling bodies. Previously, we worked with simple theoretical models leading to such equations as $v_y = - g\,t$ and $y = -1/2g\,t^2$. These equations, of course, ignore air resistance.

The reason our simple analysis succeeded is that our falling objects (golf balls, for example) were so dense and heavy, and moved at such low speeds, that the drag force due to their motion through the air was very much smaller than the force of gravity on them. At increasingly high velocities, the air drag would become larger and larger, and would no longer be negligible in comparison to mg.

This week we will use the same camera setups to explore the behavior of <u>light</u> objects – so light that the air drag forces as they accelerate will become comparable to $m\,g$. We

will let the bodies drop vertically, so that there is no *x*-component of their motion to complicate things for us.

Immediately after being released from rest, even a light falling body will accelerate according to the simple formulas used previously. This fact follows simply from the assumption that viscous drag forces are zero if the velocity of the body is zero, and rise smoothly as the body's speed increases.

As a vertically falling body accelerates, the drag force increases gradually, and always acts in the opposite direction to the velocity of the body. For a body moving vertically, the net force which it experiences is the usual *m g*, acting downward, plus the oppositely-directed viscous drag force, which we will call f_v. The equation of motion becomes

$$m \, a_y = f_{\text{total}} \ = \text{-} \, m \, g + f_v \, ,$$

where we have taken the - axis of our coordinate system to be pointing upward *(i.e., opposite to the direction of gravity)*.

At some speed, the drag force f_v will be equal to the weight *m g*, and the total force on the body will be zero. After this, it will no longer accelerate. The resulting constant speed is called the <u>terminal velocity</u>. It is approached only gradually, of course.

The goal of this lab is to study the behavior of a very light falling object, and to demonstrate the full range of behavior as it starts from rest, increases speed, and approaches terminal velocity. From this, we can learn something about the drag force, f_v. In particular, we expect you to be able to reach some conclusions about how the air drag depends on the velocity of the falling object.

This write-up for lab #3 is much shorter than for the previous labs. As you get more comfortable with the equipment, we're leaving more of the thinking to you. Take some initiative on your own to try things out. Remember to keep a good record of your work in your lab notebook.

B. The Experiment – Getting the Data

In the PreLab, you should have developed some idea about what your movie's curves of position, velocity, and acceleration should look like for a body which starts from rest and approaches terminal velocity before it leaves the camera's field of view.

Your first task is to find a body which demonstrates the expected behavior in your video. This may take several attempts, with bodies of different size and weight.

As candidate "light objects," we have provided ping pong balls, fluffy cotton balls, and some pieces of expanded styrofoam packing material. Your first goal is to find an object which displays the behavior of a body accelerating from rest and ultimately traveling at

or near its terminal velocity. An entire cotton ball, as received from the drug store, may be too heavy for our purposes. But you can pinch off smaller and smaller portions of it (taking care to keep them fluffy, and shaping them into roughly spherical shapes before dropping them). Ultimately, they should demonstrate the expected behavior. Or, you may decide that the styrofoam pieces work better for you.

In any case, your first actions should be to

- Make trial runs with a candidate object, dropping it from one hand simultaneously with dropping a golf ball from the other. Do they reach the floor at significantly different times? If not, you might want to try a lighter (smaller) object. When you observe significantly different behavior, record a qualitative impression of what you see in your notebook.

- Once you have chosen your object, open VideoPoint Capture and use the Preview Screen to check that it will be visible as it falls. You may have to increase the brightness of your image by rotating the aperture ring on the lens. Be sure to include a meter stick in the field of view.

- Set the video capture rate to 30 frames per second, so that you can observe the more complex behavior of this week's falling body.

- Take a movie, open it in VideoPoint, digitize the points, and see if you can see the initial $y = \frac{1}{2}\, g\, t^2$ behavior, followed by a transition to terminal velocity, $y = v\, t$.   You will have to look closely at your graphs, but if you can't see these regions pretty clearly in your plot of y vs. t, you should try a lighter or heavier object, and take a new movie.

- Scale your final movie, so that your analysis will be in terms of meters (and m/sec and m/sec^2) rather than in pixel units. (See the box below for an easy way to scale a movie.)

NOTE: You can use VideoPoint to make it easy to scale your movie. With the movie screen selected, click on Movie / Scale Movie. Then follow the instructions in the dialog box to enter the length on the meter stick you will be using to scale the movie, and to click on its ends. (Ignore the other options on the dialog box.) Voila! The pixel coordinates are changed to meters, both in your data table and on your graphs. It's quick and easy, but you know from your previous manual scaling work that there are hidden considerations of accuracy here.

C. Analyzing the Data

NOTE: In the following analyses, you may want to put most of your effort into looking at the <u>shape</u> of the plot of y vs. t and making linear or quadratic fits to portions of that curve.[1] It is tempting to use VideoPoint's graphs of v_y and a_y , but these are based only on the position data, and VideoPoint's numerical differentiation methods can lead to odd results at the beginning and ending points of a data sequence. If you use the velocity and acceleration graphs in reaching your conclusions, at least make sure that your thinking is confirmed on the initial y,t graph. After all, your raw data consists <u>only</u> of position versus time. Everything else (v or a) is derived from that.

In analyzing your graphs of position, velocity, and acceleration, remember that you can use VideoPoint's Zooming capability, and apply a fit function which is updated automatically per the Curve Fit parameter screen option. (See the box on the next page in case you've forgotten how to Zoom in VideoPoint.) With this capability, you can test whether you can make successful fits to portions of your data, with fit functions that don't work very well when applied to your entire data set.

- *Do your curves of position, velocity, and acceleration show the expected behavior?* If not, you may need to adjust your expectations, or think further about what may be going on. In any event, print out and discuss in your lab book what you are seeing in your data.

- What is the terminal velocity which your object demonstrates? NOTE – as always, you <u>must</u> give your estimated uncertainty when you report a measurement!   Think about the uncertainties in your scale factor, in the statistical jitter of the data points, and in your subjective ability to judge values from your graphs. If one of these is very large compared to the other two, then your work should focus on it. But you have to make the judgment!

- What was the initial acceleration which your object experienced at low velocities? *Is it at least roughly equal to 9.8 m/sec^2 ?*

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[1] Remember that it's always better to use a fit of one order higher than what you expect to find in the data. So, in the present case it would be better to transfer the data to Excel with WPTools for the fit analyses.

- Finally, what can you say about the dependence of the drag force on velocity? (Suggestion: force ties directly to acceleration, so you may want to look at a plot of acceleration *vs*. velocity.) This question may not have a simple answer. But it is worth a considerable amount of thought and discussion! Does it affect your thinking to remember that f_v is presumed to be zero when $v = 0$?

REMEMBER: Use the concepts and language of uncertainty analysis in determining and discussing your measurements and their uncertainties. Don't use vague, undefined language!

Once you have answered the above questions in your notebook, you may want to check in with your instructor to make sure you're on the right track. Finish up your analysis with anything else you think is important. Include any relevant information in your notebook, and be sure your notebook tells a coherent story about what you did and what you conclude from it.

NOTE: It is a fact of life that estimating uncertainties in measurements is difficult. Usually, there is simply no rule to guide you. But you are still responsible for estimating what your uncertainty is. Remember – you are recording what you think is the range of values for which you have a 68% probability of being "correct." That's NOT the same thing as taking an arbitrary guess without thought, or taking a large number which has a near 100% probability of being correct.

IN CASE YOU'VE FORGOTTEN HOW TO ZOOM. In this lab, you may find it useful to fit a mathematical function to only a part of your data. This can be done by zooming in on only the portion of your graph you want to fit to. If you hold down both the Ctrl key and the left mouse button while "dragging" the cursor across the diagonal of an imaginary rectangle, the graph will rescale to "Zoom In" on the region defined by the rectangle. Once you have zoomed in, the program makes the fit only to the portion of the graph showing on the screen.

To Unzoom your graph, hold down the Ctrl key and double-click anywhere on the graph.

If you uncheck the Update Automatically box when you do the fitting, the result will not change if you unzoom or otherwise change the portion of the graph being displayed.

PRELAB Problems for Lab #3: Free Fall, Terminal Velocity, etc.

In this PreLab, the major points are (a) truly understanding $f = m\,a$ in the context of a body falling in air (Parachutists know how important this is!), and (b) applying the concepts of uncertainty analysis to real situations.

1. Take a full page of paper, and make three sketch graphs, one above the other. Label them as plots of y, v_y, and a_y versus time, with time in the horizontal direction.

    a) Without worrying about the scales of each axis, sketch the <u>shape</u> of each of the curves which would result from dropping an object from rest and allowing it to accelerate to terminal velocity.

    b) Indicate on each graph the portions of the curve that are, at least roughly, parabolic, straight-line, and flat (horizontal straight-line).

    c) Where possible, indicate a value. (Hint: "g," or "zero.")

2. You have recently bought a used car, and you are curious about its fuel economy. You reset the odometer to zero the last time you filled up the tank, and this time, the attendant fills your tank with 12.5 gallons, and your odometer reads 335 miles. By comparing your odometer to highway road markers, you suspect the uncertainty in your odometer reading is about ± 3%, or ±10 miles. You are not sure whether the attendant "topped off" your tank, so you estimate that the uncertainty in your fuel measurement is ± 0.5 gallons. What is your car's fuel economy, in miles per gallon, and what's the uncertainty in it?

3. Elmer and Esmerelda have just made a scaled movie of a falling coffee filter. The later portions of the graph of velocity *vs.* time appears to be constant, and from the graph they estimate a terminal velocity of 1.22 m/sec. Of course, the velocity plotted on the graph isn't perfectly constant. From the scatter in the data, Elmer estimates the uncertainty in the plotted velocity values to be ± 0.04 m/sec. "Wait a second," says Esmerelda. "That figure depends on our measured scale factor, which we found to be 104 ± 3 pixels per meter. That also affects the uncertainty of the speed. In fact, I think the scale factor is more important than the variation between points on the velocity graph." What should Elmer and Esmerelda write for their uncertainty in the speed?

LAB #4: COLLISIONS, CONSERVATIONS, and COORDINATE SYSTEMS

BEFORE YOU COME TO LAB

- **Read Taylor's Sections 3.5 and 3.6. These cover the "final" rule for combining <u>statistically independent errors</u> in addition, subtraction, multiplication, and division. The concept of statistical independence is important. Two independent, and thus uncorrelated, variations may happen to cancel each other out in their effects, rather than always reinforcing. This leads to using a square root method to estimate their combined effects, rather than the direct sum which was used in Taylor's "provisional" rules.**
- **Read the writeup for this lab, and plan how you will approach it intellectually.**
- **Consider the optional PreLab problem set attached.**

<u>**Overview Comments**</u>: We hope that you will focus on two things in this lab. The first is the law of conservation of momentum, in which the lab question is whether certain quantities are the same before and after collisions between two moving bodies. (You may want to describe this as testing whether the <u>differences</u> in these quantities measured before and after the collision is zero.)

The second part of the lab asks you to think about motion in two dimensions, in situations where a constant vector force acts on a body, but its direction is not parallel to either of the coordinate axes. *How would you expect the graphs of position, velocity, and acceleration along the x- and y-directions to look? Would it make things simpler if you could do your analysis in a rotated coordinate system, so that the force <u>was</u> parallel to one of the coordinate axes? How do you relate the two analyses and their results?*

A. Introduction

The two parts to this lab both involve the motion of "hockey pucks" on an essentially frictionless air table. The air support cancels out the vertical force of gravity, and the pucks move freely in the horizontal plane. You will use the video camera and VideoPoint software to analyze their motion.

In the first experiment, two pucks are started off with some initial speeds and directions of motion. They then collide, and head off with different speeds and directions of motion. You know that, since there is no external horizontal force acting on the pucks, the sum of their vector momenta should be conserved *(i.e.,* should be the same before and after the collision). You will be asked to confirm this fundamental fact, by direct measurement of the components of the vector velocity (and vector momenta). Of course, your measurements can only confirm anything to within some experimental uncertainty, which will be part of your analysis.

In the second experiment, you will tilt the air table, so that there <u>is</u> a force acting on the pucks in the horizontal plane. What motion do you expect to observe, after launching a puck with some initial velocity? Think about plots of *x* versus *y*, *x versus t*, and *y versus t*. What similarities can you expect with the plots you made of a golf ball bouncing in a vertical plane, under the force of gravity? It will be well worth your time to think about these questions before arriving at the lab.

As a final step in the second experiment, we will ask you to determine what is the horizontal force acting on the puck (in direction and magnitude). As always, an analysis of experimental uncertainty is required here.

B. Conservation Laws – No External Forces

Start by making a video of a simple collision between two pucks. Be sure to include a meter stick in the camera's field of view, for use in scaling your movie. Weigh each puck to determine its mass so that you can later convert its velocity to momentum

If you need to change the brightness of your image, use a long meter stick to "nudge" the wooden extension of the lens's aperture ring.

You may want to check that the air table is close to level and, if necessary, adjust the three leveling feet on which it stands. Don't spend too much time on this – you'll never make it perfectly level.

Immediately after opening VideoPoint, you should tell the system that you want to measure the positions of <u>2</u> objects in each frame, rather than 1. Then you will want to pick off the coordinates of the 2 pucks. VideoPoint makes it easy to pick off the coordinates of Puck 1 in Frame 1, then of Puck 2 in the same frame, and then Puck 1 and Puck 2 in Frame 2, *etc*.

To do this:

- Check that, in the Options menu item on the movie screen, <u>both</u> the AutoFrame Advance and *Auto Point Advance* options have check marks beside them.

- Pick off the location of the first puck on the first frame, then click on Edit / Leave/Hide Trails.

- Move to the second puck, and pick off its location. Then click on Edit / Leave/Hide Trails again.

- The movie should automatically advance to the second frame, while showing the "trails" from Frame 1. If you continue to click first on Puck 1, and then on Puck 2, in each frame, the data sequence should continue from frame to frame without help from you. (If, at any time, you want to change whether a puck's trail is displayed or not, just use the Leave/Hide Trails button again, for whichever puck you want to affect.

- If your pickoff sequence gets confused, and you need to remove a point or to start over, you can use the Edit/Clear Selection on Frame and Edit/Clear Selection on All Frames commands to remove erroneous points. (Selecti*ng* a point may require carefully clicking on its position in the *Movie* screen. A selected point will appear with a double circle.)

After you have entered all your coordinates, you will want to make the usual graphs of *x*- and *y*-position, *x*- and *y*-velocity, *etc*. From these graphs, you can judge the components of the velocity vector of each puck just before and after their collision. (Remember how easily you can zoom in and fit to selected regions of your data.)

After converting velocities to momenta, you can see if (to within your measurement accuracy) momentum is conserved. Show your logic, and justify your conclusions, in your notebook.

SPECIAL NOTE ON UNCERTAINTY ANALYSIS:

- Although we normally expect you to explicitly consider the uncertainties in all of your measurements, you won't have time to do that in this lab. There are simply too many measurements (two components of momentum, for each of two pucks, at two different times).

- Please take time to carry out the usual uncertainty analysis and estimation for **one** of your tests of momentum conservation (*i.e.,* for either *x* or *y*). Then, for the other test, just check that your results seem sensible. Even for the first test, you can probably argue that your uncertainty analysis results (absolute or fractional uncertainty values) for one of the pucks, at one of the times, can simply be applied directly to the other puck and/or the other time. *Think about it!*

- Two hints that may also save you time: (1) Does the scaling factor for your movie matter at all, or could you consider momentum conservation in terms of kg-pixels-sec? (2) Is the uncertainty in your mass measurement so small that it can be neglected?

Do you find that the sum of the kinetic energies of the two pucks is the same after the collision as before (i.e., that kinetic energy is conserved in the collision)? If not, where do you think the energy went to?

C. ADDING AN EXTERNAL FORCE

Now, let's add a constant external force. To do this, lift the indicated leg of the air table and put the 1-1/2 inch aluminum cube under it. Now the air table has a downward slope, which will lead to a fixed force of gravity "down the slope." We want to study this force.

First, practice launching a puck so that it follows a deeply arcing path across the table. Then, take a video of such a path, and digitize the coordinates. Make the usual graphs of *x-* and *y*-position versus time, *etc.* Do you see parabolas in the *x,y* plot, and the *x,t* and *y,t* plots? Take a moment to discuss the similarities with the bouncing golf ball data you took in Lab 1.

Put block under <u>this</u> leg of the air table.

Wall
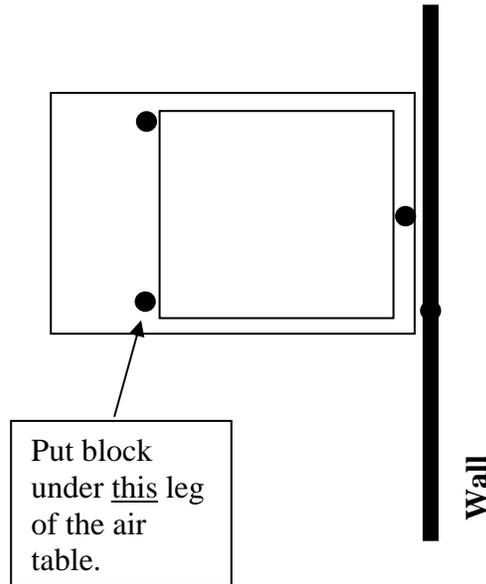
- From your graphs, determine the components of acceleration in the *x-* and *y-* directions. Multiplying these by the mass of the puck gives you F_x and F_y, the components of the force acting on the puck.
- Finally, calculate the magnitude of the force, and its direction. (With at least rough uncertainty estimates.)

D. Rotating the Coordinate System

The data taken previously with the bouncing golf ball appeared simpler that that you have just taken, in that it showed no acceleration in the *x*-direction, and simple parabolic curves in the *x,y* and *y,t* plots. But that is only because the *x-* and *y*-axes of the VideoPoint coordinate system were aligned with the direction of gravity. If the earth had been pulling the ball down-and-to-the-left, rather than straight down, things would have looked different in your plots. Similarly (and this is by logical similarity, not by accident), if your <u>camera</u> had not been aligned with gravity, your data would have appeared more complex than it did. (A few of you may have seen evidence of a small rotation of the camera when you took your golf ball video.)

In today's tilted-table experiment, we could recover the simpler description if only we could analyze the data in a coordinate system rotated to align the *y*-axis in the direction

the force produced by the tilting of the table. Surprise !!! VideoPoint has the capability of presenting our data and graphs in any rotated coordinate system !

Take a moment think in what way your graphs would be simplified if the *y*-axis was rotated to lie along the direction of the force due to the tilt of the table. What curves would be parabolic? What ones would turn into straight lines? What quantities would become zero? (Take a few minutes to discuss these things in your notebook.)

