# LAB #6: A PRECISION MEASUREMENT OF *g*

> **Do not disassemble your pendulum to measure the mass of the bob or the wire.   Instead, there are additional bobs and wires on the table in the center of the lab which you can weigh and measure.**

## Overview

So far, the experiments in Physics 103/105 Lab have addressed physical principles to an accuracy of 5 to 10%.  This is fine for getting a feel for how things work, but another objective of experimental physics is to measure precise values for constants of nature.  To make a precise measurement, one designs an experiment to minimize systematic effects or to make them easy to calculate.  In this lab you will measure *g*, the acceleration due to gravity at the Earth's surface, by timing a pendulum.  With care in your technique and attention to systematic effects you can achieve an accuracy of much better than 1%.

Uncertainty analysis is an essential part of this lab.  During this lab, you will carefully apply the methodology of combining random errors from repeated measurements using the concepts of standard deviation and standard error.   We have provided a spreadsheet to do most of the calculations for you.  But you are responsible for thinking about the results.  You may want to review Appendices B and C on Estimation of Errors in the Labs section of the Blackboard.

## Theory of a Real (Physical) Pendulum

A high-precision experiment requires unusual effort both in technique and in the underlying theory.  In this section we extend the theory of a simple pendulum to the level needed for a precise measurement of *g*.

Your textbook shows that for a simple pendulum of length $L_0$, the period $T_0$ is given by

$$T_0 = 2\pi \sqrt{\frac{L_0}{g}} \ .$$

(1)

Inverting this equation, we can calculate the value of *g* by timing the period of such a pendulum:

$$g = 4\pi^2 \frac{L_0}{T_0^2}, \quad \Rightarrow \quad \frac{\sigma_g}{g} \approx \frac{\sigma_{L_0}}{L_0}, \ \text{if} \ \frac{\sigma_{T_0}}{T_0} \ \text{is negligible.}$$

(2)

However, at the level of accuracy we are aiming for ($\sigma_g / g = 0.001$), we may not assume that our pendulum is a *simple* pendulum. There are two important effects that make the simple pendulum assumption break down. Our *physical* pendulum is 1) not a point mass suspended by a massless string, and 2) not a true harmonic oscillator (because the restoring term in the equation is not <u>exactly</u> proportional to the displacement). If we were to neglect either of these two effects, we would find that our measurement of $g$ was systematically off.



Light Beam
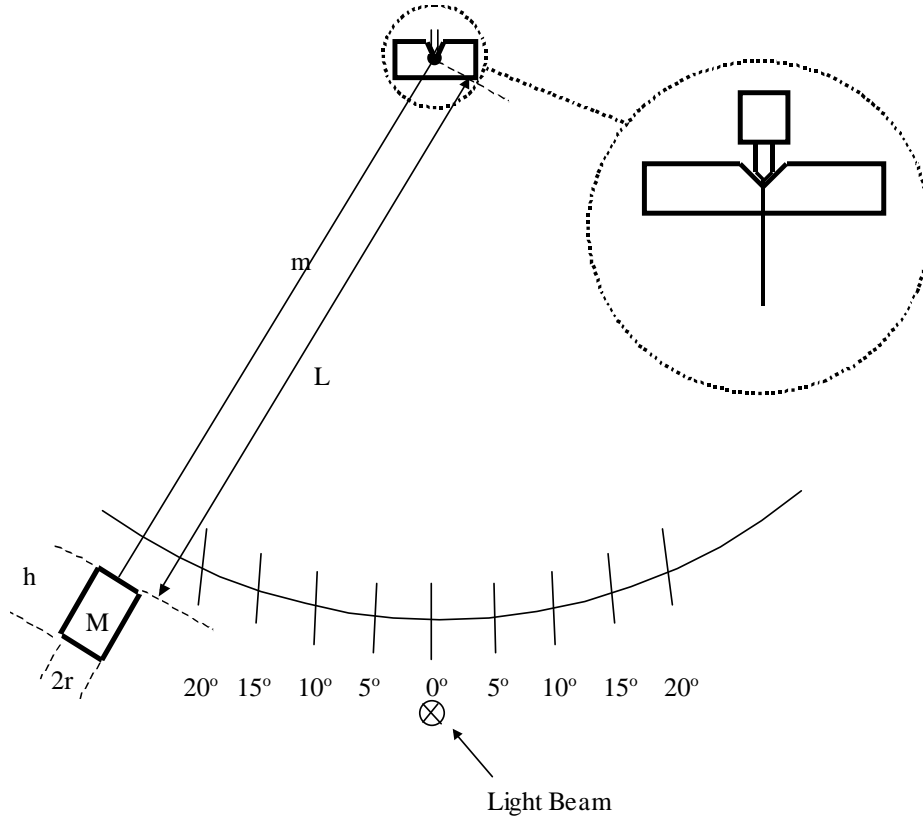
Consider the consequences of the first effect. For a physical pendulum of total mass $M_T$, moment of inertia $I$ about the pivot point, and distance $D_{cm}$ between the pivot point and the center of mass, the torque equation about the pivot point is

