LAB #5: ROTATIONAL MOTION


BEFORE YOU COME TO LAB:

- **Consider the "optional" PreLab problem set attached.   Note that you need the answer to problem 2 to complete the Lab.**

This week you will do two experiments involving rolling objects for which friction can be largely ignored. The setup for each experiment is different; pick whichever one you want to do first, and go to one of the setups for that experiment at the start of the lab. (Be flexible.) The two projects in this week's Lab might be relevant to engineers designing theme park rides, courses for downhill ski races, *etc.* They will remind you of roller coaster loops and of racers taking to the air after speeding across a bump.

A: Rolling Off a Log

You may have already worked out the problem of an object sliding off a frictionless circular track. The object leaves the track at an angle $\theta = \cos^{-1}(2/3) = 48.2°$.   In this Lab you minimize friction by letting an object *roll* down the circular track. In the Prelab problems you are encouraged to show that the theoretical expression for the angle at which a rolling object of mass m, radius r, and moment of inertia $I = kmr^2$ about its center leaves the

track is $\theta = \cos^{-1}\dfrac{2}{3+k}.$ Hence, $k = \dfrac{2}{\cos\theta} - 3,$ and

$$\sigma_k = \frac{2\sin\theta}{\cos^2\theta}\sigma_\theta.$$

The idea of this experiment is to work out a technique whereby you can measure as accurately as possible the angle where the rolling object just leaves the track, and thereby determine the k factor (and uncertainty on this) for a solid cylinder, a hollow cylinder, and a sphere.

You may find that using chalk dust, or wetting the object before you start it rolling, causes it to leave a track which answers the question.   Alternatively, you can make a movie and use VideoPoint to determine the takeoff angle.

If you choose to use VideoPoint, it will be convenient to convert to polar coordinates from the usual Cartesian *(x,y)* coordinate system. Then, after moving the origin of the
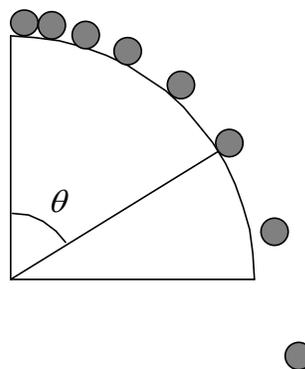
coordinate system to the center of the track, you can plot *radius* versus time to see if you can determine the takeoff angle.

Using **VideoPoint** with polar coordinates

To change to polar coordinates, double-click on the origin and make the change in the first screen shown below. The origin can be moved to the center of the track simply by clicking-and-dragging on it. Then you can choose to plot radius or angle when you make your graphs.(See the second screen shown.

However you choose to do your analysis, once you have your technique down for one object, try it on the others, and think about any differences you find.
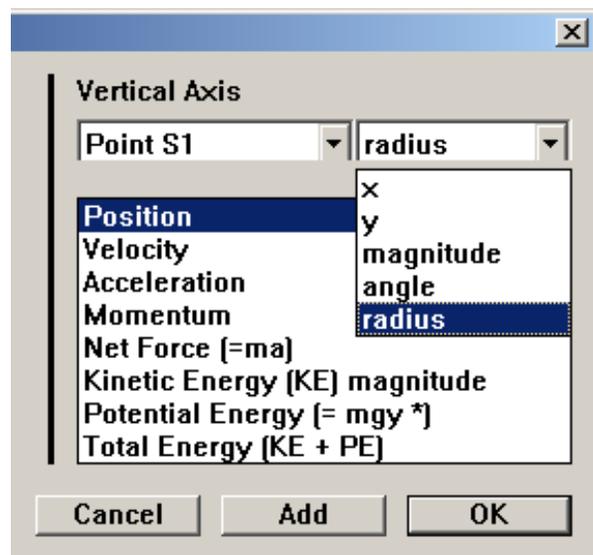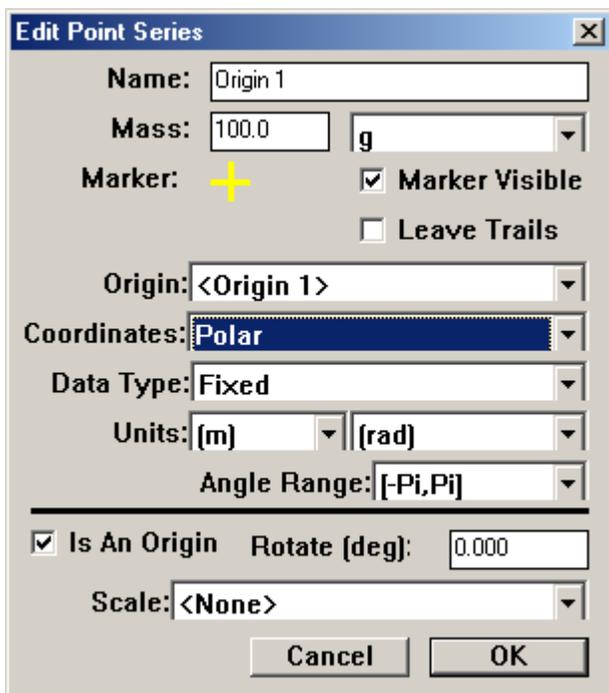
You can (with practice) balance a ball by spinning it on your fingertip.   Can you balance a ball on a log by spinning it?