After organizing your thoughts on what to expect, check that your graph's fit control boxes have the Update Automatically box checked. Then get ready to watch how they change as we rotate the coordinate system. *This is really neat !!!*

To manually rotate coordinate systems in VideoPoint:

- With the *Movie* window active, click on the origin of the *x,y* coordinate system. A small circle should appear, centered on the origin, and with a dot where it intersects the *x*-axis.
- Now "click and hold" on the small dot which appeared. While still holding down on your mouse button, move the cursor out along the *x*-axis, and then upward in the *y*-direction.
- The coordinate axes should rotate in the rotational direction in which you move the cursor.
- Repeat as desired. Note the effects on your graphs and fit parameters. (See note below for more precise control of final rotation angle.)

Rotate the axes and watch the behavior of your graphs and fit parameters. What angle does it take to produce simple behavior analogous to the one-dimensional acceleration situation with the bouncing golf ball? Summarize the results in your lab book. (What is the general behavior of your graphs as you rotate the coordinates? What is the magnitude of the measured acceleration in the simplified system? What is its direction, as described in the original coordinate system? Do these agree with the data from your work in the previous section?)

NOTE: After you have used the cursor-based rotation to determine approximately what rotation angle gives the simple results, you can control the angle more precisely by editing the number in the Rotate (deg) figure in the box which appears when you double-click on the origin of the axes in the movie screen.

If you have time, you can calculate the expected force (or acceleration) from the geometry of the tilted table. You know how high you lifted one of the feet, and you can measure the geometry of the pattern of the three feet. It will take some trigonometry to devise the formula for the predicted angle and magnitude of the acceleration vector, but

you could get a pretty good idea of what these should be simply by making scaled drawings.

PRELAB Problems for Lab #4: Collisions, Conservations, and Coordinate Systems

1. Referring to the figure on the following page, make sketch plots of the behavior of v_x and v_y for *each* of the two pucks as a function of time. Although you don't know the scales of either position or time, be careful to get the signs correct, and to show at least roughly correct relative magnitudes.

2. Avik and Venus are calculating the momentum and kinetic energy of a puck on their air table. The puck is moving parallel to the x axis with a speed of 12.0 ± 0.2 m/sec, and its mass is 30.00 ± 0.01 grams.

   a) What is the x-component of the momentum of the puck, and what is its uncertainty? How much would the uncertainty change if the mass were precisely known (*i.e.,* if the uncertainty in the mass was negligibly small) ?

   b) What is the kinetic energy of the puck, and what is its uncertainty? How does the fractional uncertainty in kinetic energy compare to the fractional uncertainty in momentum?

Figure for PreLab #4. This is the <u>final</u> frame of a video of two colliding pucks. The *y*-axis points toward the top of the page, and the *x*-axis points to the right.

LAB #5: INCLINED PLANES AND ENERGY CONSERVATION

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BEFORE YOU COME TO LAB:

- **Deduce an expression for the acceleration *a* of a cart of mass *M* that moves up or down a ramp of angle θ to the horizontal, if it is subject to a rolling-friction force according to eq. (1).**

- **Read Taylor's Section 3.11, on the "General Formula for Error Propagation." This section deals with how to work with functions that are not simply sums, differences, products, or quotients. The language used is that of partial differentiation in calculus. Here, the effect of varying one of the independent variables is handled by differentiating the function of interest, while holding all of the other variables fixed (*i.e.*, treating them as constants in the differentiation process.)   If the language is unfamiliar and makes you uncomfortable, just remember this basic concept.**

- **Also, think about how you would use error bars in judging the adequacy of fits on graphs.   (The question might be whether a graph's fitted line passes through the error bars for roughly 68% of the points.) You may want, either in lab or before, to look at Excel's capability to add error bars to graphs. (See the box on the third page of this procedure.)**

- **Note especially the warning that follows implicitly from all of our error analysis discussions – if there is a <u>systematic</u> deviation of your points from a fitted line (rather than a random "jitter"), your data is trying to tell you something!**

- **Read the writeup for this lab, and plan how you will approach it intellectually. You will be "teasing out" some subtleties of an apparently simple motion.**

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A. Introduction

In this lab, you will use your cameras and VideoPoint software to study motion along an inclined plane. You may be surprised by some of the nuances you can capture with your video cameras. Take time to examine your data carefully, and you will uncover some

subtle physics. You may even discover a few things that will give you a head start on material for upcoming weeks.

By this time, you are starting to develop some very sophisticated tools for evaluating experimental uncertainties. In this lab, more than the earlier ones, you'll find that we've really raised the bar for error analysis. Remember, the uncertainty is an important part of every measurement you make.

This week's lab is both longer and harder than last week's. Think about it ahead of time and come to lab ready to hit the ground running. But don't worry if you don't get to the extension projects; as long as you work hard for three hours, you won't be penalized.

You have been studying the conservation of mechanical energy – for example, the sum of kinetic energy ($1/2\ m\ v^2$) and gravitational potential energy ($m\ g\ h$). You know that such things as sliding and rolling friction are non-conservative forces, and that they "steal" mechanical energy from an otherwise isolated system. If, for example, there is a constant (vector) rolling friction force $\boldsymbol{f}_r$ acting on a body that moves a (vector) distance $\boldsymbol{L}$, the amount of mechanical energy lost (and converted to thermal energy) is just $\boldsymbol{f}_r \cdot \boldsymbol{L}$. So, frictional forces affect not only accelerations (simply because they <u>are</u> forces), but also mechanical energies (because they are non-conservative).

A rolling wheel is subject to energy loss associated with deformation of the wheel (or the ball bearings in the wheel) and/or the surface on which the wheel rolls. The rolling friction can be modeled as being proportional to the normal force F_n between the wheel and the surface on which it rolls, directed opposite to the velocity of the center of the wheel, with magnitude

$$f_r = \mu_r\, F_n\,, \qquad\qquad (1)$$

where μ_r is the (small) coefficient of rolling friction.
http://en.wikipedia.org/wiki/Rolling_resistance

In this lab, you will study the effects of f_r on a cart which is started from rest at the lower end of a tilted ramp, slows down as it moves upward until it stops and reverses direction, and returns to the lower end of the ramp. The major questions will focus on the effect of f_r on the cart's acceleration, and on its kinetic energy.

B. Things to Do

For this lab, it will be important to level your camera. VideoPoint assumes that gravity points along the negative *y*-axis. Any sidewise tilt of the camera and its coordinate axes will lead to wrong calculations of gravitational potential energy.

You will find a ball hanging from a string at your station, and you can later use VideoPoint to check that the string is really parallel to the system's *y*-axis. (And, if the camera is not level, you can use VideoPoint's coordinate rotation capability to make things look like it was.) <u>You will probably find that it's easier just to level your camera from the start using one of the bubble levels at the center table</u>.

In order to compute the cart's kinetic energy, you will need to weigh the cart, and to tell VideoPoint what the mass was. Balances are in the lab, and VideoPoint lets you record the results.

To assign a mass to your cart after you have made a movie and digitized its coordinates, go to the View / Coordinate System screen and change the value of S1's mass from its default value of 100 g to whatever you measure your cart's mass to be.

1. Rolling a Cart on the Ramp

You will find a cart and a ramp at your station. Practice giving the cart a quick push so that it rolls up the ramp, turns around near the top of the ramp, and comes down again. Predict qualitatively what the graphs of position, velocity, and acceleration as functions of time will look like.

- With one end of the ramp up about 5 or 6 inches higher than the other end, roll the cart up the ramp, and let it roll back down by itself. Record a good movie, and scale it in VideoPoint. Determine the angle of tilt of the ramp by looking at a plot of *x*- versus *y*-position. Calculate the acceleration you would expect in the absence of friction. (Be sure to say whether you are calculating the acceleration along the ramp, or that in the *x*-direction.)

- Now make the usual graphs of position, velocity, and acceleration versus time. (You may find it useful to plot the <u>magnitudes</u> of velocity and acceleration, rather than just their *x*- and *y*-components.) *How well do your graphs agree qualitatively with your predictions?*

In particular, do your data show, within their accuracy, any difference between the acceleration of the cart when it is going upward and its acceleration when it is going downward? Either way, compare what you see with the difference you would expect to see, assuming a value of μ_r of 0.02 or less.

- Now let's make some new plots – ones involving <u>energy</u>. It will be convenient to plot the various forms of energy on the same graph.

To make a single graph plotting kinetic energy, potential energy, and total energy, after choosing Point S1 / Magnitude for the vertical axis variable, hold the Shift key down while clicking on Kinetic Energy, Potential Energy, and then on Total Energy.

Do your graphs suggest that total mechanical energy is conserved, or not?

If you plotted the energies with time as the horizontal variable, it might be difficult to judge two instants when the cart is at the same height, and therefore to know if its kinetic energy is, or is not, different when it returns to any particular height. To address this

difficulty, <u>make a plot of the energies using Point S1 / X / Position as the horizontal variable</u>.

If you have troubles understanding this new plot, watch what happens on the graphs as you "play the movie" and the plots show which points correspond to the successive frames.

- You should now be able to see clearly how much mechanical energy the cart lost in its motion. Use this figure, and the distance traveled, to calculate values for f_r and μ_r, given that energy transfer is equal to force times distance.

- With f_r and μ_r now known from the measured <u>energy</u> losses, go back and calculate the expected effect of friction on the <u>acceleration</u> of the cart as it travels up, and as it travels back down. Compare this with your graphs of acceleration, and discuss. This is a great example of how energy considerations and Newton's laws can work together.

TO ADD ERROR BARS TO EXCEL GRAPHS:

- After creating a graph, just double-click on one of your data points. Excel will bring up a dialog box to allow you to choose to plot error bars in *x* or *y*, or both. You will have to tell Excel to extend them in the positive, negative, or both directions from the point.

- The magnitude of the errors plotted can be specified as a fixed absolute number, or as a fixed percentage of each value plotted. It can even be specified point by point in a separate column of the data table.

2. Extension Project – Rolling Other Stuff on the Ramp

What do you suppose would happen if you rolled a ball, a hollow "pipe," or a solid circular cylinder down the ramp? If you roll two of them down side by side, which one gets to the bottom first? Is mechanical energy conserved here? If not, where is it going?

Make movies of a couple of these objects rolling down the ramp, and look at the acceleration and energy graphs. Compare the acceleration of these objects to that of your cart. *What's going on here? -- There's more to consider than meets the eye!*

PRELAB Problems for Lab #5: Inclined Planes and Energy Conservation

1. An ideal cart of mass $m = 250$ grams rolls on massless, frictionless wheels on a ramp tilted from the horizontal by an angle $\theta = 5.73°$.

> a. Draw a force diagram, showing the gravitational ($m\,g$) and normal force (N) acting on the cart in the absence of friction. Do they depend on whether the cart is accelerating along the ramp, or not?

> b. Draw force diagrams, with the rolling frictional force f_r included, for the cases where the cart is (i) moving upward along the ramp, and (ii) moving back down the ramp.

> c. Calculate the magnitudes of the cart's acceleration along the ramp, in the three cases shown in your diagrams. (Assume that the coefficient of rolling friction is $\mu_r = 0.02$.) What percentage changes in acceleration are involved in the three cases?

> d. Calculate the kinetic energy which the cart would gain in traveling a distance $L = 1.5$ meters down the ramp.

> e.   How much mechanical energy would be dissipated (transferred into heat energy) by the motion described in the preceding two questions?   What percentage of the kinetic energy of the immediately preceding question is this?

2. Measure (or estimate) the following things and describe briefly your technique for each measurement. Report the best value, absolute uncertainty, and relative uncertainty for each measurement. Remember to be sensible about significant figures!

> a. The length of one of your shoes.

> b. The time it takes for a sheet of paper to float to the ground from your desk.

> c.   The temperature in the room you are in right now. (Chances are you don't have a thermometer handy—that's okay. Just take a wild guess, and then estimate your uncertainty in that guess. Remember, you are just shooting for a 68% confidence level. Professional scientists and engineers make rough calculations like this all the time: you might as well get used to it!)

LAB #6: TWO NICE EXPERIMENTS IN ROTATIONAL MOTION

BEFORE YOU COME TO LAB:

- **Read Taylor's Section 4.6, on "Systematic Errors." This section deals with how to work with uncertainties that are not due simply to random measurement variations.**

- **Consider the optional PreLab problem set attached. NOTE that one of the PreLab problems assigned is from the book by Taylor.**

This week you will do two experiments involving rolling objects. The setup for each experiment is different; pick whichever one you want to do first, and go to one of the setups for that experiment at the start of the lab. (Be flexible.)

Overview Comments: The two projects in this week's lab might be relevant to engineers designing theme park rides, courses for downhill ski races, *etc.* They will remind you of roller coaster loops and of racers taking to the air after speeding across a bump.

As you think about the relevant formulas, you will find it interesting to consider the effects of considering the effects of letting the moment of inertia, *I*, become either very small or very large relative to $m\,r^2$. Which case leads to the right answer for objects which slide without rotating? Were you justified in neglecting the rotation of the wheels on the carts which you rolled down a ramp in a previous lab? How could you make an object which has a moment of inertia much, much larger than $m\,r^2$? *Hint – you might have seen such an object in a lecture demonstration.*

- **Before you start taking data,** take just a few minutes to discuss with your partners the answers to the questions in the Overview Comments, and to quickly record your conclusions. Also, discuss what would happen in each of the two experiments if an object with $I \gg m\,r^2$ were to be used. Would the angle θ in Part 6A approach 0° , or 90° ? Would the height *h* in Part 6B approach zero, or infinity? [You may want to check your understanding and conclusions with your Instructor before moving on.]

LAB #6: THINGS TO DO

I. Experiment #6A: Rolling Off a Log

You already worked out in the Learning Guide the problem of an object sliding off a frictionless circular track (Learning Guide 3, problem VI). The object leaves the track at an angle $\theta = \cos^{-1}(2/3) = 48.2°$ degrees. In the lab it's easier to let an object *roll* down the circular track. In your prelab assignment, you will show that the theoretical expression for the angle at which a rolling object of mass *m*, radius *r*, and moment of inertia *I* about its center leaves the track is

$$\theta = \cos^{-1}\left[(2/3)\left(1 + \frac{1}{3}\frac{I}{mr^2}\right)^{-1} \right] .$$
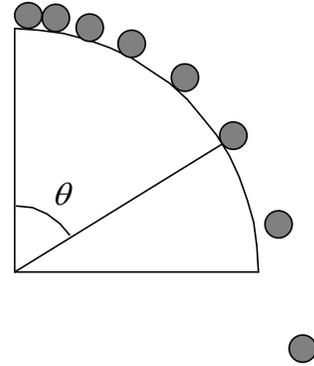
The idea of this experiment is to work out a technique whereby you can measure as accurately as possible the angle where the rolling object just leaves the track. You may find it useful to use your video cameras and VideoPoint for this, or you may find a better way: it's up to you. Try to get results for a few different rolling objects: a solid cylinder, a hollow cylinder, and a sphere.

After a few trial runs in which you estimate the angle at which the object leaves the track, stop and think how you can determine this angle with decent accuracy. You may find that using chalk dust, or wetting the object before you start it rolling, causes it to leave a track which answers the question. Alternatively, you can make a movie and use VideoPoint to determine the takeoff angle.

If you choose to use VideoPoint, it will be convenient to convert to polar coordinates from the usual Cartesian *(x,y)* coordinate system. Then, after moving the origin of the coordinate system to the center of the track, you can plot *radius* versus time to see if you can determine the takeoff angle. (See below for converting to polar coordinates in VideoPoint.)

<u>Using VideoPoint with polar coordinates – possibly useful in analyzing circular motion:</u>

To change to polar coordinates, double-click on the origin and make the change in the first screen shown below. The origin can be moved to the center of the track simply by clicking-and-dragging on it. Then you can choose to plot radius or angle when you make your graphs. (See the second screen shown.)

However you choose to do your analysis, once you have your technique down for one object, try it on the others, and think about any differences you find.
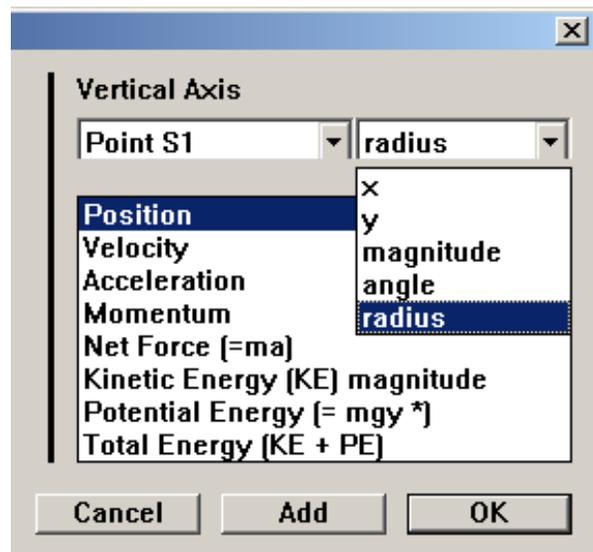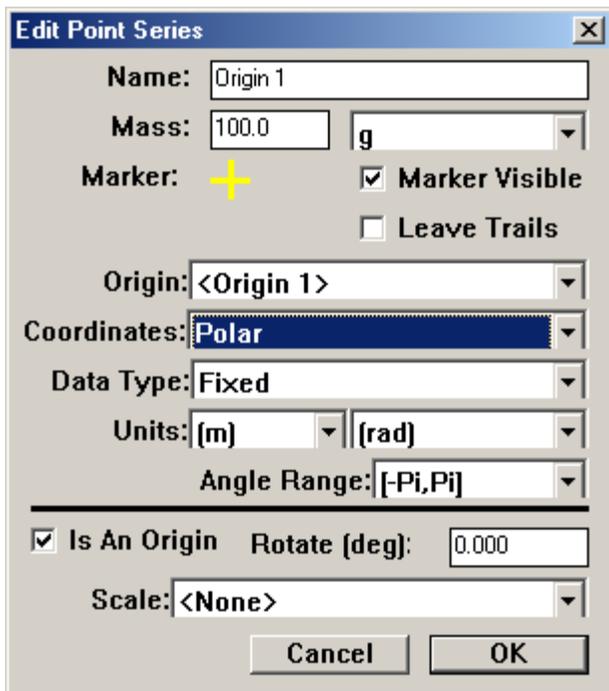
II. EXPERIMENT # 6B: Loop-the-Loop
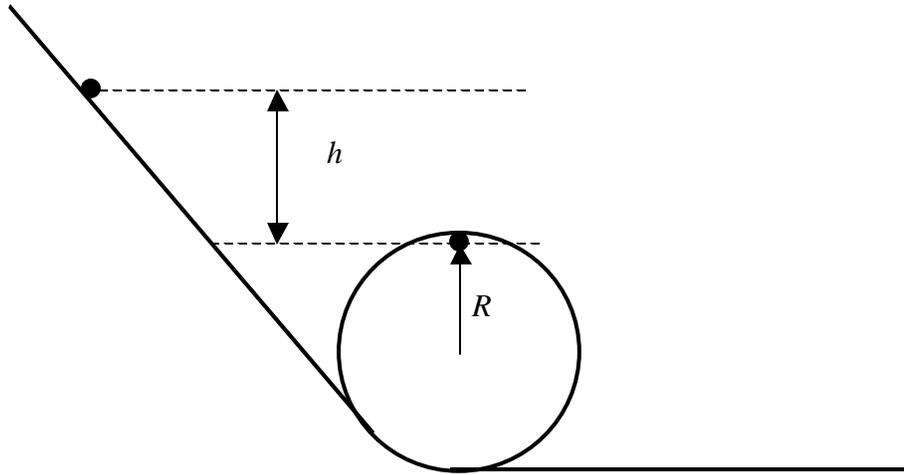
You will roll a ball down a ramp, and it will pick up enough speed to "loop the loop." The question you will answer is how far up the ramp you must start it for it to successfully make it around the loop without leaving the track. Think about what height h above the top of the loop you must start it from. How fast must it be going at the top of the loop to just stay in contact? Then, what height h will give the ball this critical speed? How does h depend on the radius of the loop R? Does h depend on anything else?