$$\tau = -D_{cm}M_T g \sin\theta \approx -D_{cm}M_T g\theta = I\alpha = I\frac{d^2\theta}{dt^2}, \tag{3}$$

so $\theta \approx \theta_0 \cos\omega t$ where the angular frequency is given by $\omega = \sqrt{\dfrac{D_{cm}M_T g}{I}}$, and the period $T_0$ for small oscillations is given

by $\qquad T_0 = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I}{M_T D_{cm} g}}, \qquad$ and hence $\qquad g = \dfrac{4\pi^2}{T_0^2}\dfrac{I}{M_T D_{cm}}. \tag{4}$

You may approximate your pendulum as a cylindrical bob of mass $M$, height $h$ and radius $r$, suspended on a long thin wire of mass $m$. Let the distance from the pivot to the top of the cylindrical bob be $L$. (See the sketch above.) The moment of inertia of a thin disc

about a diameter is $dI = dm\, r^2 / 4$ (which follows from the momentum of inertia $dm\, r^2 / 2$ about its axis using the so-called perpendicular axis theorem). Then using the parallel axis theorem, the moment of inertia of the bob (= a stack of thin discs) about the pivot point is given by

$$I_{bob} = \int \left[ \frac{dm\, r^2}{4} + dm\left(L+h-x\right)^2 \right] = \int_0^h \frac{M\, dx}{h} \left[ \left(L+h-x\right)^2 + \frac{r^2}{4} \right] = M\left( L^2 + Lh + \frac{h^2}{3} + \frac{r^2}{4} \right).$$

so that $I = I_{bob} + I_{string} = M\left( L^2 + Lh + \frac{h^2}{3} + \frac{r^2}{4} \right) + \frac{m\, L^2}{3} = ML^2\left( 1 + \frac{h}{L} + \frac{h^2}{3L^2} + \frac{m}{3M} \right).$ \hfill (5)

The distance $D_{cm}$ from the pivot to the center of mass of the bob is given by

$$M_T D_{cm} = M\left( L + \frac{h}{2} \right) + \frac{m\, L}{2} = ML\left( 1 + \frac{h}{2L} + \frac{m}{2M} \right). \hfill (6)$$

Substituting equations (5) and (6) into (4), we obtain

$$g = \frac{4\pi^2 L}{T_0^2} \left( \frac{1 + \dfrac{h}{L} + \dfrac{h^2}{3L^2} + \dfrac{r^2}{4L^2} + \dfrac{m}{3M}}{1 + \dfrac{h}{2L} + \dfrac{m}{2M}} \right). \hfill (7)$$

Because the torque equation (3) is not exactly the form of the equation for a simple harmonic oscillator, the period $T$ is depends on the maximum angle $\theta_0$ of the motion of the pendulum. A more advanced analysis shows that

$$T = T_0\left( 1 + \frac{1}{4}\sin^2 \frac{\theta_0}{2} + \frac{9}{64}\sin^4 \frac{\theta_0}{2} + \ldots \right), \hfill (8)$$

where $T_0$ is period assuming the system to be a simple harmonic oscillation, as in eq. (6). Remember, $T$ is the period that you measure, and $T_0$ from equation (6) is the period that you would have measured for your physical pendulum if $\theta_0$ were very small.