B: Loop-the-Loop

You can roll a ball down a ramp, and it will pick up enough speed to "loop the loop." Determine experimentally the minimum height h such that the ball loops the loop without leaving the track. From this measurement (and its uncertainty), determine the *k* factor of the ball (and the uncertainty on *k*).
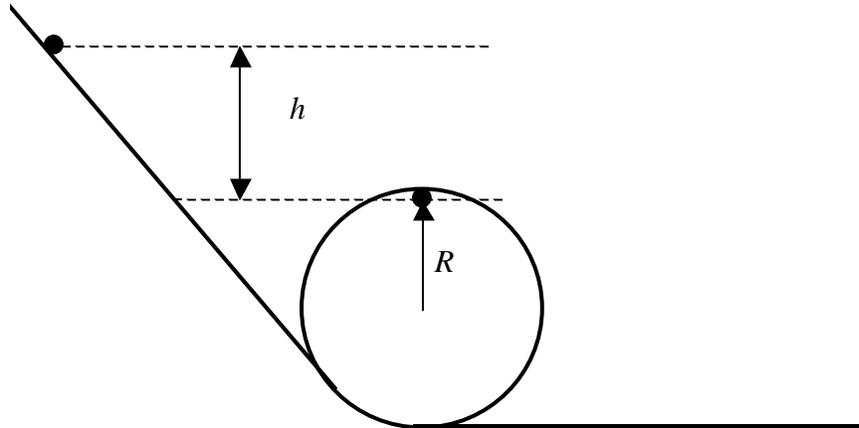
C. Angular Momentum Toys

Take some time during the Lab to play with the angular momentum toys:

- Rotating stool + hand weights or bicycle wheel
- Bicycle wheel by itself: hold it with two hands, or via the rope on one handle.

Spin the wheel up by hand (your Lab partner can help with this.

For the rotating stool + hand weights, think about conservation of angular momentum:

$$L = I\omega \left(\approx 2Wr\omega \right) = I'\omega' \left(\approx 2Wr'\omega' \right).$$

For the spinning bicycle wheel think about the vector equation for changes in angular momentum due to torques: $\dfrac{d\mathbf{L}}{dt} \left(\approx I \dfrac{d\boldsymbol{\omega}}{dt} \right) = \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}.$

When holding the bicycle wheel by the rope such that its angular velocity vector $\boldsymbol{\omega}$ is horizontal, it precesses about the vertical with angular velocity Ω according to $I\dfrac{d\boldsymbol{\omega}}{dt} = \boldsymbol{\Omega} \times I\boldsymbol{\omega} = \mathbf{r} \times m\mathbf{g}.$ Since vectors $\boldsymbol{\omega}$ and Ω are perpendicular, and vectors $\mathbf{r}$ and $\mathbf{g}$ are perpendicular, the equation of motion simplifies to $\Omega = \dfrac{mgr}{I\omega}.$ Can you verify that the faster you spin the wheel, the slower it precesses? Can you keep track of the vector cross products to predict whether the precession is clockwise or counterclockwise, as viewed from above?

PRELAB Problems for Lab #5: Rotational Motion

1. An object of radius r, mass m, and moment of inertia $I = kmr^2$ about its axis starts from rest at the top of a large cylinder of radius R and rolls down. At what angle θ from the vertical does the object leave the surface of the large cylinder? (Hint: This problem is of the example of a frictionless mass that slide down (and falls off a "log"), which you solved using conservation of energy and $F = mv^2 / R$. In this problem, you have an extra energy term due to the ball's rotation, which you can relate to its translation using $v = \omega r$.)
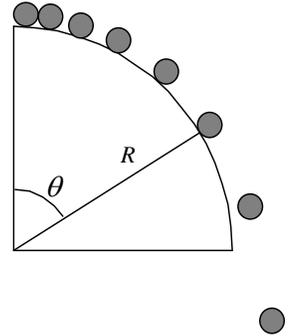
2. A ball has radius r, mass m, and moment of inertia $I = kmr^2$ about its center. The ball starts at rest, rolls without slipping down a ramp, and does a loop-the-loop of radius R. (See the figure for Experiment B.) What is the height h above the top of the loop that the ball must be started at to make it around the loop-the-loop without leaving the track? (Hint: This problem is similar to the loop-the-loop for a frictionless mass, which you also solved using conservation of energy. In this Prelab problem, you have an extra energy term associated with the ball's rotation, which you can relate to the ball's translation using $v = \omega r$.) Give an expression for the k factor in terms of m, h, r and R, and for the uncertainty σ_k due to the uncertainty σ_h in a measurement of height h.