Tips on Technique

- What's the best way to tell whether the ball has left the ramp? Would your video cameras and VideoPoint help, or can you think of a better way?

- The ball you'll use is probably too big to treat as a point particle in your calculations. When you make a measurement of, say, the height of the ball, where should you measure to? The top of the ball? The bottom of the ball? Or maybe some other, *particular, special point associated with the ball*? (Big hint.) How should you measure the radius of the loop? To the inside edge? The outside edge? *Or some other way*? (Again, big hint.)

- This experiment is actually a lot more subtle than you might initially suspect. Your first calculation of the height h needed will probably not match your experiment within your range of uncertainty. (Yes, your *uncertainty*, which you should of course be calculating.) What other refinements can you make to either the experiment or your calculations? For instance, is any kinetic energy being lost to friction? How might you measure the fraction lost to friction?

PRELAB Problems for Lab #6: Two Nice Experiments in Rotational Motion

1. An object of radius r, mass m, and moment of inertia I about its axis starts from rest at the top of a large cylinder of radius R and rolls down. At what angle θ from the vertical does the object leave the surface of the large cylinder? (Hint: This problem is similar to Learning Guide 3, Problem VI, which you solved using conservation of energy. In this problem, you have an extra energy term for the energy of the ball's rotation, which you can relate to its translation using $v = \omega r$.)
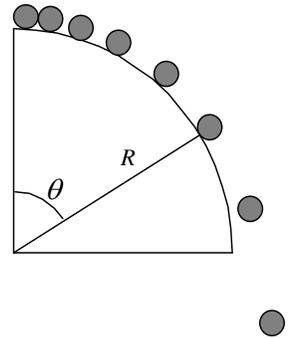
2. A ball has radius r, mass m, and moment of inertia I about its center. The ball starts at rest, rolls without slipping down a ramp, and does a loop-the-loop of radius R. (See the figure for Experiment 6B.) What is the height h above the top of the loop that the ball must be started at to make it around the loop-the-loop without leaving the track? (Hint: This problem is similar to Learning Guide 2, Problem VIII, which you also solved using conservation of energy. In this Prelab problem, however, you have an extra energy term associated with the ball's rotation, which you can relate to the ball's translation using $v = \omega r$.)

3. Do Taylor's Problem 4.24.

LAB #7: Fluids

 BEFORE YOU COME TO LAB:

- **Read Taylor's Section 5.1, on "Histograms and Distributions." This section covers the basic concepts of distributions, of which the familiar Gaussian "bell curve" is only the most familiar example.**

- **Consider the optional PreLab problem set attached. NOTE that the PreLab problems are assigned from Taylor.**

Overview Comments:

Fluids can move in complicated ways. Infinitesimal "point masses" and rigid extended bodies can also move, but they are relatively simple objects. For point masses, only the three coordinates giving their positions are required in order to describe their motion. For extended bodies only the three angles which define their orientations are required in addition.

For fluids, things are different. Any part of the material, whether it be the entire sample or a mathematically described small portion of it, can move, and can also change shape. But each portion has mass and inertia, and can change its momentum only in response to forces. Understanding how the forces that arise <u>between</u> various portions of a fluid sample affect their shapes and motion leads to complicated problems in <u>fluid mechanics</u>. The applications of this field of study are legion. You can't study intergalactic gas clouds, or the flow of oil in an Alaskan pipeline, or the aerodynamic forces that hold a supersonic airplane up, without getting into it.

In this lab, you will look at some of the results of fluid mechanics, dealing largely with the effects of viscous flow. These involve the dissipative "frictional" forces which arise in fluid motion (except in the case of superfluids). Try to think about what is happening from the point of view of a small element of the fluid, and of Newton's Laws applied to that element.

P.S. Did you know that a cubic meter of air weighs almost three pounds? No wonder it takes strength to hold your arm out the window of a moving car – it takes force to make all that air get out of the way!

I. Flow of a Viscous Fluid in a Circular Pipe

It is a remarkable fact that fluid immediately adjacent to an immobile surface, such as the wall of a pipe, always has zero velocity. In order for fluid some distance y from the surface to flow at velocity v, a force must be applied:

$$F = \frac{\eta A v}{y}$$

where A is the area of the surface (or, equivalently, the area of the layer of fluid), and η is the coefficient of viscosity. Fluid flow through a circular pipe is slightly more complicated. The flow rate, Q, through a circular pipe of radius R and length L is given by Poiseuille's law:

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L}$$

where ΔP is the difference in pressure at the ends of the pipe. The R^4 dependence might seem surprising. A factor of R^2 comes from the area of the pipe; another factor of R comes from the above equation for the viscous force, since Q is proportional to the velocity which, for a given force, is proportional to distance $y = R$; a final factor of R comes from the circular shape of the pipe.
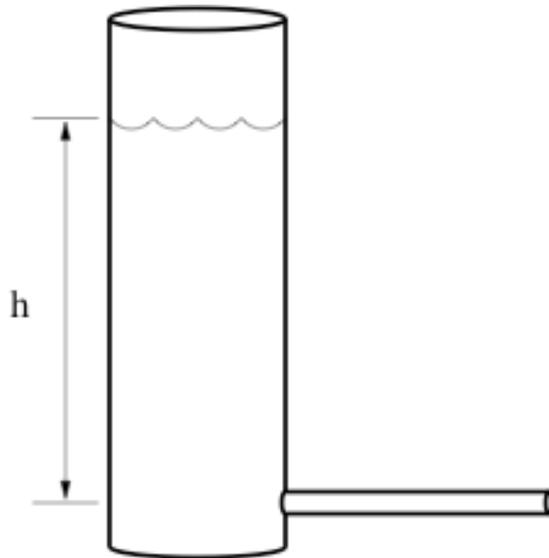

Figure 1: Apparatus for the first part of the experiment. The vertical cylinder is partly filled with oil. It is open to the atmosphere at the top.

Specific Instructions:

Use the apparatus shown in Figure 1 to test Poiseuille's law and to measure the viscosity of a fluid. The fluid is heavy machine oil, which fills the large vertical cylinder. Its weight produces the pressure at the bottom of the cylinder and, therefore, at one end of the small horizontal tube. The other end of the horizontal tube is at atmospheric pressure. Thus the pressure difference across the length of the small tube is $\Delta P = \rho\, g\, h$, where h is the height of the fluid above the tube.

Find the density of the oil using a scale and a graduated cylinder.

Measure the flow rate in each of the three available tubes (radii 0.370, 0.307 and 0.242 cm), using a stopwatch and a graduated cylinder.

Hints: Keep the small tube horizontal to minimize the effect of gravity on the flow. Measure the height of the fluid in the vertical cylinder before and after the oil flows out, and use the average value. From which point should the height be measured?

Analysis:

First use your data to test the assertion that Q is proportional to R^4. Although it isn't strictly true, assume that each tube has the same length L. Then you can reformulate Poiseuille's equation as:

$$Q = \text{Constant x } R^{\alpha}$$

You want to check that α is 4. Do this by analyzing your measurements of Q using logarithmic plotting: make a plot (using **Excel**) with the quantities ($\log Q$) and ($\log R$) on the two axes, and extract from it the value of the exponent α.

Next, find the viscosity η. For this part of the analysis, assume that the exponent $\alpha = 4$. Rework Poiseuille's equation to extract the value of the coefficient of viscosity, and use your three measurements of Q to calculate three values of η. Are the values close to each other? Formal uncertainty analysis isn't required, but look for a procedural error if the measurements are grossly different.

II. Terminal Velocity

An object falling through a viscous fluid feels three forces. Gravity pulls the object downward:

$$F_{\text{grav}} = \rho V\, g$$

where ρ and V are the density and volume of the object, respectively, and g is gravitational acceleration. The buoyant force pushes the object upward:

$$F_{\text{buoy}} = \rho_f \, V \, g$$

where ρ_f is the density of the fluid. Finally, there is a drag force opposing the motion of the object. Stokes' law gives the drag force on a spherical object of radius R moving with velocity v in a viscous medium:

$$F_{\text{drag}} = 6 \pi \eta R v$$

where R is the radius of the sphere. When these three forces balance, no net force acts on the sphere, so it falls with constant velocity, called "terminal velocity". Combine the expression of the three forces acting on the spherical object to derive the expression of the "terminal velocity".

Specific Instructions and analysis

Test the equation you just derived by measuring the terminal velocity of small lead spheres falling through the oil you analyzed in the first part of the experiment (for lead, ρ_f = 11.7 g cm^{-3}).

Measure the diameter of one of the spheres, taking an average of several measurements if it isn't really spherical. Measure the velocity of the sphere falling through the oil using a stopwatch. Repeat the experiment for at least three different spheres. Are the measured values close to the values predicted by your equation?

III. Buoyant Force

Figure 2: Apparatus for the final part of the Lab.

The density of gas in a helium balloon is less than the density of the surrounding air, so the balloon feels a net upward force. The buoyant force (ρ_{air} = 1.29 kg m^{-1} at 1 atm pressure) can be balanced by hanging a mass below the balloon as in figure 2.

The total weight is:

$$W_{total} = \left(m_1 + m_{string} + m_{balloon} + m_{He} \right) g$$

where m_1 is the mass hanging below the balloon, m_{string} is the mass of the string, $m_{balloon}$ is the mass of the (empty) balloon, and m_{He} is the mass of the helium within the balloon.

The masses of the balloon, string, and hanging weight can be measured on scales, but for the mass of the helium you have to rely on measurements of volume and pressure. Given that the atomic mass of helium is 4, if there are n moles of helium in the balloon, the mass is $m_{He} = 4.00 \ g \cdot n$.

The ideal gas law relates n to the pressure, volume, and temperature of the balloon (P, V, and T) and the universal gas constant: $P \ V = n \ R \ T$. Solving for n and substituting R = 8.3145 J mol^{-1} K^{-1} and T = 293K (approximate room temperature) allows you to calculate the mass.

Specific Instructions and analysis

Measure the mass of the empty balloon. Fill it with helium, and after stopping the flow of helium, measure the pressure within the balloon before tying off the end of the balloon. You may need the following conversion factors: 1 psi = 6985 Pa, 1 atm = 1.013 x 10^5 Pa. Also, remember to add the atmospheric pressure to the "gauge pressure" reading on the pressure meter.
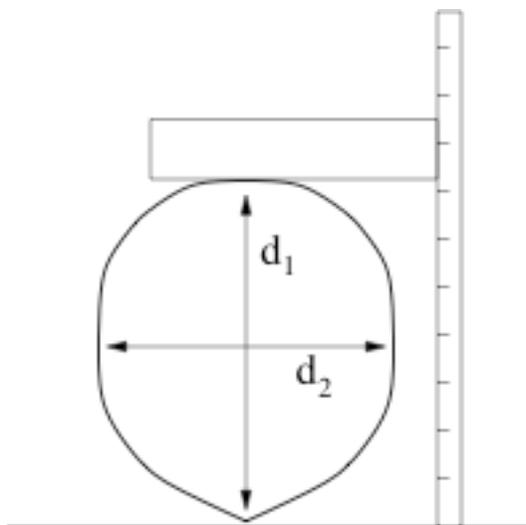
Figure 3: Measuring the dimensions of a balloon.

Next measure the volume of the balloon. One way of doing this is to put it on a table, hold a meter stick vertically next to it, and use a wooden board to help measure its size on the meter stick. (See figure 3.) You can estimate the size of the balloon from the dimensions d_1 and d_2.

Cut a piece of string a couple of feet long, measure its mass and tie it to the bottom of the balloon. Finally, tie a 5-g hanger to the string and keep adding weights to the hanger until the balloon is in equilibrium. To fine-tune the hanging weight, you may want to use small paper clips (about 0.3 g each) or pieces of tape. After you have achieved equilibrium, detach the hanger and its weights and measure their mass on a scale.

Now you have all the pieces of data you need to test the buoyancy formula. Calculate the buoyant force and the weight. *Are they close to each other?*

PRELAB Problems for Lab #7: Fluids

1) Work problem 5.4 of Taylor.

2) Work Problem 5.6 of Taylor.

LAB #8: COUPLED PENDULUMS AND NORMAL MODES

BEFORE YOU COME TO LAB:

1. **Read Knight's Chapter 14 (Oscillations), as assigned for lecture. Note that the equations describing Simple Harmonic Motion have applications in physics and engineering extending far beyond the simple systems described.**

2. **Read Taylor's Section 3.2, on "The Square Root Rule for a Counting Experiment."   While not specifically relevant to this week's experiment, the argument that counts of things (such as nuclear decays, occurrences of rare diseases in human populations, *etc*.) have uncertainties of the order of the square root of the counts is of crucial importance in making both scientific and political decisions.**

3. **Read the lab writeup. Focus on the relationship between the physical phenomena and the equations as you carefully go through the Introduction and then through Appendix I of the writeup.**

4. **Consider the optional PreLab problem set attached.**

A. Introduction

In this week's lecture and lab, you will be studying the motions of simple harmonic oscillators. The set of concepts involved in Simple Harmonic Motion (SHM) is one of a handful of extremely useful building blocks for many advanced areas in physics. Electrical, acoustical, and optical systems oscillate with SHM, completely analogous to the mass-plus-spring and pendulum systems you'll be looking at during these two weeks. Your radio selects the station you want by using electrical resonance tuned to the desired frequency. Some theoretical physicists in search of Grand Unified Theories (GUTs) have proposed describing the smallest fundamental particles as comprised of oscillating "strings." [1] Many physicists use their physical intuition about how systems with springs and masses are likely to behave in order to predict how many different systems will act. Contrary to its name, "physical intuition" isn't always an innate characteristic; a person

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[1] In this case, the word "string" is used evocatively rather than literally.

develops physical intuition with experience. This lab gives you the chance to develop such intuition about SHM in simple mechanical systems.

This week's lab is a study in describing seemingly-complex motion with simple equations. The techniques underlying the work will recur throughout your scientific and technical futures. Our focus is first on the equations describing Simple Harmonic Motion (SHM). These follow from any situation in which there is

> (a) an equilibrium situation ($x = 0$, say) generated by a restoring term which is <u>linear</u> in the variable:

$$F = - k\, x, \quad \text{for example, for a spring.}$$

> (b) an "inertial term" (the mass, say) which prevents the restoring term from easily and immediately pushing the system back to its equilibrium situation. Newton tells us that

$$F = m\, a = m\, d^2x/dt^2 .$$

Combining the above terms gives us the basic equation of SHM:

$$m\ d^2x/dt^2 = - k\, x .$$

Any system described by an equation of this form will exhibit SHM behavior, including having simple sinusoidal solutions, a characteristic frequency leading to resonance behavior, *etc.* To a mathematician, this is an example of a <u>linear differential equation</u>, in which the unknown function and its derivatives appear only in the first degree. (There are no squares, cubes, square roots, or strange functions of x involved.)

Linear differential equations of the above form, which pervade science and engineering, have one wonderful characteristic – <u>their solutions may be superposed</u>. That is, if there are two functions *u(t)* and *v(t),* each of which satisfy the equation, then any linear sum of the two functions is also a solution.

As you may know, Fourier Analysis shows that complex functions can be described as sums of sine and cosine functions. Electrical engineers build their careers on this simple fact. The response of a system described by the SHM model to any (additional) driving force *f(t)* can be described by investigating its response to a family of sinusoidal driving functions which, when summed, equate to the complicated function *f(t).* As you would expect, a sine function tuned to the system's resonant frequency will produce a huge response, and other frequencies will have less effect. (This is how you select the radio station you want, starting from the fact that a whole host of stations are broadcasting simultaneously, with their outputs superposed linearly in the air.)

More generally, it is common in physics to find that the equations describing a complex system allow not one, but a whole host of resonant frequencies. The overtones produced by a musical instrument are such frequencies, and their relative frequencies and amplitudes are what makes one instrument have a characteristic "sound."

In this week's lab, we will study a fairly simple system – two identical pendulums, connected by a weak spring. In the absence of the spring, the two would swing at the same sinusoidal frequency, independent of each other. Adding the spring produces two characteristic frequencies, which in turn lead to a complex motion, which is not a simple sinusoid. But the motion can be analyzed as a sum of two sinusoidal motions, each of which obeys the simple equations of SHM, and oscillates at its characteristic frequency.

As you pursue your interest in science and engineering, you will find that the approach to analyzing a complicated system often hinges on an ability to look for characteristic frequencies, and to determine the patterns of motion which generate simple sinusoidal behavior at those frequencies. With these solutions in hand, then, linearity of the equations allows any general motion to be described as a sum of these simple motions. This is the concept of <u>normal modes</u>, which refers to the set of patterns which each leads to a simple sinusoidal variation in time.

All of this math may seem a little complicated. It is not really necessary to carrying out the lab. But it is the purpose of doing the lab.

You will want to observe the various patterns of motion of the two-pendulum system, and record carefully descriptions of the patterns of motion that you see. Using a stopwatch, you can determine the frequencies associated with the various patterns which occur. Then, using a simple computer model which sums and graphs the combined effects of two sinusoids, you can simulate the patterns that you observe, and other patterns as well (for example, the first overtone in a musical instrument).

B. The Apparatus and the Measurements

This system consists of two pendulums connected at their centers of mass with a spring, as drawn in Figure 1. Using a stopwatch, measure the frequencies of the pendula without the spring, and then the frequencies of the normal modes. (See Appendix A.)

After you have analyzed the simple patterns of motion, try displacing one pendulum while holding the other one fixed. Then release them simultaneously. Describe the resulting motion. Be quantitative -- do the two frequencies you observe in this situation agree with what you expected from your measurements of the normal modes, based on Appendix A?

Take a VideoPoint movie of the complex motion, using a capture rate of 5 or 6 frames per second. (Since we will be interested mainly in frequencies, there is no need to include a meter stick in the field of view.) Make sure that the video covers at least one complete cycle of the motion. Pick off the positions of some identifiable point on one of the pendulums. (There may be a lot of points, but since we are interested in frequencies more than in exact positions, you can click rapidly through the video.)