The following table gives you an idea of how big the difference between $T$ and $T_0$ can be, according to eq. (8).

| $\theta_0$ | $T/T_0$ | $\theta_0$ | $T/T_0$ |
| --- | --- | --- | --- |
| 0° | 1.00000 | 10° | 1.00191 |
| 5° | 1.00048 | 11° | 1.00231 |
| 6° | 1.00069 | 12° | 1.00275 |
| 7° | 1.00093 | 13° | 1.00323 |
| 8° | 1.00122 | 14° | 1.00374 |
| 9° | 1.00154 | 15° | 1.00430 |

# LAB #6: THINGS TO DO

## I. Measuring the Period

Measure the period of your pendulum using the light-activated electronic timer on your table. On period is a full cycle (= two half cycles), such that the bob passes throught the photogate in the same direction at the beginning and end of a period.

Set the mode switch to P so that the pendulum turns the timer on and off with successive passes. To time multiple periods, hold down the telegraph key on your table to keep the timer from being switched on and off with every pass. Set the switch on the back of the timer to manual reset to keep the display from resetting to zero immediately after every measurement. The timer can measure 1, 10, 100, or 1000 second intervals. Use the 100 second interval.

Measure the period for $\theta_0 = 10°$, timing the pendulum for a total of ten periods in a single measurement (rather than making 10 measurements of a single period), and dividing that result by 10. Estimate the random error by repeating the measurement about five times. Feel free to use Excel to help you with averaging or other calculations. After using your computer to calculate $g$ in part III below, go back and repeat the measurement for $\theta_0 = 5°$ and 15°.

**Tips on Technique:**

- Be certain to time 10 complete periods (= an even number of half periods); otherwise the placement of the light beam affects the measurement. If your value of $g$ is off by more that 2%, you likely did not measure 10 complete measurements.
- Center the light beam to make the durations of half cycles the same on both sides. There would be a small systematic shift in the measurement of the period due to the damping of the amplitude, if the light beam is not centered.
- Do not use amplitudes below 5°. Even if the light beam is centered, the finite size of the light beam causes a systematic error that is significant for small amplitudes.
- Check that when you release the bob it swings to a maximum angle close to the desired release angle. If your release is "hesitant", you may need to release the bob at a slightly larger angle than what you desire to achieve during the motion.

## II. Other Measurements

Measure the lengths and masses needed to calculate $g$ from equation (7). Measure the effective length of the pendulum wire from the pivot point to the top of the bob (which length is slightly longer than that of the physical wire itself) using a metal 2-meter ruler. (Wooden ones warp and shrink.) Measure $L$ from the bottom of the V notch at the pivot point to the top of the mass. Do not rest the ruler on top of the mass as its weight may elongate the wire. Keep the ruler vertical to be than 1/2° during the measurement.

**Do not disassemble your pendulum to measure the mass of the bob or the wire. Instead, there are additional bobs and wires on the table in the center of the lab which you can weigh and measure.**

# III. Using the Computers to Do the Tedious Calculations

Calculating *g* and doing error propagation with something as messy as equation (7) is, to put it nicely, the perfect job for a computer. We have programmed an Excel worksheet to help you with your calculations. An image of the worksheet is appended at the end of this write-up, and a .xls file is posted on Blackboard, so you can check it out before you get to Lab.

In the Physics 103 folder, double-click on Precision Measurement of g (Rev C).xls to open the worksheet. Since you will be adding your own data to it, you may want to click on File → Save As to allow you to rename and save a copy of the worksheet in the student data folder for your lab period. Then, any time you save the file, it will be saved there under its new name.

Many of the cells in the original worksheet have been "write protected" to keep you from accidentally changing or deleting the parts that do your calculations for you. You can type your values into the yellow, boxed regions, and you can also do any additional calculations to the side or below the ready-made part, which is contained in columns A-Q and rows 1-66.