Make a plot of position *versus* time. Do the general characteristics agree with those expected? Does the video confirm the frequencies you measured by stopwatch?
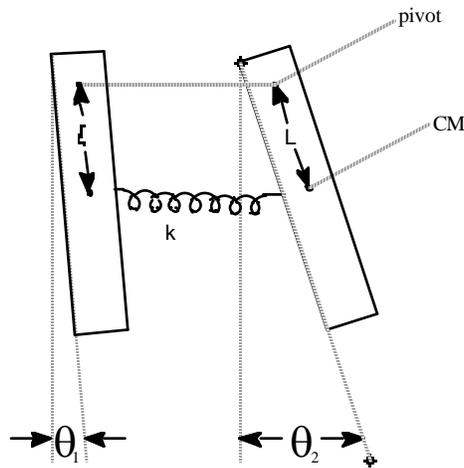
Figure 1: Coupled pendulums; each pendulm is made from a length of 2x4 A spring connects the two pendulums at their centers of mass (CMs).

C. Computer Modeling of Your Results

An Excel program, Normal Modes Modeling, can be found in your Physics 103 folder. It provides a fun way of confirming that two sinusoidal functions can generate the complex behavior that you observed with the two pendulums.

The program calculates the function

$$X(t) = A1 * \cos(2\ pi * F1 * t) + A2 * \cos(2\ pi * F2 * t) .$$

There are two terms, with amplitudes A1 and A2, oscillating at frequencies F1 and F2. You can type in the two frequencies you observed for your normal modes, and see if the model duplicates the more complex behavior. (For present purposes, just set A1 = A2 = 1.)

The following figure shows an example Excel screen. Amplitudes and frequencies can be changed at will. Note also that you may need to change the maximum value of the range of time plotted in order to see the entire behavior.

Use the modeling program to confirm that the frequencies you measured in the complex motion can generate the pattern that you saw in your VideoPoint graph, and that the video's frequencies agree with your stopwatch measurements and with the theory. Note that coordinates on the graph may be "picked off" by moving your cursor onto the curve and pausing. Clicking on the curve, and then clicking again after the little squares have appeared, will allow using the right and left arrow keys on the keyboard to move the point "picked off" in small sideways steps.
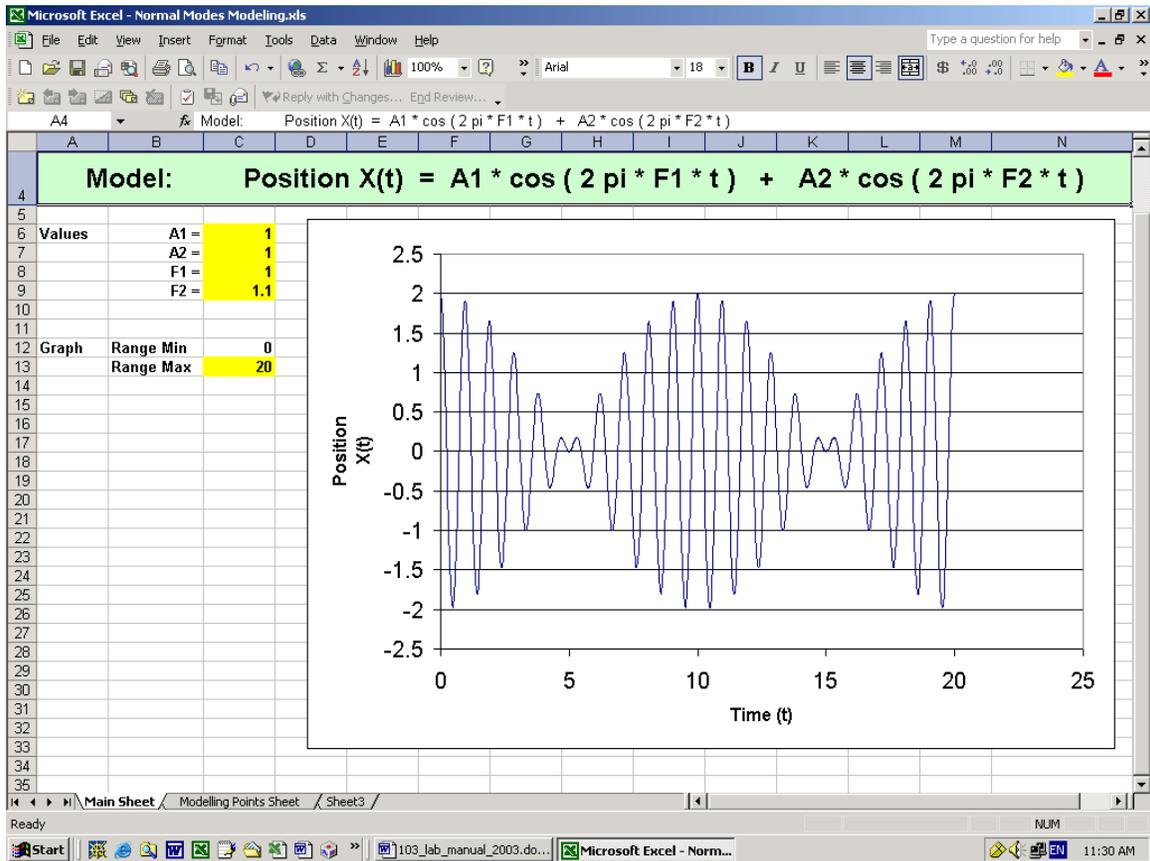
D. Further Computer Modeling

While you have the Excel program running, use it to look at other interesting phenomena, involved in such things as musical instruments and their "tone." In each case, print out a copy of the computer's result and write down in your notebook a summary of what you find notable in the waveforms. Note that, up to now, you have been looking at two frequencies which are close to each other.

In the following, you may want to make the amplitude of the higher-frequency term smaller than that of the fundamental.

(a) Look at two frequencies which are exactly a factor of two apart. These are most relevant to such instruments as the flute and the organ, in which the overtones tend to be almost exact multiples of the fundamental note being played.

(b) Look at two frequencies which are almost, but not quite, a factor of two apart. (Say, with a frequency ratio of 2.2 to 1.) Such waveforms are perhaps relevant to the sound produced by a saxophone, in which the tapered bore causes the overtones to not be exact multiples of the fundamental note.

(c) What about non-integer ratios of frequencies? Think about what period of time makes such a system return to its initial state. (Use simple integer ratios first, such as 3:2.)

Appendix I: Coupled Oscillations

Consider the case of the two pendulums coupled with a spring at their centers of mass (Figure 1). We will explicitly assume $L_1=L_2=L$. Positive angles are counterclockwise motions. There are two torques on each pendulum. The weight of a pendulum gives a torque, which tries to restore the pendulum to vertical,

$$\tau_w = -(m\,g)\,L\,sin(\theta) \approx -m\,g\,L\theta,$$

using the small-angle approximation $sin(\theta) \approx \theta$. The spring provides a restoring torque on the *first* pendulum which acts to bring the separation between the centers of mass of the pendulums to the unstretched length of the spring, L_0:

$$\tau_s = -[k\,(L_s\text{-}L_0)]\cdot L \approx -k\,L[L\,sin(\theta_1) - L\,sin(\theta_2)] \approx -k\,L^2(\theta_1\text{-}\theta_2),$$

where L_s is the stretched length of the spring, and the first approximation is that the stretched spring makes a small angle to the horizontal, and the second approximation is that both θ_1 and θ_2 are small. (The latter condition implies the former.) The restoring torque on the second pendulum is equal in magnitude, but in the opposite direction.

Newton's law in angular variables applied to each pendulum gives the following equations of motion:

$$I_1\alpha_1 = I_1\frac{d^2\theta_1}{dt^2} = -m_1\,g\,L\,\theta_1 - k\,L^2(\theta_1\text{-}\theta_2), \tag{1}$$

$$I_2\alpha_2 = I_2\frac{d^2\theta_2}{dt^2} = -m_2\,g\,L\,\theta_2 + k\,L^2(\theta_1\text{-}\theta_2). \tag{2}$$

Notice that neither of these equations describes simple harmonic motion because of the spring coupling terms. However, if we consider the case of two symmetric pendulums, *so that* $I_1 = I_2 = I$ and $m_1 = m_2 = m$, then we can get two new equations from (1) and (2) which DO describe simple harmonic motion. We get the first by adding together (1) and (2):

$$I\frac{d^2}{dt^2}(\theta_1 + \theta_2) = -m\,g\,L(\theta_1+\theta_2), \tag{3}$$

and the second by subtracting (2) from (1):

$$I\frac{d^2}{dt^2}(\theta_1 - \theta_2) = -m\,g\,L(\theta_1\text{-}\theta_2) - 2\,k\,L^2(\theta_1\text{-}\theta_2). \tag{4}$$

Equation (3) is an equation for simple harmonic motion in the variable $u = \theta_1+\theta_2$, which has solution (up to an arbitrary phase angle) $u(t) = u_{max}\,cos(\omega_u\,t)$, where $\omega_u^2 = m\,g\,L\,/\,I$.

Similarly, equation (4) describes simple harmonic motion in the variable $v = \theta_1 - \theta_2$, and has solution $v(t) = v_{max} \cos(\omega_v t)$, where $\omega_v^2 = (m\, g\, L + 2\, k\, L^2) / I$. The solutions $u(t)$ and $v(t)$ are called **normal modes** = motions in which both pendula move at the same frequency. The normal mode $u(t)$ is excited when both boards are displaced equally in the same direction. The spring then has no effect. The normal mode $v(t)$ is excited by starting the boards with equal and opposite displacements of each board. The frequency for this mode is higher because this motion exercises the spring.

The variables that we observe directly are $\theta_1(t)$ and $\theta_2(t)$. In the simple case where $u_{max} = v_{max} = A$, when the arbitrary phases are both zero, the motion of $\theta_1(t)$ is:

$$\theta_1(t) = \frac{1}{2}(u + v) = \frac{1}{2}\, A\, [\cos(\omega_u t) + \cos(\omega_v t)],$$

which can be rewritten with the help of the trigonometric identity for the addition of cosines as:

$$\theta_1(t) = A\left(\cos\left(\frac{(\omega_u + \omega_v)}{2}t\right)\cos\left(\frac{(\omega_u - \omega_v)}{2}t\right)\right).$$

Think of this as periodic motion with angular frequency $\omega_{av} = (\omega_u + \omega_v)/2$, with an amplitude that varies periodically with a slower frequency, $\Delta\omega/2 = (\omega_u - \omega_v)/2$. Similarly, (using the trig identity for the difference between two cosines), the expression for the motion of the second pendulum is:

$$\theta_2(t) = \frac{1}{2}(u\text{-}v) = -A\left(\sin\left(\frac{(\omega_u + \omega_v)}{2}t\right)\sin\left(\frac{(\omega_u - \omega_v)}{2}t\right)\right)$$

One of these functions is plotted in Figure 4 for the case of a weak spring coupling,

$$K << m\, g\, / L,$$

so that $|\Delta\omega| << \omega_u$.

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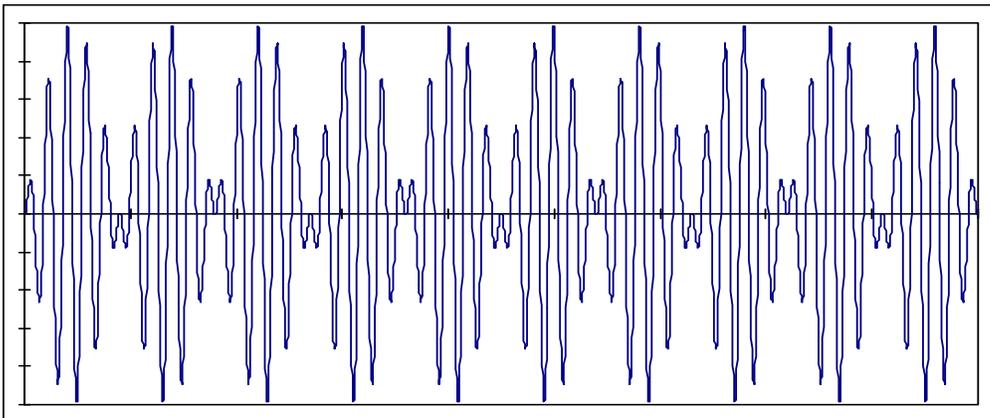
The oscillations of one pendulum in the case of weak coupling. The vertical axis gives the amplitude in arbitrary units while the horizontal axis measures time (in arbitrary units).

PRELAB Problems for Lab #8: Coupled Pendulums and Normal Modes

1. Consider a simple pendulum consisting of a mass m suspended on a string of length L.

   (a) What is the pendulum's natural frequency if $L = 1$ m?

   (b) What length L for a clock pendulum would make the clock "tick" once every
       second, if the tick occurs every time the clock's pendulum passes through its
       equilibrium position?

2. Two identical carts are placed on a frictionless air track, so that they are free to move
along a horizontal line. Weak springs of negligible mass are used to join the two cars and
to connect each of the cars with a fixed point its end of the air track.   The three springs
have identical spring constants k.   Each car has a mass M.

   (a) What will be the frequency of motion if each cart is moved some distance D
       to the right of its equilibrium position, and they are simultaneously released
       from that position? (Hint: Think of the springs, and use the similarity of the
       two cart's situations.)

   (b) What will be the frequency of motion if, instead of the above, one cart is
       moved a distance D to the right of its equilibrium position, and the other cart
       is moved a distance D to the left?

LAB #9: A PRECISION MEASUREMENT OF *g*

BEFORE YOU COME TO LAB:

- **Read the Taylor's example on "Measurement of *g* with a Simple Pendulum," in his Section 3.9. This will get you started on the uncertainty analysis of this week's lab.**

- **Consider the optional PreLab problem set attached. Note that one of the PreLab problems is assigned from Taylor.**

Overview Comments:

So far, the experiments in Physics 103 Lab have addressed physical principles to an accuracy of 5 to 10%. This is fine for getting a feel for how things work, but another objective of experimental physics is to measure precise values for constants of nature. To make a precise measurement, one designs an experiment to minimize systematic effects or to make them easy to calculate. In this lab you will measure *g*, the acceleration due to gravity at the Earth's surface, by timing a pendulum. With care in your technique and attention to systematic effects you can achieve an accuracy of much better than 1%.

Uncertainty analysis is an essential part of this lab. We have provided a spreadsheet to do most of the calculations for you. But you are responsible for thinking about the results. You may want to do a quick review of your Error Analysis material before coming to lab. During this lab, you will carefully apply the methodology of combining random errors from repeated measurements using the concepts of standard deviation and standard error.

Light Beam
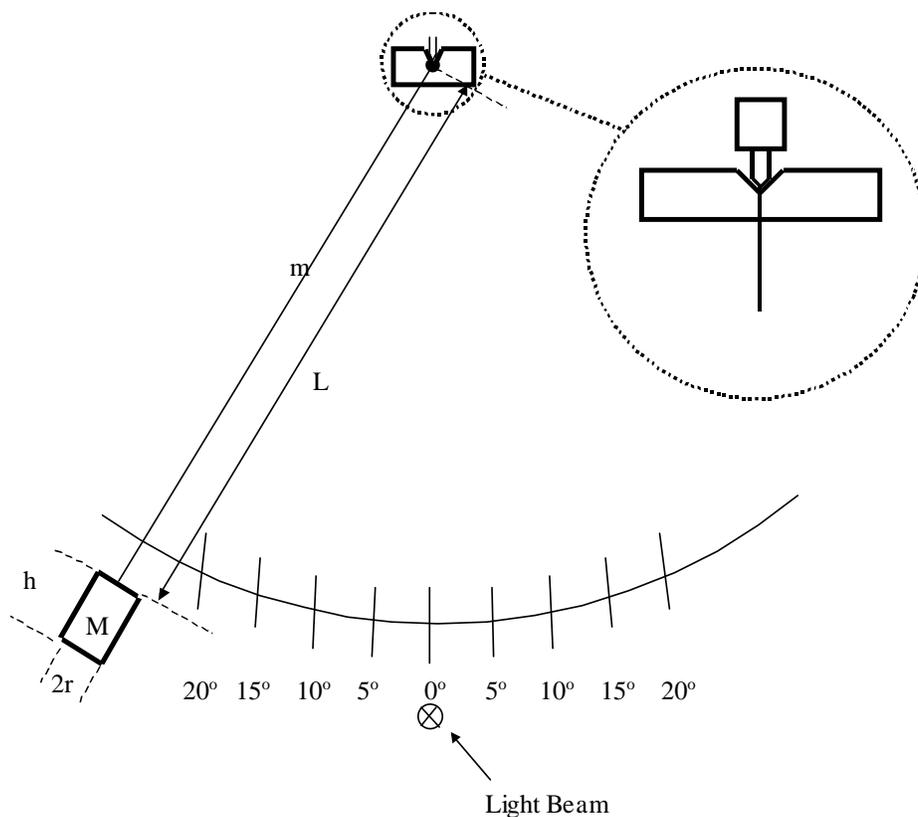
Theory of a Real (Physical) Pendulum

A high-precision experiment requires unusual effort both in technique and in the underlying theory. In this section we extend the theory of a simple pendulum to the level needed for a precise measurement of g.

Your textbook shows that for a simple pendulum of length L_0, the period T_0 is given by

$$T_0 = 2\pi\sqrt{\frac{L_0}{g}} \ . \tag{1}$$

Inverting this equation, we can calculate the value of g by timing the period of such a pendulum:

$$g = 4\pi^2 \frac{L_0}{T_0^2} \ . \tag{2}$$

However, at the level of accuracy we are aiming for, we may not assume that our pendulum is a *simple* pendulum. There are two important effects that make the simple pendulum assumption break down. Our *physical* pendulum is 1) not a point mass suspended by a massless string, and 2) not a true harmonic oscillator (because the restoring term in the equation is not <u>exactly</u> proportional to the displacement). If we were to neglect either of these two effects, we would find that our measurement of g was systematically off.

Consider the consequences of the first effect. For a physical pendulum of total mass M_T, moment of inertia I about the pivot point, and distance D_{cm} between the pivot point and the center of mass, the period T_0 for small oscillations is given by

$$T_0 = 2\pi \sqrt{\frac{I}{M_T D_{cm} g}} \, . \tag{3}$$

You may approximate your pendulum as a cylindrical bob of mass M, height h and radius r, suspended on a long thin wire of mass m. Let the distance from the *pivot* to the top of the cylindrical bob be L. (See the sketch on the previous page.) You should be able to derive the expression for the moment of inertia I of this physical pendulum

$$I = I_{bob} + I_{string}$$

$$I = M\left[\left(L + \frac{h}{2}\right)^2 + \frac{r^2}{4} + \frac{h^2}{12}\right] + \frac{m L^2}{3} \, . \tag{4}$$

(The moment of inertia of a cylinder about a line perpendicular to its axis through its center of mass is $I = M\left(\frac{r^2}{4} + \frac{h^2}{12}\right)$, and the $M\left(L + \frac{h}{2}\right)^2$ term comes from the parallel axis theorem.)

To evaluate the $M_T D_{cm}$ term in the denominator of equation (3), we need to consider both the mass of the bob and the wire, and find the distance from the pivot to their center of mass:

$$M_T D_{cm} = (M + m) D_{cm}$$

$$= (M + m)\left(\frac{M\left(L + \frac{h}{2}\right) + m\frac{L}{2}}{(M + m)}\right) \tag{5}$$

$$= M\left(L + \frac{h}{2}\right) + \frac{m L}{2} \, .$$

Substituting equations (4) and (5) into (3), the period of oscillation becomes

$$T_0 = 2\pi \sqrt{\frac{L}{g}\left[\frac{M\left[\left(1 + \frac{h}{2L}\right)^2 + \frac{r^2}{4L^2} + \frac{h^2}{12L^2}\right] + \frac{m}{3}}{M\left(1 + \frac{h}{2L}\right) + \frac{m}{2}}\right]} \, . \tag{6}$$

Solving this for g and combining a few terms gives us

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$$g = \frac{4\pi^2 L}{T_0^2} \left(\frac{1 + \dfrac{h}{L} + \dfrac{h^2}{3L^2} + \dfrac{r^2}{4L^2} + \dfrac{m}{3M}}{1 + \dfrac{h}{2L} + \dfrac{m}{2M}} \right). \qquad (7)$$

Next, consider that the pendulum is not a true harmonic oscillator. For a true harmonic oscillator, the equation for theta would be

$$\frac{d^2\theta}{dt^2} + \frac{4\pi^2}{T_0^2}\theta = 0, \qquad \text{(true harmonic oscillator)} \qquad (8)$$

which has a solution

$$\theta(t) = \theta_m \cos\left(\frac{2\pi}{T_0}t\right), \qquad \text{(true harmonic oscillator)} \qquad (9)$$

where the amplitude θ_m is the maximum value of θ, and the period T_0 is independent of the amplitude. In reality, the equation $\tau = I\alpha$ for a pendulum leads to

$$\frac{d^2\theta}{dt^2} + \frac{4\pi^2}{T_0^2}\sin(\theta) = 0, \qquad \text{(actual pendulum)} \qquad (10)$$

for which the actual period of oscillation T does depend on the amplitude θ_m. (For small oscillations, we often make the approximation $\sin(\theta) \approx \theta$, but for this measurement we will need to be more precise.) To get an expression for the period T as a function of the amplitude θ_m, one solves equation (10), but this leads to complicated elliptic integrals which are most easily treated by numerical approximations. These give for the actual period T a series expansion in the variable $\sin^2(\theta_m/2)$:

$$T = T_0\left[1 + \frac{1}{4}\sin^2\left(\frac{\theta_m}{2}\right) + \frac{9}{64}\sin^4\left(\frac{\theta_m}{2}\right) + \ldots\right], \qquad (11)$$

where T_0 is determined from equation (6). Remember, T is the period that you measure, and T_0 from equation (6) is the period that you would have measured for your physical pendulum if θ_m were very small.

Just to give you an idea of how big a correction the difference between T and T_0 really is, here's a Table of T/T_0 vs. θ_m from Equation (11).

θ_m	T/T_0	θ_m	T/T_0
0°	1.00000	10°	1.00191
5°	1.00048	11°	1.00231
6°	1.00069	12°	1.00275
7°	1.00093	13°	1.00323
8°	1.00122	14°	1.00374
9°	1.00154	15°	1.00430

LAB #9: THINGS TO DO

I. Measuring the Period

Measure the period of your pendulum using the light-activated electronic timer on your table. Set the mode switch to "P" so that the pendulum turns the timer on and off with successive passes. To time multiple periods, hold down the telegraph key on your table to keep the timer from being switched on and off with every pass. Set the switch on the back of the timer to "manual reset" to keep the display from resetting to zero immediately after every measurement. The timer can measure 1, 10, 100, or 1000 second intervals. Which one should you use?

Measure the period for $\theta_\mathrm{m} = 10°$, timing the pendulum for ten periods. Estimate the random error by repeating the measurement about five times. How should you estimate the uncertainty of your time measurements? Feel free to use Excel to help you with averaging or other calculations. After using your computer to calculate g in part III below, go back and repeat the measurement for $\theta_\mathrm{m} = 5°$ and 15°.

Tips on Technique: Be certain to time complete cycles; otherwise the placement of the light beam affects the measurement. (Why?) There is another systematic effect due to the damping of the amplitude, if the light beam is not centered. Center the light beam by making the times for half cycles the same on both sides. Finally, even if the light beam is centered, the finite size of the light beam gives this systematic error if the amplitude is too small. **Do not use amplitudes below 5°.**

II. Making Other Measurements

Now, you will measure the lengths and masses needed to calculate g from equation (7). Measure the length of the pendulum wire with a metal 2-meter ruler. (Wooden ones warp and shrink.) Measure L from the bottom of the V notch at the pivot point to the top of the mass. Do not rest the ruler on top of the mass as its weight may elongate the wire.

Do not disassemble your pendulum to measure the mass of the bob or the wire. (This will add more kinks to your wire, and make your life more difficult than it has to be.) Instead, there are additional bobs and wires on the table in the center of the lab

which you can weigh and measure. Think about how accurately you need to determine *r, h,* and the masses.

III. Using the Computers to Do the Tedious Calculations

Calculating *g* and doing error propagation with something as messy as equation (7) is, to put it nicely, the perfect job for a computer. To help you out, we have programmed an Excel worksheet to help you with your calculations. A copy of the worksheet is appended at the end of this write-up, so you can check it out before you get to lab.

In the Physics 103 folder, double-click on Precision Measurement of g (Rev C).xls to open the worksheet. Since you will be adding your own data to it, you may want to click on File → Save As to allow you to rename and save a copy of the worksheet in the student data folder for your lab period. Then, any time you save the file, it will be saved there under its new name.

Many of the cells in the original worksheet have been "write protected" to keep you from accidentally changing or deleting the parts that do your calculations for you. You can type your values into the yellow, boxed regions, and you can also do any additional calculations to the side or below the ready-made part, which is contained in columns A-Q and rows 1-66.

Let's start by understanding the spreadsheet's logic:

In section (a) of the worksheet, you will want to put something recognizable for a group name and date so that you and others will be able to recognize it later, for instance as printouts become scattered around the lab.

In section (b) of the worksheet, you can enter your raw data for all measured quantities (L, M, h, r, m, T, and θ_m), along with their absolute uncertainties. The worksheet automatically computes the relative (percentage) uncertainty for you.

In section (c), the worksheet shows you the Amplitude Correction Factor T/T_0 that comes from your pendulum not being a true harmonic oscillator. It also calculates the "corrected period" T_0 of your pendulum in the limit of very small oscillations.

In section (d), the worksheet calculates the individual geometric correction factors related to your pendulum being a "physical pendulum" rather than an idealized point mass. Each one of the terms A, B, C, D, E, and F corresponds to one of the terms in equation (7). The worksheet also calculates an overall "mass and dimension" correction factor, and combines this with the length L and the corrected period T_0 to give you the best value of *g* from your measurements according to equation (7).

In section (e), the worksheet calculates the absolute and relative uncertainty in *g* from the uncertainties in each of the original measurements, and combines them in quadrature (i.e., by taking the square root of the sum of the squares) to give the final

uncertainty in g from your experiment (as implied by your original uncertainty estimates).

Start by entering your basic measurements into section (b) of the worksheet. Do you get a reasonable value for g?

Next, play around with the numbers a little bit. Do the geometric (mass and dimension) correction factors behave as they should? Does the corrected period give reasonable answers? (When the computer is doing all of the heavy lifting for you, it's easy for the lab to turn into a "plug and chug" activity, with your brains totally tuned out. Don't let that happen!)

Next, add your uncertainties to the worksheet. Do they propagate through as they should? Which ones are the biggest sources of error?

Once you have put your numbers in for one set of measurements at $\theta_m = 10°$ and are satisfied with your results, repeat the experiment with $\theta_m = 5°$ and with $\theta_m = 15°$. Your calculated value for T_0 should not vary with θ_m; if it does, you have a systematic error.

IV. Points to Ponder

These are things you should make a particular point of pondering in your notebook. The TA's will be looking for these when they check the notebooks.

1. About how big does your θ_m have to be for the period T of your pendulum to differ from the small amplitude period T_0 by 0.1%? By 1%?

2. Which leads to the largest "correction" in your calculation: the fact that your pendulum is not a true harmonic oscillator, or the fact that it is not a simple (point mass) pendulum?

3. In section (b) of the spreadsheet, you estimated errors for all of your measured quantities, and the spreadsheet calculated percentage errors for you. Rank these from most accurate to least accurate.

4. Each of the uncertainties in your measured quantities contributes some uncertainty to the calculation of g, as is shown in section (e) of the spreadsheet. Rank these from the largest contribution to the total uncertainty in g to the smallest contribution. Are these the same as your rankings as for question 3? Why or why not?

5. Suppose you could spend five hours and \$100 in the lab cutting any one of your measured uncertainties in half. (That is, with 5 hours of your time and \$100 you could decrease the uncertainty in L, M, h, r, m, T, or θ_m by a factor of two.) Which one would give you the "biggest bang for your buck" in terms of reducing your overall uncertainty in g?

6. The premise of question 5 is kind of bogus, in that you probably can't guarantee that one afternoon and $100 would cut the uncertainty in any one of your variables by a factor of 2. In reality, some of those uncertainties would be easy to improve, some wouldn't be. So where would you spend your time? What would be the next thing you would do to make a significant improvement in some part of your measurement to reduce your overall uncertainty in g? (There's no one right answer here; this is your chance to think and be creative.)

V. The "Right" Answer

This section violates a fundamental law of Physics 103/105 Lab, namely: there is no "right" answer. But it turns out that what was at the time the world's best measurement of *g* was made in Palmer Hall (now Frist Campus Center) in 1963, when the Physics Department was still housed there. Jim Faller, working in room 130, got

$$g = 980.1604 \pm 0.0007 \text{ cm/sec}^2 \qquad \text{(J. E. Faller, Ph. D. Thesis, Princeton, 1963).}$$

In 1998, Physics 103/105 labs moved from Palmer Hall to McDonnell Hall. *How much do you think Jim Faller's measurement would have changed had he made it on the second floor of McDonnell Hall? Could his experiment tell the difference? Can yours?*

We only have a "right" answer because Faller worked for 3 years and used a very fancy technique. Incidentally, he built a "suitcase" version of his apparatus, which he carried around comparing values of *g* in standards labs all over the world — London, Paris, Geneva. Anyway, what is the *discrepancy* between your value of *g* and Faller's? *Do they agree within the errors?*

VI. Report your Values to Your TA

When you are satisfied with your results and understand the uncertainty that the spreadsheet has calculated for you, report your results to your TA. Your TA will tabulate the results for each group, and will give each of you two graphs showing your results plotted along with those of your classmates. At the end of the week, we will collect the values for all of the lab sections and post them in the hallway, and possibly on the course web site.

How do your results compare to those of your classmates? Do your results agree to within your uncertainty? If not, then why? These kinds of "friendly" discussions are what good science is all about.

Calculation Spreadsheet for Precision Measurement of "g"

(Physics 103

NOTE: Input data only into cells in the boxed regions (yellow on the screen). All other entries are calculated

(a) Group Name, Date, etc.

Enter Group Identification and date here.

(b) Measurements and Estimated Errors:

			Best Value		**+/-**	**Estimated Error**		**(Per Cent)**
Wire length	L	=	1.00000	m	+/-	0.01000	m	1.000%
Bob mass	M	=	1.00000	kg	+/-	0.01000	kg	1.000%
Bob length	h	=	0.10000	m	+/-	0.00100	m	1.000%
Bob radius	r	=	0.10000	m	+/-	0.00100	m	1.000%
Wire mass	m	=	0.01000	kg	+/-	0.00010	kg	1.000%
Period	T	=	3.00000	sec	+/-	0.00100	sec	0.033%
Amplitude	θ_m	=	10	deg	+/-	1.0	deg	10.000%

(c) Calculation of Corrected (i.e., small-amplitude) Period:

Factor	T/T_0	=	1.00191	--	+/-	0.00038	--	0.038%
Corrected period	T_0	=	2.99429	sec	+/-	0.00152	sec	0.051%

(d) Calculation of "g":

			Best Value			Uncertainty		(Per Cent)
First term ($4\pi^2 L / T_0^2$)		=	4.40324	m/sec^2	+/-	0.04415	m/sec^2	1.003%

Correction terms for dimensions and masses (best estimates only):

A	=	h / L	=	0.10000
B	=	$h^2 / 3 L^2$	=	0.00333
C	=	$r^2 / 4 L^2$	=	0.00250
D	=	$m / 3 M$	=	0.00333
E	=	$h / 2 L$	=	0.05000
F	=	$m / 2 M$	=	0.00500

Overall mass and dimension correction factor:

$$(1 + A + B + C + D) / (1 + E + F) \quad = \quad 1.05134$$

Best Value of "g" (first term multiplied by overall correction factor):

$$g \quad = \quad \underline{4.62931 \ \text{m/sec}^2}$$

(e) Uncertainty in "g" -- Propagation of Errors:

Source of Error		% Error (Original Estimates)		Propagated Errors in Value of "g" Absolute		(Percent)
Wire length	L	1.000%	+/-	0.04383 m/sec^2		0.9467%
Bob mass	M	1.000%	+/-	0.00008 m/sec^2		0.0017%
Bob length	h	1.000%	+/-	0.00226 m/sec^2		0.0488%
Bob radius	r	1.000%	+/-	0.00021 m/sec^2		0.0045%
Wire mass	m	1.000%	+/-	0.00008 m/sec^2		0.0017%
Measured Period	T	0.033%	+/-	0.00308 m/sec^2		0.0666%
Amplitude	θ_m	10.000%	+/-	0.00336 m/sec^2		0.0763%
Quadratic Sum of Errors =			+/-	**0.044123 m/sec^2**		**0.9531%**

Rev C -- 11/15/01

PRELAB Problems for Lab #9 ; A Precision Measurement of *g*

1. To measure *g* in lab this week, you will time the period of a pendulum. Suppose that you have measured a pendulum's period five times, and obtained the following five values (in seconds): 2.871, 2.866, 2.871, 2.873, and 2.869. What is your best estimate of the actual period of the pendulum and what is the statistical uncertainty in that value? (Make whatever reasonable assumptions you need to make about the distribution being smooth and symmetric and morally upright; this isn't a trick question.)

2. Work Taylor's Problem 3.40 (on his page 88).

3. Suppose that for an amplitude of 0.1° your pendulum has a period of 2.0000 sec. What will the period of your pendulum be when it swings with an amplitude of 10° ? (Hint: the values shown in the Table of T / T_0 *vs.* θ_m appearing after Eqn. (11) in the lab manual will be useful here. If your calculations become laborious, you are probably barking up an unnecessarily inconvenient tree.)

4. Consider two pendulums, each consisting of a heavy solid brass sphere tied to a massless string. One pendulum is a sphere of radius 2 cm tied to a string of length 98 cm, the other pendulum is a sphere of radius 5 cm tied to a string of length 95 cm. For both pendulums, the distance between the pivot and the center of mass of the sphere is exactly 1 meter. Based on the theory of pendulums developed in the write-up for this lab, which pendulum has the greater period? Or are they the same? Explain briefly.

LAB #10: THE SPEED OF SOUND AND SPECIFIC HEATS OF GASES

This week you will measure the velocity of sound in a gas, and you will weigh the gas in order to find its density. These measurements will lead to a determination of C_P/C_V, the ratio of specific heats at constant pressure and constant volume, and thus should allow you to determine the molecular structure of the gas. It is a beautiful example of how measurements of macroscopic lengths, masses, and forces provide information about molecules having sizes and masses of 10^{-10} meters and 10^{-23} grams!

Let's quickly review how these quantities relate -- that is, how the speed of sound ultimately relates to degrees of freedom and specific heat. The speed v of longitudinal waves is given by

$$v = \sqrt{\frac{B}{\rho}}$$

where ρ is the density of the medium and B is the bulk modulus, defined by the relationship

$$\Delta P = -B \frac{\Delta V}{V}$$

The bulk modulus is a property of a substance: it describes the fractional change in the volume when the pressure is increased by an amount ΔP.

Now consider sound traveling in an ideal gas. As an opening exercise, start with the ideal gas law and by differentiating show that $B = P$. What assumptions did you make to get this result?

$B = P$ is not correct, however. To see why, note that sound waves propagate as a series of compressions and expansions. These variations in the medium take place so rapidly that there is no time for heat transfer from one part of the medium to another. Such a $Q = 0$ process is called adiabatic. In an adiabatic process the quantity PV^γ remains constant. Starting with $PV^\gamma = $ constant (rather than $PV = $ constant) we can show that

$$B = \gamma P, \quad \text{where } \gamma \text{ is the ratio } C_P / C_V \quad .$$

From this it follows that the speed of sound is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

GAS SAFETY

Tanks of various gases will be available in the lab. Choose one of these for your measurement of the speed of sound and, of course, use the same gas for measurement of the density. The tanks contain gas at high pressure: *do not try to move the tanks*. Every tank has three valves: one on the throat of the bottle, which allows gas to enter a large reduction valve in the regulator, which bleeds the gas slowly out of a small needle valve to which the rubber tubing is attached. <u>You need to use only the needle valve. The others have been adjusted</u>: <u>***do not tamper with them***</u>. The pressure has been set high enough to give a flow that will fill your flask in a reasonable time. If someone should increase the pressure it could cause a flask to blow up! When you open the needle valve you should feel a gentle breeze of gas flowing out of the rubber tube when held near your lips. Keep the needle valve closed when you are not taking gas, open it when you need some.

Velocity of sound

The apparatus for measuring the speed of sound is a vertical cylinder of gas. The length of the column of gas is adjusted by changing the height of a metal reservoir of oil. A loudspeaker is attached to the top of the cylinder, and it generates sound from an electrical audio oscillator. The loudspeaker also responds to sound vibrations (like a microphone) and thus the sound waves set up in the cylinder of gas affect it also. This action is detected by a meter. When the cylinder is in resonance you will get a significant change in the reading of the meter included in the speaker circuit.
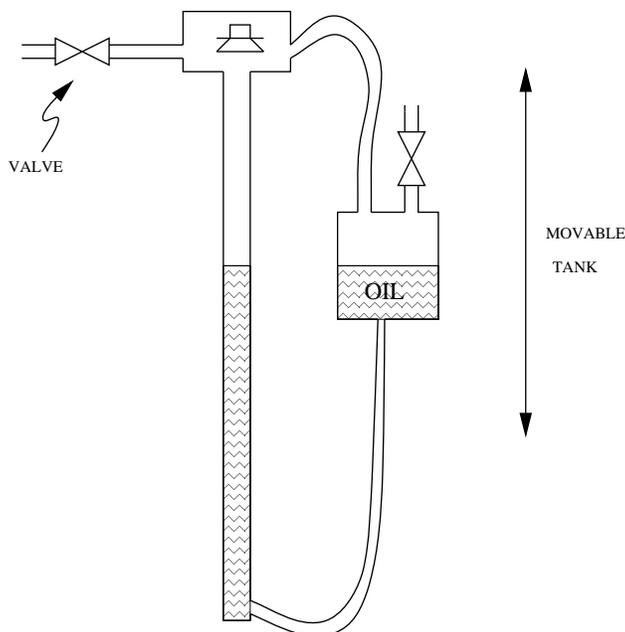
As a preliminary step, turn on the oscillator at a convenient frequency (1000 cycles/sec) and let the oil descend from the top down about 60 cm. Watch the meter and listen to the sound. You should be able to find several positions of the oil level at which resonance occurs. You will want to adjust the output level of the oscillator so that the meter reading is near the end of the scale when the system is off resonance. When the

system is at a resonance, the meter reading should change by about 10 or 15%. You might find that a different frequency (10-20%) will give a stronger indication of resonance. Explore a bit.