Let's start by understanding the spreadsheet's logic:

In <u>section (a)</u> of the worksheet, you will want to put something recognizable for a group name and date so that you and others will be able to recognize it later, for instance as printouts become scattered around the lab.

In <u>section (b)</u> of the worksheet, you can enter your raw data for all measured quantities (*L*, *M*, *h*, *r*, *m*, *T*, and $\theta_0$), along with their absolute uncertainties. The worksheet automatically computes the relative (percentage) uncertainty for you.

In <u>section (c)</u>, the worksheet shows you the Amplitude Correction Factor $T/T_0$ that comes from your pendulum not being a true harmonic oscillator. It also calculates the "corrected period" $T_0$ of your pendulum in the limit of very small oscillations.

In <u>section (d)</u>, the worksheet calculates the individual geometric correction factors related to your pendulum being a "physical pendulum" rather than an idealized point mass. Each one of the terms A, B, C, D, E, and F corresponds to one of the terms in equation (7). The worksheet also calculates an overall "mass and dimension" correction factor, and combines this with the length *L* and the corrected period $T_0$ to give you the best value of *g* from your measurements according to equation (7).

In <u>section (e)</u>, the worksheet calculates the absolute and relative uncertainty in *g* from the uncertainties in each of the original measurements, and combines them in quadrature (*i.e.*, by taking the square root of the sum of the squares) to give the final

uncertainty in $g$ from your experiment (as implied by your original uncertainty estimates).

Start by entering your basic measurements into section (b) of the worksheet. If you do not get a reasonable value for $g$ (between 9.77 and 9.83 m/s$^2$), check for mistakes in your data entry. *Did you actually measure 10 full periods, and not 9 ½ or 10 ½? Did you actually measure the effective length of the pendulum, or only the physical length of the wire? Was the pivot wedge seated in the V-notch so that the pendulum swung freely and reliably?*

Next, think about the numbers reported by the spreadsheet. Do the geometric (mass and dimension) correction factors behave as they should? Does the corrected period give reasonable answers? (When the computer is doing all of the heavy lifting for you, it's easy for the lab to turn into a "plug and chug" activity, with your brains totally tuned out. Don't let that happen!)

Next, add your uncertainties to the worksheet. Do they propagate through as they should? Which ones are the biggest sources of error?

Once you have put your numbers in for one set of measurements at $\theta_0 = 10°$ and are satisfied with your results, repeat the experiment with $\theta_0 = 5°$ and with $\theta_0 = 15°$. Your calculated value for $T_0$ should not vary with $\theta_0$; if it does, you have a systematic error somewhere, which you should try to locate.

## IV. Points to Ponder

These are things you should make a particular point of pondering in your notebook. The AI's will be looking for these when they check the notebooks.

1. About how big does your $\theta_0$ have to be for the period $T$ of your pendulum to differ from the small amplitude period $T_0$ by 0.1%? By 1%?

2. Which leads to the largest "correction" in your calculation: the fact that your pendulum is not a true harmonic oscillator, or the fact that it is not a simple (point mass) pendulum?

3. In section (b) of the spreadsheet, you estimated errors for all of your measured quantities, and the spreadsheet calculated percentage errors for you. Rank these from most accurate to least accurate.

4. Each of the uncertainties in your measured quantities contributes some uncertainty to the calculation of $g$, as is shown in section (e) of the spreadsheet. Rank these from the largest contribution to the total uncertainty in $g$ to the smallest contribution. Are these the same as your rankings as for question 3? Why or why not?

5. Suppose you could spend five hours and $100 in the lab cutting any one of your measured uncertainties in half. (That is, with 5 hours of your time and $100 you could

decrease the uncertainty in $L$, $M$, $h$, $r$, $m$, $T$, or $\theta_m$ by a factor of two.) Which one would give you the "biggest bang for your buck" in terms of reducing your overall uncertainty in $g$?