You are now ready to fill the cylinder with gas from one of the tanks. Note that you will need to flush out the air (or other gas) previously there. Raise the oil as high as possible during flushing to facilitate this. Be sure to open and close the valves in the right order so that the pressure in the tube is not raised above atmospheric pressure.

Lower the oil level from the top and note the approximate location of the first three resonances. Then raise the oil slowly through each resonance. (Upward motion makes is easier to determine the level of oil, as you will discover on trying it both ways.) Find the position of resonance as accurately as possible.

Compute the speed of sound from the spacing between resonances and the frequency of the audio oscillator.

Density of the gas

The density is found by weighing a known volume of the gas at atmospheric pressure. Two flasks of equal size are provided, one coated with plastic and the other clear.

Evacuate the coated flask with the vacuum pump (you can tell when this is achieved when a sharp clacking sound of the pump replaces the initial slurping sound). Weigh the evacuated flask (be careful not to drop it, the plastic coating has been put on to minimize hazard). Then fill the flask with your gas and reweigh.

Use the clear flask to find the volume by filling it with water to the proper level (remember the rubber stopper in the plastic coated flask). Measure the volume of water.

You can check your measurement of the density by comparing to the density of an ideal gas of particular molar mass: $p V = n R T$ and so $M n / V = M p / R T$, where M is the molar mass. Consider such gases as Ar, N_2, and CO_2. What density does the ideal gas law predict?

From the measured density and speed of sound for your gas, and the pressure of the atmosphere in the lab, determine the value of γ for your gas.

Tips on technique

- A good way find a resonance position is for one person to slowly raise the oil level while another records the meter readings. The resonance position can be found by graphing the meter reading versus position.

- You must measure the change in weight (few grams) of the evacuated and filled flask, which can be done by moving the rider on the balance. Be sure that the weights in the balance pan are not changed while you fill the flask with gas.

- To fill the flask, open the needle valve, repeat the lip test, and if a gentle flow is found attach the hose to your flask and open the stopcock. It will be filled when the hissing noise at the needle valve stops. Shut the stopcock. Shut the needle valve and remove the tubing. The pressure in the flask is slightly above atmospheric. Open the stopcock slightly and gas will hiss out allowing the pressure to drop to normal. Weigh the filled flask. After a minute, open the stopcock again briefly and close. Reweigh. Repeat after another minute. Reweigh. If the weight has not changed again you are finished. If it has, repeat again. Why does the weight change at all?

What gas did you use?

The tanks will be labeled so that they can be distinguished, but the labels indicating what gases they contain will be hidden. Some time into the lab your AI will provide you with a list of gases which might have been in the tanks. Can you determine from your measurements of density and γ which gas you used? How confident are you? I.e., what's your error, and how well can you distinguish between monatomic, diatomic, and polyatomic gases?

Figure from Tipler's 4th Edition (p. 492) – The first four harmonics of standing waves on a string fixed at both ends.
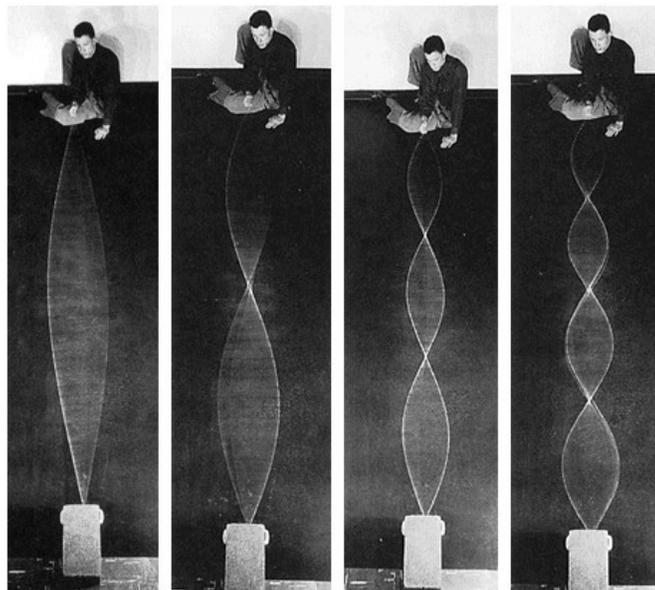
<u>REFERENCE INFORMATION</u>

The following pages contain general information which you may find useful. They include:

1. A table from Tipler, on specific heats of gases. Values of γ may be derived from this data.

2. A table of densities of various gases, directly relevant to your calculation of v.

3. A Periodic Table of the Elements, giving the atomic weights of the various elements. This will be useful if you want to consider gas densities from the point of view of the perfect gas law, $PV = nRT$.

You may recall that 1 atm = 101.3 kPa = 76.00 cm Hg .

Table 18-3 from Tipler (5th edition):

Molar Heat Capacities in J/mol·K of Various Gases at 25°C

Gas	c_p'	c_v'	c_v'/R	$c_p' - c_v'$	$(c_p' - c_v')/R$
Monatomic					
He	20.79	12.52	1.51	8.27	0.99
Ne	20.79	12.68	1.52	8.11	0.98
Ar	20.79	12.45	1.50	8.34	1.00
Kr	20.79	12.45	1.50	8.34	1.00
Xe	20.79	12.52	1.51	8.27	0.99
Diatomic					
N_2	29.12	20.80	2.50	8.32	1.00
H_2	28.82	20.44	2.46	8.38	1.01
O_2	29.37	20.98	2.52	8.39	1.01
CO	29.04	20.74	2.49	8.30	1.00
Polyatomic					
CO_2	36.62	28.17	3.39	8.45	1.02
N_2O	36.90	28.39	3.41	8.51	1.02
H_2S	36.12	27.36	3.29	8.76	1.05

DENSITY AND SPECIFIC GRAVITY OF GASES AND VAPORS

Name	Formula	Density g./l 0°C 760 mm	Density lbs./ft.³ 32°F, 1 atm.	Specific gravity Air = 1	Specific gravity O₂ = 1
Acetylene..............	C_2H_2	1.173	0.07323	0.9073	0.8208
Air....................	....	1.2929	.08071	1.0000	0.9047
Ammonia...............	NH_3	0.7710	.04813	0.5963	0.5395
Argon.................	A	1.7837	.11135	1.3796	1.2482
Arsenic fluoride........	AsF_5	7.71*	.481*	5.96*	5.40*
Arsenic hydride........	AsH_3	3.484*	.2175*	2.695*	2.438*
Boron fluoride..........	BF_3	2.99*	.187*	2.31*	2.09*
Butane (n)............	C_4H_{10}	2.5190†	.15725†	2.0854†	1.8868†
Butane, iso-...........	C_4H_{10}	2.673	.1669	2.067	1.870
Carbon dioxide.........	CO_2	1.9769	.12341	1.5290	1.3834
Carbon monoxide.......	CO	1.2504	.07806	0.9671	0.8750
Carbon oxysulfide......	COS	2.72	.170	2.10	1.90
Chlorine...............	Cl_2	3.214	.2006	2.486	2.249
Chlorine dioxide........	ClO_2	3.09[11]	.193[11]	2.39[11]	2.16[11]
Chlorine monoxide......	Cl_2O	3.89	.243	3.01	2.72
Cyanogen..............	C_2N_2	2.335*	.1458*	1.806*	1.634*
Dimethylamine.........	$(CH_3)_2NH$	1.966[17]	.1227[17]	1.521[17]	1.376[17]
Ethane................	C_2H_6	1.3566	.08469	1.0493	0.9493
Ethylene..............	C_2H_4	1.2604	.07868	0.9749	0.8820
Fluorine...............	F_2	1.696	.1059	1.312	1.187
Germanium hydride (di-germane).............	Ge_2H_6	6.74[20]	.421[20]	5.21[20]	4.72[20]
Germanium tetrahydride	GeH_4	3.420	.2135	2.645	2.393
Helium................	He	0.17847	.01114	0.13804	0.12489
Hydrogen..............	H_2	0.08988	.005611	0.06952	0.06290
Hydrogen bromide.......	HBr	3.6445	.2275	2.8189	2.5503
Hydrogen chloride.......	HCl	1.6392	.10233	1.2678	1.1471
Hydrogen iodide........	HI	5.7891	.3614	4.4776	4.0510
Hydrogen selenide......	H_2Se	3.670	.229	2.839	2.568
Hydrogen sulfide.......	H_2S	1.539	.09608	1.190	1.077
Hydrogen telluride......	H_2Te	5.81	.363	4.49	4.07
Krypton...............	Kr	3.708	.2315	2.868	2.595
Methane...............	CH_4	0.7168	.04475	0.5544	0.5016
Methylamine...........	CH_3NH_2	1.396	.08715	1.080	0.9769
Methyl chloride........	CH_3Cl	2.3076	.1441	1.7848	1.6148
Methyl ether..........	$(CH_3)_2O$	2.1098	.1317	1.6318	1.4764
Methyl fluoride........	CH_3F	1.5452	.09646	1.1951	1.0813
Neon..................	Ne	0.90035	.05621	0.69638	0.63004
Nitric oxide...........	NO	1.3402	.08367	1.0366	0.9378
Nitrogen..............	N_2	1.25055	.07807	0.96724	0.87510
Nitrogen (atm.)........	....	1.2568	.07846	0.9721	0.8795
Nitrosyl chloride.......	NOCl	2.992	.1868	2.314	2.094
Nitrosyl fluoride.......	NOF	2.176*	.1358*	1.683*	1.523*
Nitrous oxide..........	N_2O	1.9778	.1235	1.5297	1.3840
Nitroxyl chloride.......	NO_2Cl	2.57*	.160*	1.99*	1.798*
Nitroxyl fluoride.......	NO_2F	2.90	.181	2.24	2.03
Oxygen................	O_2	1.42904	.08921	1.10527	1.0000
Ozone.................	O_3	2.144	.1338	1.658	1.500
Phosphine.............	PH_3	1.5294	.09548	1.1829	1.0702
Phosphorus fluoride.....	PF_3	3.907*	.2439*	3.022*	2.734*
Phosphorus oxyfluoride..	POF_3	4.8	.30	3.7	3.4
Phosphorus pentafluoride	PF_5	5.81	.363	4.494	4.066
Propane...............	C_3H_8	2.0096	.1254	1.554	1.407

* Temperature not stated, probably 20°C.
† Both butane and air at 710 mm.

DENSITY AND SPECIFIC GRAVITY OF GASES AND VAPORS (Continued)

Name	Formula	Density g./l 0°C 760 mm	Density lbs./ft.³ 32°F, 1 atm.	Specific gravity Air = 1	Specific gravity O₂ = 1
Radon.................	Ra	9.73	.607	7.526	6.809
Silicane, chloro-.......	SiH_3Cl	3.03	.189	2.34	2.12
Silicane, chloromethyl...	SiH_2ClCH_3	3.64	.227	2.82	2.55
Silicane, dichloromethyl.	$SiHCl_2CH_3$	5.3	.33	4.1	3.7
Silicane, dimethyl.......	$SiH_2(CH_3)_2$	2.73	.170	2.11	1.91
Silicane, methyl........	SiH_3CH_3	2.08	.130	1.61	1.46
Silicane, trifluoro-......	$SiHF_3$	3.86	.241	2.99	2.70
Silicon fluoride.........	SiF_4	4.684	.2924	3.623	3.278
Silicon hexahydride......	Si_2H_6	2.85	.178	2.204	1.994
Silicon tetrahydride.....	SiH_4	1.44	.0899	1.114	1.008
Stibine (15°C, 754 mm)..	SbH_3	5.30	.331	4.10	3.71
Sulfur dioxide..........	SO_2	2.9269	.1827	2.2638	2.0482
Sulfur fluoride.........	SF_6	6.50*	.406*	5.03*	4.55*
Sulfuric oxyfluoride......	SO_2F_2	3.72*	.232*	2.88*	2.60*
Trimethylamine.........	$(CH_3)_3N$	2.580	.1611	1.996	1.805
Trimethyl boron........	$(CH_3)_3B$	2.52	.157	1.95	1.76
Tungsten fluoride.......	WF_6	12.9	.805	9.98	9.03
Xenon.................	Xe	5.851	.3653	4.525	4.094

* Temperature not stated, probably 20°C.

Periodic Table

World Wide Web version prepared by G. P. Moss

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This version of the Periodic table is based on that recommended by the Commission on the Nomenclature of Inorganic Chemistry and published in *IUPAC Nomenclature of Inorganic Chemistry, Recommendations 1990.* Atomic weights are quoted to five significant figures. For more precise values see the table of 1999 recommended values (see *Pure Appl. Chem.*, 2001, **73**, 667-683; agreed 2001 update from *Chem. Internat.* 2001, **23**, 179). The names of elements 104-109 were agreed in 1997 (see *Pure Appl. Chem.,* 1997, **69**, 2471-2473). For elements with no stable nuclides the mass of the longest-lived isotope has been quoted in brackets. However three such elements (Th, Pa, and U) do have a characteristic terrestrial isotopic composition, and for these an atomic weight is tabulated. The names for elements 110 to 114 are temporary and are based on the 1978 recommendations (see *Pure Appl. Chem.,* 1979, **51**, 381-384). The approved symbol for element 110 is included.

1																	18
1 **H** 1.0079	2											13	14	15	16	17	2 **He** 4.0026
3 **Li** 6.941	4 **Be** 9.0122											5 **B** 10.811	6 **C** 12.011	7 **N** 14.007	8 **O** 15.999	9 **F** 18.998	10 **Ne** 20.180
11 **Na** 22.990	12 **Mg** 24.305	3	4	5	6	7	8	9	10	11	12	13 **Al** 26.982	14 **Si** 28.086	15 **P** 30.974	16 **S** 32.065	17 **Cl** 35.453	18 **Ar** 39.948
19 **K** 39.098	20 **Ca** 40.078	21 **Sc** 44.956	22 **Ti** 47.867	23 **V** 50.942	24 **Cr** 51.996	25 **Mn** 54.938	26 **Fe** 55.845	27 **Co** 58.933	28 **Ni** 58.693	29 **Cu** 63.546	30 **Zn** 65.409	31 **Ga** 69.723	32 **Ge** 72.64	33 **As** 74.922	34 **Se** 78.96	35 **Br** 79.904	36 **Kr** 83.798
37 **Rb** 85.468	38 **Sr** 87.62	39 **Y** 88.906	40 **Zr** 91.224	41 **Nb** 92.906	42 **Mo** 95.94	43 **Tc** (98)	44 **Ru** 101.07	45 **Rh** 102.91	46 **Pd** 106.42	47 **Ag** 107.87	48 **Cd** 112.41	49 **In** 114.82	50 **Sn** 118.71	51 **Sb** 121.76	52 **Te** 127.60	53 **I** 126.90	54 **Xe** 131.29
55 **Cs** 132.91	56 **Ba** 137.33	57-71 *	72 **Hf** 178.49	73 **Ta** 180.95	74 **W** 183.84	75 **Re** 186.21	76 **Os** 190.23	77 **Ir** 192.22	78 **Pt** 195.08	79 **Au** 196.97	80 **Hg** 200.59	81 **Tl** 204.38	82 **Pb** 207.2	83 **Bi** 208.98	84 **Po** (209)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** (226)	89-103 ＃	104 **Rf** (261)	105 **Db** (262)	106 **Sg** (266)	107 **Bh** (264)	108 **Hs** (277)	109 **Mt** (268)	110 **Ds** (281)	111 **Uuu** (272)	112 **Uub** (285)		114 **Uuq** (289)				

* Lanthanide series	57 **La** 138.91	58 **Ce** 140.12	59 **Pr** 140.91	60 **Nd** 144.24	61 **Pm** (145)	62 **Sm** 150.36	63 **Eu** 151.96	64 **Gd** 157.25	65 **Tb** 158.93	66 **Dy** 162.50	67 **Ho** 164.93	68 **Er** 167.26	69 **Tm** 168.93	70 **Yb** 173.04	71 **Lu** 174.97
＃ Actinide series	89 **Ac** (227)	90 **Th** 232.04	91 **Pa** 231.04	92 **U** 238.03	93 **Np** (237)	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259)	103 **Lr** (262)

Return to <u>Atomic weight table</u>

PRELAB Problems for Lab #10 ; The Speed of Sound and Specific Heats of Gases

1. In this experiment, you will vary the height of the oil in a column of air, producing an air column with varying length. At certain lengths, acoustic resonances will occur for a given frequency of sound, f. Given two different (consecutive) heights h_1 and h_2, of the oil in the column that produce resonance, derive an expression for the speed of sound, v, in terms of f, h_1 and h_2.

2. What are the resonant wavelengths, λ_n, for standing sound waves in a small-diameter pipe of length L having one end open and the other end closed ? Assume that there is a displacement node at the closed end of the pipe, and a pressure node at the open end, and write down a few words as to why you think these assumptions are reasonable. (Tipler's pages 517 and 518 are relevant to this problem.)

3. Assume that various traveling waves can exist in the same space, described by the equations

$$\psi_1 (x,t) = A \sin (\ 7 \text{ m}^{-1} \cdot x \ - \ 520 \text{ sec}^{-1} \cdot t) \qquad ,$$

$$\psi_2 (x,t) = A \sin (\ 7 \text{ m}^{-1} \cdot x \ + \ 520 \text{ sec}^{-1} \cdot t) \qquad ,$$

$$\psi_3 (x,t) = A \sin (- 7 \text{ m}^{-1} \cdot x \ - \ 520 \text{ sec}^{-1} \cdot t) \qquad , \qquad\qquad \text{and}$$

$$\psi_4 (x,t) = A \sin (- 7 \text{ m}^{-1} \cdot x \ + \ 520 \text{ sec}^{-1} \cdot t) \qquad .$$

 (a) Which waves are traveling in the direction of the positive x axis, and which in the negative x direction?

 (b) What are the frequencies f and wavelengths λ of the waves? What are their speeds of propagation?

 (c) What are the angular frequencies (ω) and wave numbers (k) of the waves?

 (d) Are any of the four wave functions identical with each other? Explain why or why not. (A is the same in each case.) You may want to consider the behavior of the four functions in the vicinity of $x = 0$, $t = 0$.

> Note: It is irrelevant to answering these questions whether the ψ's (and A) describe sound waves, the electric fields of a radio or light wave, or a quantum-mechanical wave function. Superposition, and the conditions leading to standing waves and resonance, are important aspects of any wave phenomenon.

Appendix A
Data Analysis with **Excel**

This Appendix is provided primarily for students who are not frequent users of the common Excel spreadsheet program. We hope that it will be useful as tool for gaining confidence in using this important tool. For some students, it may serve mainly as a reminder of how to do things that you have done before.

Even experienced Excel users, however, should use this Appendix as an introduction to some added software, called WPTools, which extends Excel's graphing capabilities to provide extra information useful in scientific applications. (WP stands for Workshop Physics.)

Students with access to Excel on their own computers, or through the campus systems, can run through almost all of the material outside the lab. Only the WPTools material will be unavailable.

Computers are used for data analysis in any modern physics laboratory, and Princeton's teaching labs are no exception. We have built our data analysis systems around the program Excel, which is widely used on and off campus. We've added some Workshop Physics (WP) tools to make graphing data easier, and to let you do regression calculations with uncertainties, but otherwise we are using the standard, off-the-shelf software. If you are already familiar with Excel, great! If not, we'll give brief instructions here. Like any software, it can be confusing at first, so don't hesitate to ask your instructors and your fellow students for help. If you can, play around with the program a bit before lab, to get comfortable with it.

A.1   Starting Things Up

- If the computer isn't already on, turn it on and wait for it to boot up.
- If the Physics 103 window isn't already open, double click on its icon to open it.
- Double click on Excel with WPTools (Look for the X logo.) to get the program running.

A.2   Entering Data: a Simple Example[1]

When Excel is started up, you need to open a spreadsheet to work in. If you are asked if you want to reopen WPTools, click No. Then go to File → New and click on OK to open a new Workbook. (If you wanted to open a pre-existing spreadsheet, you would use the File → Open menu command; if you want to save a new spreadsheet, use the File → Save menu command. Since we'll be working with fairly small data sets, neither of these is really necessary for your lab work.)

Let's assume that you have some actual data. If not, just write down a list of 10 random pairs of numbers between 0 and 100 in increasing order. Then write down a second such list beside the first, to generate two lists of numbers, each in increasing order. Call the two lists *x* and *y*.   Go to Excel, and start entering the data in the upper left-most cell (called A1). To do this, move the cursor to this cell and click on it with the left mouse button. Enter the first *x*-data value here, pressing Enter when you are done. The cell below A1 (called A2) will automatically become the selected cell. Enter your second *x*-value here. Work down the first data column in this way. If you need to correct any of your entries, simply move the cursor, click on the relevant cell, and reenter the number.

Once you've entered the first column of numbers, move the cursor to the top cell of the second column (cell B1) and click on the left button to select it. Enter the first *y* data value here. Then press Enter and type your subsequent *y* values in the cells below.

Now that your two columns of data are in the computer, select them. Do this by moving the cursor to the top left cell, pressing *and holding* the left mouse button, *dragging* the cursor to the lowest filled cell in the second column, and *then releasing* the mouse button. The entire block of numbers will now be *selected*, as indicated by a change in color or a border. Now go to the WPTools pull-down menu and select Linear Fit. Immediately, you will get a graph of your data, along with the values and uncertainties of the best-fit straight line. After admiring this for a minute, print out a copy.

A.3   Calculations in Excel

Sometimes you will want to transform your raw data in some way before plotting it. For example, you may have entered two columns of data as above, but you may want to convert the *y*-values from inches to meters.   This is where a spreadsheet program becomes really handy. Select a blank cell somewhere on the sheet. (Cell C1 would be a good choice.) Instead of entering a number in this cell, enter the formula = 0.0254 * B1 and press Return. Excel will not only display the calculated result in cell C1, but it will also remember the formula. This is useful for two reasons: First, if you decide to change the value in cell B1, the number in cell C1 will automatically be updated. Second, you can copy the formula in cell C1 into other cells, transforming the rest of column B to use

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[1] Note: Menu commands are described as follows: File → Open means move the cursor to the word File on the line near the top of the screen, press and hold the left mouse button, drag the cursor down to the word Open, and release the button.

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it. To do this, first select cell C1. The cell becomes outlined, and a little square appears in the lower right corner of the outline. Move the cursor onto this square, press and hold the left mouse button, and move the cursor down several lines. Then release the button. Voila! Excel will use the same formula to multiply all the cells in column B by 0.0254.

Excel can do much more complicated arithmetic – for example, you could use the formula = sqrt (A1) * B1 to take the square root of the values of cells in column A, multiply them by the values in column B, and put the result in some other column.

You might also want to take differences between successive items in your data list. If you type into cell C2 the formula = B2 − B1, and then use the little square to fill this formula into cells B3, B4, *etc.,* then you will have the differences in column C.

If you want to do a transformation like this, and then you want to do a plot or a curve fit, the columns of data you want to plot may not be adjacent to each other. No problem! Let's say you want to plot the data in cells A1 to A10 on the horizontal axis, and that in cells C1 to C10 on the vertical axis. First select cells A1-A10. (Go to A1, hold down the left mouse button, drag the cursor to A10, and then release the mouse button.) Then hold down the Ctrl key and select cells C1-C10. Now both regions, A1-A10 and C1-C10 have been selected, but not cells B1-B10. Run the WPTools → Linear Fit routine, and you will get the plot you want.

A.4 Accumulating Values: Excel Tricks

There will be times in the physics lab when you want to accumulate sums of a series of values. For example, you might have measured a series of time intervals,

$$\Delta t_1 = \text{time interval between event 1 and event 2,}$$

$$\Delta t_2 = \text{time interval between event 2 and event 3,}$$

$$\Delta t_3 = \text{time interval between event 3 and event 4,}$$

etc.

You may wish to convert these into a continuous time scale. In other words, you may want to declare that t is equal to 0 at the time of event 1, and then find values for

$$\text{time of event 2} = \Delta t_1$$

$$\text{time of event 3} = \Delta t_1 + \Delta t_2$$

$$\text{time of event 4} = \Delta t_1 + \Delta t_2 + \Delta t_3$$

This is easy to do. Say that $\Delta t_1, \Delta t_2$, etc., are in cells A1, A2, *etc.,* and we want to put the accumulated times in column B. First put a 0 in cell B1 (since we are taking $t = 0$ at the

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time of the first event). Then go to cell B2 and enter the formula = SUM ($A$1:A1). (The SUM function simply adds up the cells in a specified range.) The usefulness of the $ notation becomes apparent when you want to calculate the rest of the time values. Select B2, move the cursor to the square in the lower right hand corner of the cell border, press and hold the left mouse button, drag the cursor down several cells, and release the button. The cells in column B are now filled with SUM functions, but in a special way. The $A$1 keeps the first cell in the range fixed at A1, because of the $ signs. But the second part of the function call changes from A1 to A2 to A3, *etc.* In other words, cell B3 now reads = SUM ($A$1:A2), cell B4 reads = SUM ($A$1:A3), and so on. These are exactly the formulae we want for the event time calculations, so column B is now filled with your calculated time values.

A.5   Further Notes About the Workshop Physics Excel Routines.

- Use the WPTools → Polynomial Fit menu command, and set Order = 2 to fit lines of the form $y = a_0 + a_1 x + a_2 x^2$.

- If you enter non-numerical text in the cell above each column of data, it will be used to label the horizontal and vertical axes on the plot.

- The plotting and fitting routines always use the first selected column for the horizontal (x) points, and the second selected column for the vertical (y) points. So the order of selecting columns matters. (If the columns are adjacent, and selected both at once, the left-hand column is plotted horizontally.)

- Empty rows are usually ignored, but partially empty rows may corrupt the fit.

- To delete a plot, select it (move the cursor onto it and click once); then press the Delete key. To delete a column of the data sheet, select the entire column by clicking on the letter at the top, and use Edit → Delete.

- If data are modified after running a fit, the associated plot will be automatically updated, but the fit parameters will NOT be re-calculated. Usually it is best to delete both the old plot and its fit parameters after updating data.

Appendix B

Estimation of Errors

While the subject of **error analysis** can become quite elaborate, we first emphasize a basic but quite useful strategy, discussed in secs. B.1-2. Then, we distinguish between **random** (or **statistical** uncertainties and **systematic** uncertainties in sec. B.3. Random uncertainties follow the famous bell curve, as sketched in secs. B.4-5. The important distinction between the uncertainty on a single measurement, and the uncertainty on the average of many repeated measurements is reviewed in secs. B.7-7. The subject of **propagation of errors** on measured quantities to the error on a function of those quantities is discussed in sec. B.8.

B.1   67% Confidence

Whenever we make a measurement of some value v, we would also like to be able to say that with 2/3 probability the value lies in the interval $[v - \sigma, v + \sigma]$. We will call σ the **uncertainty** or **error** on the measurement. That is, if we repeated the measurement a very large number of times, in about two thirds of those measurements the value v would be in the interval stated.

B.2   A Simple Approach

Repeat any measurement three times, obtaining a set of values $\{v_i\}$, $i = 1, 2, 3$. Report the average (mean),

$$\bar{v} = \frac{1}{N} \sum_{i=1}^{N} v_i \qquad \text{(for } N = 3\text{)}, \tag{B.1}$$

as the best estimate of the true value of v, and the uncertainty σ as

$$\sigma = \frac{v_{\max} - v_{\min}}{2}. \tag{B.2}$$

If you take more than three measurements, you can still implement this procedure with the aid of a **histogram**. Divide the range of observed values of v into 5-10 equal intervals (called **bins**). Located the bin that contains each measurement, and draw a box one unit high above that bin. Stack the boxes on top of one another if more than one measurement falls in a bin. To estimate the error, determine the interval in v that contains the central 2/3 of the measurements, *i.e.*, the central 2/3 of the boxes you just drew, and report the error as 1/2 the length of this interval.

B.3   Random and Systematic Uncertainties

The uncertainty in a measurement of a physical quantity can be due to intrinsic random uncertainty (colloquially: error) as well as to systematic uncertainty.

Random uncertainties lead to difference in the values obtained on repetition of measurements. Systematic uncertainties cause the measurement to differ from its ideal value by the same amount for all repetitions of the measurement.

Random uncertainties can arise from vibrations of the components of a set-up driven by random thermal fluctuations, random noise in the electronics, and/or many other small but uncontrolled effects including quantum fluctuations.

In principle, the effect of random uncertainties can be made as small as desired by repetition of the measurements, such that the dominant uncertainty is due to systematic effects (which can only be reduced by designing a better measurement apparatus).

B.4   The Bell Curve

In many cases when a measurement is repeated a large number of times the distribution of values follows the bell curve, or Gaussian distribution:

$$P(v) = \frac{e^{-(v-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\sigma},$$ (B.3)

where $P(v)dv$ is the probability that a measurement is made in the interval $[v, v + dv]$, μ is true value of the variable v, and σ is the standard deviation or uncertainty in a single measurement of v. See Figure B.1.
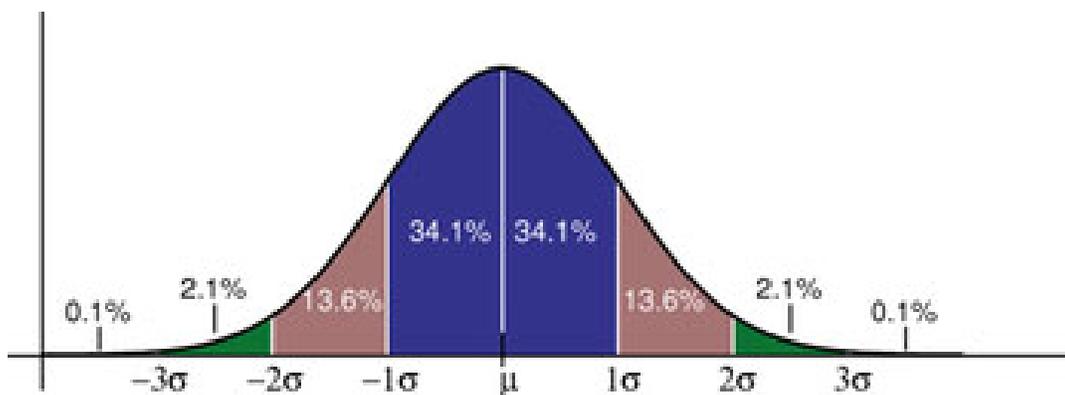

Figure B.1: The probability distribution measurements of a quantity with true value μ and Gaussian uncertainty σ of a singe measurement. About 68% of the measurements would fall in the interval between $\mu - \sigma$ and $\mu + \sigma$, and 95% would fall in the interval $\mu \pm 2\sigma$.

The Table lists the confidence that a single measurement from a Gaussian distribution falls within various intervals about the mean. If the 100 students in Ph101 each make 100

Table B.1: The probability (or confidence) that a measurement of a Gaussian-distributed quantity falls in a specified interval about the mean.

Interval	Confidence
$\pm\sigma$	68%
$\pm 2\sigma$	95%
$\pm 3\sigma$	99.7%
$\pm 4\sigma$	99.994%

measurements during these lab sessions, then 10,000 measurements will be taken in all. The Table tells us that if those measurements have purely Gaussian 'errors', then we expect one of those measurements to be more than 4σ from the mean.

B.5 Estimating Uncertainties When Large Numbers of Measurements Are Made

One can make better estimates of uncertainties if the measurements are repeated a larger number of times. If N measurements are made of some quantity resulting in values v_i, $i = 1, ...N$ then the mean is, of course,

$$\bar{v} = \frac{1}{N} \sum_{i=1}^{N} v_i, \tag{B.4}$$

and the standard deviation of the measurements is

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (v_i - \bar{v})^2}. \tag{B.5}$$

Calculus experts will recognize that the operation $(1/N) \sum_{i=1}^{N}$ becomes $\int P(v)\, dv$ in the limit of large N. Then, using the Gaussian probability distribution (B.3) one verifies that

$$\bar{v} = \langle v \rangle = \int_{-\infty}^{\infty} v P(v)\, dv, \quad \text{and} \quad \sigma^2 = \left\langle (v - \bar{v})^2 \right\rangle = \int_{-\infty}^{\infty} (v - \bar{v})^2 P(v)\, dv. \tag{B.6}$$

B.6 The Uncertainty on Mean of a Uniformly Distributed Quantity

Not all measurable quantities follow the Gaussian distribution. A simple example is a quantity with a uniform distribution, say with values v equally probable over the interval $[a, b]$. It is clear that the average measurement would be $(a + b)/2$, but what is the uncertainty of the measurement? If we adopt the simple prescription advocated in secs. B.2 we would

report the uncertainty as $(b-a)/3$ since $2/3$ of the time the measurement would fall in an interval $2(b-a)/3$ long. If instead we use the calculus prescription for σ given in eq. (B.6) we find that

$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{b-a}{3.46}, \tag{B.7}$$

which result is often used by experts.

B.7   The Uncertainty in the Mean

Thus far we have considered only the uncertainty or spread in measured values of some quantity v. A related but different question is: what is the uncertainty on our best estimate of v (which is just the mean value of our measurements, $\bar{v} = (1/N)\sum v_i$)?

The uncertainty on the mean $\bar{v}$ is surely less that the uncertainty, σ, on each measurement v_i. Indeed, the uncertainty on the mean is given by

$$\sigma_{\bar{v}} = \frac{\sigma}{\sqrt{N}}, \tag{B.8}$$

where σ is our estimate of the measurement error obtained from one of the methods sketched previously.

Appendix C illustrates eq. (B.8) using measurements of g from past Ph101 labs.

B.8   The Uncertainty on a Function of Several Variables (Propagation of Error)

In many cases we are interested in estimating the uncertainty on a quantity f that is a function of measured quantities a, b, ... c. If we know the functional form $f = f(a, b, ...c)$ we can estimate the uncertainty σ_f using some calculus. As a result of our measurements and the corresponding 'error analysis' we know the mean values of a, b, ... c and the error estimates σ_a, σ_b, ... σ_c of these means. Our best estimate of f is surely just $f(a, b, ...c)$ using the mean values.

To estimate the uncertainty on f we note that the change in f due to small changes in a, b, ... c is given by

$$\Delta f = \frac{\partial f}{\partial a}\Delta a + \frac{\partial f}{\partial b}\Delta b + ... + \frac{\partial f}{\partial c}\Delta c. \tag{B.9}$$

If we just averaged this expression we would get zero, since the 'errors' Δa, ... Δc are sometimes positive, sometimes negative, and average to zero. Rather, we square the expression for Δf, and then average.

$$\Delta f^2 = \left(\frac{\partial f}{\partial a}\right)^2 \Delta a^2 + ... + \left(\frac{\partial f}{\partial c}\right)^2 \Delta c^2 + ... + 2\frac{\partial f}{\partial a}\frac{\partial f}{\partial c}\Delta a\Delta c + ... \tag{B.10}$$

On average the terms with factors like $\Delta a\Delta c$ average to zero (under the important assumption that parameters a, b, ... c are independent). We identify the average of the squares of

the changes relative to the mean values as the squares of the errors: $\langle \Delta a^2 \rangle = \sigma_a^2$, *etc.* This leads to the prescription

$$\sigma_f^2 = \left(\frac{\partial f}{\partial a} \right)^2 \sigma_a^2 + ... + \left(\frac{\partial f}{\partial c} \right)^2 \sigma_c^2 + ... \tag{B.11}$$

Some useful examples are

$$f = a \pm b \pm ... \pm c \qquad \Rightarrow \qquad \sigma_f = \sqrt{\sigma_a^2 + \sigma_b^2 + ... + \sigma_c^2}, \tag{B.12}$$

and

$$f = a^l b^m ... c^n \qquad \Rightarrow \qquad \frac{\sigma_f}{f} = \sqrt{l^2 \left(\frac{\sigma_a}{a} \right)^2 + m^2 \left(\frac{\sigma_b}{b} \right)^2 + ... + n^2 \left(\frac{\sigma_c}{c} \right)^2}, \tag{B.13}$$

where l, m and n are constants that may be negative.

For more detailed and rigorous analyses one can consult, for example:

- P.R. Bevington and D.K. Robinson, *Data Reduction and Error Analysis for the Physical Science*, 2nd ed. (McGraw-Hill, New York, 1992).

- J.R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (University Science Books, 1997).

Appendix C

Standard Deviation of the Mean of g

Suppose you make N repeated measurements of a quantity g, such as the acceleration due to gravity. How well is the value of g determined by these measurements?

For example, during the 2006 sessions of Ph101 Lab 3 a total of 37 different measurements of g were made, as shown in the histogram Fig. C.1.
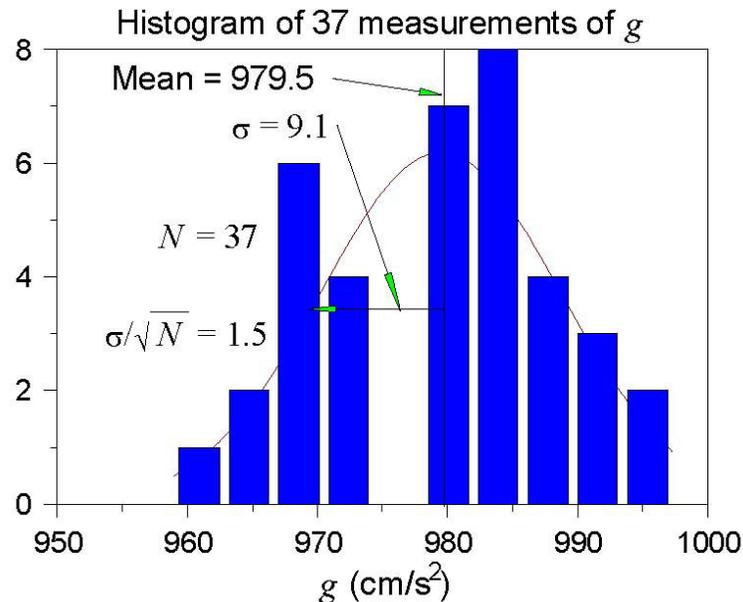

Figure C.1: Histogram of the values of g measured in the 2006 Ph101 Lab 3. The horizontal axis is g, and the vertical axis is the number of times a value of g was reported to lie with the range of g corresponding to the width of a vertical bar.

A histogram is a graph containing M vertical bars in which the height of a bar indicates the number of data points whose value falls within the corresponding "bin", i.e., within the interval $[g_j - \Delta/2, g_j + \Delta/2]$, where $g_j, j = 1, M$ and the centers of the M bins and Δ is the bin width. One can make a histogram of a data set $\{g_i\}$ using Excel/Tools/Data Analysis/Histogram. *Enter the data $\{g_i\}$ in one column of an* Excel *spreadsheet. Click on the* Input Range: *box of the* Histogram *window; then click and hold the left mouse button on the first data point, and drag the mouse to the last data point to enter the cell addresses of the data. Click on* Chart Output *and then* OK *to create a basic histogram. If the number/spacing of "bins" chosen by* Excel *is awkward, fill a new column with a linear series of 5-10 steps that begins near the lowest g_i and ends near the highest; create a new histogram with the* Excel *addresses of the first and last elements of the bin list in the box* Bin Range:.

The mean value $\bar{g}$ is calculated according to

$$\bar{g} = \frac{\sum_{i=1}^{N} g_i}{N}, \tag{C.1}$$

and was found to be $\bar{g} = 939.5$ cm/s^2 for the data shown in Fig. C.1.

The distribution of the value of g is approximately Gaussian, and the standard deviation of this distribution is calculated according to

$$\sigma_g = \sqrt{\frac{\sum_{i=1}^{N}(g_i - \bar{g})^2}{N-1}}, \tag{C.2}$$

with the result that $\sigma_g = 9.1$ cm/s^2.

The standard deviation σ_g is a good estimate of the uncertainty on a single measurement of g. However, after 37 measurements of g, the uncertainty on the mean value $\bar{g}$ is much smaller than σ_g.

An important result of statistical analysis is that the standard deviation (*i.e.*, the uncertainty) of the mean of the N measurements is related to the standard deviation of the distribution of those measurements by,

$$\sigma_{\bar{g}} = \frac{\sigma_g}{\sqrt{N}}. \tag{C.3}$$

For the data shown in Fig. C.1, where $N = 37$, we obtain

$$\sigma_{\bar{g}} = \frac{9.1}{\sqrt{37}} = 1.5 \text{ cm/s}^2. \tag{C.4}$$

That is, we can report the result of all 37 measurements of g as

$$g = 979.5 \pm 1.5 \text{ cm/s}^2. \tag{C.5}$$

As a check that eq. (C.3) is valid, we can analyze the data another way. Namely, we can first calculate the means $\bar{g}_i$ for the 5 different sessions of Ph101 Lab 3. Then, we can make a histogram of these 5 values, as shown in Fig. C.2.

The mean of the 5 means is 979.6 cm/s^2, which is essentially identical to the mean of the 37 individual measurements of g. The standard deviation of the 5 means shown in Fig. C.2 is calculated to be 1.6 cm/s^2, which is essentially identical to the previous calculation (C.4) of the standard deviation of the mean.

Concluding Remarks: If N were much larger than what we have here, the histogram C.1 would approach the Gaussian distribution (the bell-curve) shown in Appendix B. The peak in the histogram would be very close to the mean value $\bar{g}$ of the measurements, which represents the best estimate of g from the data. The standard deviation $\sigma_g \approx \text{width}/2$ is a measure of the uncertainty of a single measurement,[1] while $\sigma_g/\sqrt{N}$ is the uncertainty on the best estimate $\bar{g}$.

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[1] Strictly speaking, the full width at half maximum of a Gaussian distribution is $2.35\sigma_g$.
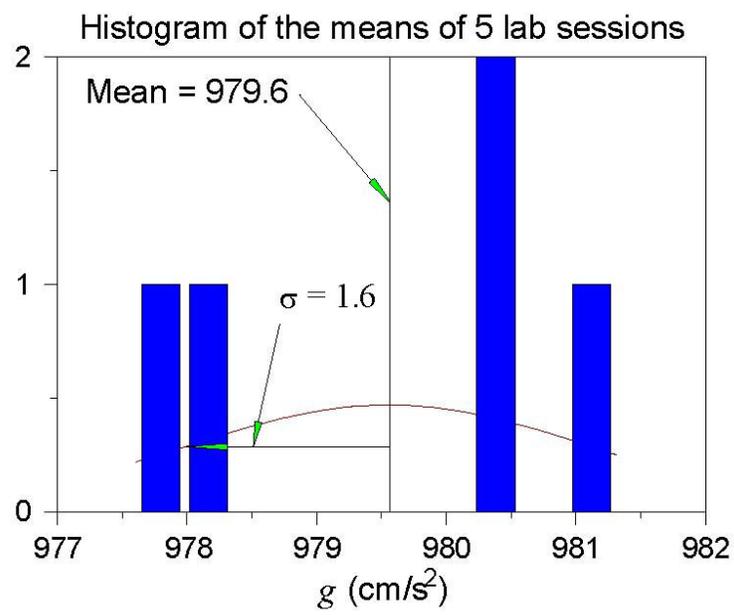
Figure C.2: Histogram of the mean values of g measured in the 5 sessions of Ph101 Lab 3 in 2006.

Appendix D

Polynomial Fits in WPtools

D.1 Polynomial Regression

In this technical Appendix we sketch the formalism used in the **polynomial regression** method for fitting data. This is a generalization of the method of linear regression.

We start with a set of data (x_j, y_j), $j = 1, ...m$, and we wish to fit these data to the nth-order polynomial

$$y(x) = \sum_{i=0}^{n} a_i x^i. \tag{D.1}$$

In general each measurement y_j has a corresponding uncertainty σ_j. That is, if the measurements were repeated many times at coordinate x_j the values of y_j would follow a gaussian distribution of standard deviation σ_j. We indicate in sec. D.2 how the program WPtools proceeds in the absence of input data as to the σ_j.

Because of the uncertainties in the measurements y_j we cannot expect to find the ideal values of the coefficients a_i, but only a set of best estimates we will call $\hat{a}_i$. However, we will also obtain estimates of the uncertainties in these best-fit parameters which we will label as $\sigma_{\hat{a}_i}$.

The best-fit polynomial is then

$$\hat{y}(x) = \sum_{i=0}^{n} \hat{a}_i x^i. \tag{D.2}$$

The method to find the $\hat{a}_i$ is called least-squares fitting as well as polynomial regression because we minimize the square of the deviations. We introduce the famous **chi square**:

$$\chi^2 = \sum_{j=1}^{m} \frac{[y_j - \hat{y}(x_j)]^2}{\sigma_j^2} = \sum_{j=1}^{m} \frac{\left(y_j - \sum_{i=0}^{n} \hat{a}_i x_j^i\right)^2}{\sigma_j^2}. \tag{D.3}$$

Fact: $\exp(-\chi^2/2)$ is the (un-normalized) probability distribution for observing a set of variables $\{y_j(x_j)\}$ supposing the true relation of y to x is given by eq. (D.2).

A great insight is that $\exp(-\chi^2/2)$ can be thought of another way. It is also the (un-normalized) probability distribution that the polynomial coefficients have values a_i when their best-fit values are $\hat{a}_i$ with uncertainties due to the measurements $\{y_j\}$. Expressing this in symbols,

$$\exp(-\chi^2/2) = \text{const} \times \exp\left(-\sum_{k=0}^{n} \sum_{l=0}^{n} \frac{(a_k - \hat{a}_k)(a_l - \hat{a}_l)}{2\sigma_{kl}^2}\right), \tag{D.4}$$

or equivalently

$$\chi^2/2 = \text{const} + \sum_{k=0}^{n}\sum_{l=0}^{n} \frac{(a_k - \hat{a}_k)(a_l - \hat{a}_l)}{2\sigma_{kl}^2}. \tag{D.5}$$

The uncertainty on $\hat{a}_k$ is σ_{kk} in this notation. In eqs. (D.4) and (D.5) we have introduced the important concept that the uncertainties in the coefficients $\hat{a}_k$ are correlated. That is, the quantity σ_{kl}^2 is a measure of the probability that the values of a_k and a_l both have positive fluctuations at the same time. In fact, σ_{kl}^2 can be negative indicating that when a_k has a positive fluctuation then a_l has a correlated negative one.

One way to see the merit of minimizing the χ^2 is as follows. According to eq. (D.5) the derivative of χ^2 with respect to a_k is

$$\frac{\partial \chi^2/2}{\partial a_k} = \sum_{l=0}^{n} \frac{a_l - \hat{a}_l}{\sigma_{kl}^2}, \tag{D.6}$$

so that all first derivatives of χ^2 vanish when all $a_l = \hat{a}_l$. That is, χ^2 is a minimum when the coefficients take on their best-fit values $\hat{a}_i$. A further benefit is obtained from the second derivatives:

$$\frac{\partial^2 \chi^2/2}{\partial a_k \partial a_l} = \frac{1}{\sigma_{kl}^2}. \tag{D.7}$$

In practice we evaluate the χ^2 according to eq. (D.3) based on the measured data. Taking derivatives we find

$$\frac{\partial \chi^2/2}{\partial \hat{a}_k} = \sum_{j=1}^{m} \frac{\left(y_j - \sum_{i=0}^{n} \hat{a}_i x_j^i\right)\left(-x_j^k\right)}{\sigma_j^2} = \sum_{i=0}^{n}\sum_{j=1}^{m} \frac{\hat{a}_i x_j^i x_j^k}{\sigma_j^2} - \sum_{j=1}^{m} \frac{y_j x_j^k}{\sigma_j^2}, \tag{D.8}$$

and

$$\frac{\partial^2 \chi^2/2}{\partial \hat{a}_k \partial \hat{a}_l} = \sum_{j=1}^{m} \frac{x_j^k x_j^l}{\sigma_j^2} \equiv M_{kl}. \tag{D.9}$$

To find the minimum χ^2 we set all derivatives (D.8) to zero, leading to

$$\sum_{i=0}^{n}\sum_{j=1}^{m} \frac{x_j^i x_j^k}{\sigma_j^2}\hat{a}_i = \sum_{j=1}^{m} \frac{y_j x_j^i}{\sigma_j^2} \equiv V_k. \tag{D.10}$$

Using the matrix M_{kl} introduced in eq. (D.9) this can be written as

$$\sum_{i=0}^{n} M_{ik}\hat{a}_i = V_k. \tag{D.11}$$

We then calculate the inverse matrix M^{-1} and apply it to find the desired coefficients:

$$\hat{a}_k = \sum_{l=0}^{n} M_{kl}^{-1} V_l. \tag{D.12}$$

Comparing eqs. (D.7) and (D.9) we have

$$\frac{1}{\sigma_{kl}^2} = M_{kl}. \tag{D.13}$$

The uncertainty in best-fit coefficient $\hat{a}_i$ is then reported as

$$\sigma_{\hat{a}_i} = \sigma_{ii} = \frac{1}{\sqrt{M_{ii}}}. \tag{D.14}$$

D.2 Procedure When the σ_j Are Not Known

This method can still be used even if the uncertainties σ_j on the measurements y_j are not known. When the functional form (D.1) correctly describes the data we claim that on average the minimum χ^2 has value $m - n - 1$.[1] To take advantage of this remarkable result we suppose that all uncertainties σ_j have a common value, σ. Then

$$\chi^2 = \sum_{j=1}^{m} \frac{[y_j - \hat{y}(x_j)]^2}{\sigma^2} \approx m - n - 1, \tag{D.15}$$

so that

$$\sigma_j = \sigma = \sqrt{\frac{\sum_{j=1}^{m}[y_j - \sum_{i=0}^{n} \hat{a}_i x_j^i]^2}{m - n - 1}}. \tag{D.16}$$

In practice it appears that the error estimates from this procedure are more realistic if a fit is made using a polynomial with one order higher than needed for a 'good' fit to the data.

Using eq. (D.16) as the estimate of the uncertainty σ on each of the measurements y_j, the matrix M_{kl} of eq. (D.9) becomes

$$M_{kl} = \frac{m - n - 1}{\sum_{j'=1}^{m}[y_{j'} - \sum_{i'=0}^{n} \hat{a}_{i'} x_{j'}^{i'}]^2} \sum_{j=1}^{m} x_j^k x_j^l. \tag{D.17}$$

The estimate (D.14) of the uncertainty on the fit coefficient $\hat{a}_i$ is now given by

$$\sigma_{\hat{a}_i} = \frac{1}{\sqrt{M_{ii}}} = \sqrt{\frac{\sum_{j'=1}^{m}[y_{j'} - \sum_{i'=0}^{n} \hat{a}_{i'} x_{j'}^{i'}]^2}{(m - n - 1) \sum_{j=1}^{m} x_j^{2i}}} \tag{D.18}$$

When WPtools performs a polynomial regression it generates a plot of the data points and the best-fit curve, along with numerical values of various parameters associated with the fit. Figure D.1 gives an example of a fit to a set of 8 data points of the form $y = x^2$. The fit is to the form $y = a_0 + a_1 x + a_2 x^2$. The fit coefficients are $a_0 = -0.4107$, $a_1 = -0.3274$ and $a_2 = 1.1964$. The uncertainties (standard errors) on the fit coefficients are reported as $SE(a_0) = 4.0070$, $SE(a_1) = 2.0429$ and $SE(a_2) = 0.2216$, as calculated according to eq. (D.18). *Note that the uncertainties on coefficients a_1 and a_1 are larger than the coefficients themselves, which tells us that these coefficients are indistinguishable from zero.*

Also indicated on the plot are the values $R^2 = 0.9915$ and $\sigma = 2.8721$. The latter is the uncertainty in the data points $\{y_j\}$, calculated according to eq. (D.16) with $m = 8$ and $n = 2$. The quantity R^2 is defined by

$$R^2 = \frac{\sum_{j=1}^{m}[\hat{y}(x_j) - \overline{y}]^2}{\sum_{j=1}^{m}[y(x_j) - \overline{y}]^2}, \tag{D.19}$$

where the average $\overline{y} = \sum_{j=1}^{m} y(x_j)/m$. This is a measure of the "goodness of fit". If the fit is perfect then $\hat{y}_j = y_j$ for all j and $R^2 = 1$. It is not obvious, but $R^2 \leq 1$ always. The extreme case of $R^2 = 0$ occurs when the fit has the trivial form $\hat{y}(x) = \overline{y}$ for all x, which in general is a bad fit. The qualitative conclusion is that if R^2 is not close to 1, the fit results are to be regarded with suspicion.

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[1] The whole fitting procedure does not make sense unless there are more data points (m) than parameters ($n + 1$) being fitted.

Series1(X)=a_0+$\Sigma a_n(X)^n$

R^2=0.9915
σ =2.8721

a_0=-0.4107
a_1=-0.3274
a_2=1.1964

• Series1
— Series1 Fit

SE(a_0)=4.0070
SE(a_1)=2.0429
SE(a_2)=0.2216
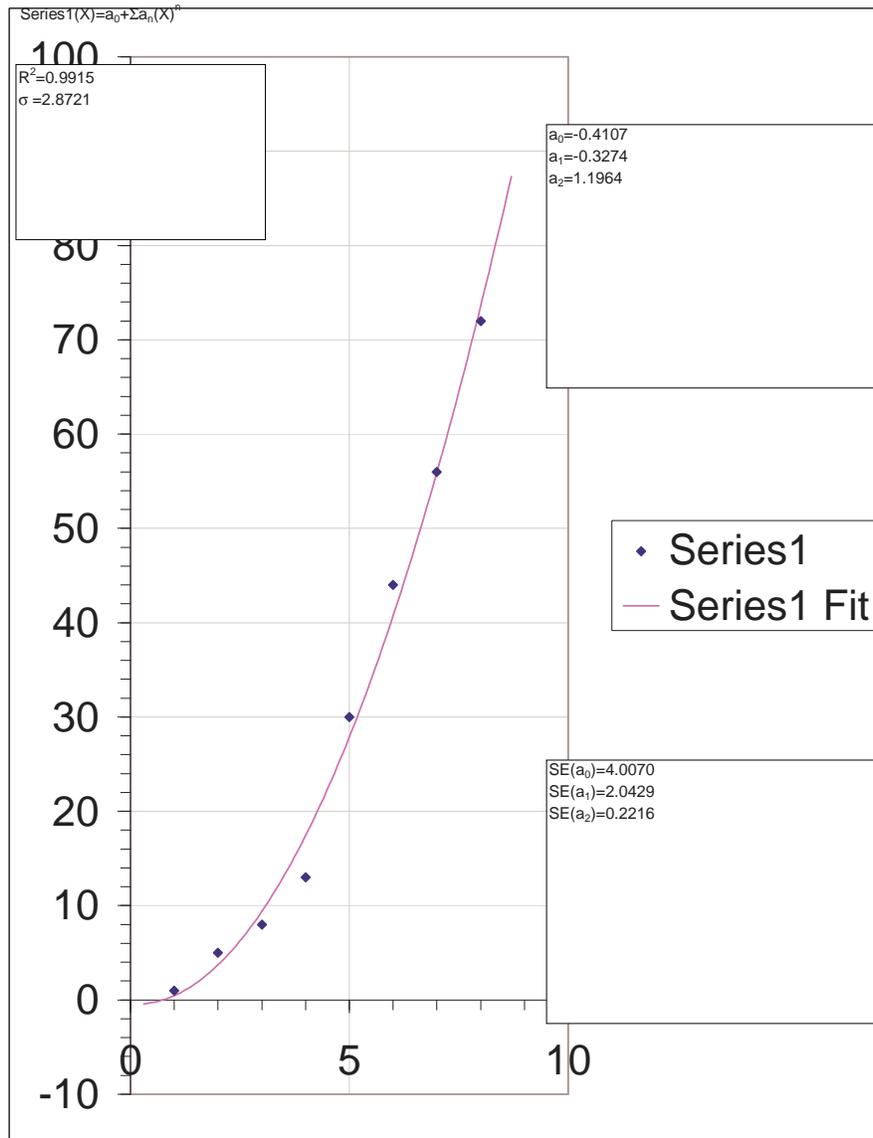
Figure D.1: Sample plot from WPtools Polynomial Fitting.