6. The premise of question 5 is kind of bogus, in that you probably can't guarantee that one afternoon and $100 would cut the uncertainty in any one of your variables by a factor of 2. In reality, some of those uncertainties would be easy to improve, some wouldn't be. So where would you spend your time? What would be the next thing you would do to make a significant improvement in some part of your measurement to reduce your overall uncertainty in g? (There's no one right answer here; this is your chance to think and be creative.)

## V. The "Right" Answer

This section violates a fundamental law of Physics 103/105 Lab, namely: there is no "right" answer. But it turns out that what was at the time the world's best measurement of $g$ was made in Palmer Hall (now Frist Campus Center) in 1963, when the Physics Department was still housed there. Jim Faller, working in room 130, got

$g = 980.1604 \pm 0.0007$ cm/sec$^2$ \hspace{1cm} (J. E. Faller, Ph. D. Thesis, Princeton, 1963).

In 1998, Physics 103/105 labs moved from Palmer Hall to McDonnell Hall. *How much do you think Jim Faller's measurement would have changed had he made it on the second floor of McDonnell Hall? Could his experiment tell the difference? Can yours?*

We only have a "right" answer because Faller worked for 3 years and used a very fancy technique. Incidentally, he built a "suitcase" version of his apparatus, which he carried around comparing values of $g$ in standards labs all over the world — London, Paris, Geneva. Anyway, what is the *discrepancy* between your value of $g$ and Faller's? *Do they agree within the errors?*

## VI. Report your Values to Your AI

When you are satisfied with your results and understand the uncertainty that the spreadsheet has calculated for you, report your results to your AI, who will tabulate the results for each group, and will give each of you two graphs showing your results plotted along with those of your classmates. At the end of the week, we will collect the values for all of the lab sections and post them in the hallway, and possibly on the course web site.

How do your results compare to those of your classmates? Do your results agree to within your uncertainty? If not, then why? These kinds of "friendly" discussions are what good science is all about.

**Calculation Spreadsheet for Precision Measurement of "g"**

(Physics 103/105)

*NOTE: Input data only into cells in the boxed regions (yellow on the screen). All other entries are calculated automatically. Except for the boxed regions, all the cells in Rows 1 through 64 and in columns A through Q are write-protected. If you need to do calculations other than those that have been provided for, you must use cells outside this region.*

*(a)* **Group Name, Date, etc.** | 12/5/2006

*(b)* **Measurements and Estimated Errors:**

| | | | Best Value | | +/- | Estimated Error | | (Per Cent) |
|---|---|---|---|---|---|---|---|---|
| Wire length | L | = | 1.95300 | m | +/- | 0.00050 | m | 0.026% |
| Bob mass | M | = | 1.20610 | kg | +/- | 0.00005 | kg | 0.004% |
| Bob length | h | = | 0.07200 | m | +/- | 0.00050 | m | 0.694% |
| Bob radius | r | = | 0.02550 | m | +/- | 0.00025 | m | 0.980% |
| Wire mass | m | = | 0.00370 | kg | +/- | 0.00010 | kg | 2.703% |
| Period | T | = | 2.83390 | sec | +/- | 0.00005 | sec | 0.002% |
| Amplitude | $\theta_0$ | = | 15.0 | deg | +/- | 0.3 | deg | 2.000% |

*(c)* **Calculation of Corrected (i.e., small-amplitude) Period:**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Amplitude Correction Factor | $T/T_0$ | = | 1.00430 | -- | +/- | 0.00017 | -- | 0.017% |
| Corrected period | $T_0$ | = | 2.82177 | sec | +/- | 0.00049 | sec | 0.017% |

*(d)* **Calculation of "g":**

| | Best Value | | Uncertainty | (Per Cent) |
|---|---|---|---|---|
| First term ( $4\pi^2 L / T_0^2$ ) = | 9.68322 | m/sec$^2$ +/- | 0.00343 m/sec$^2$ | 0.035% |

Correction terms for dimensions and masses (best estimates only):

| | | | | |
|---|---|---|---|---|
| A | = | $h / L$ | = | 0.03687 |
| B | = | $h^2 / 3L^2$ | = | 0.00045 |
| C | = | $r^2 / 4L^2$ | = | 0.00004 |
| D | = | $m / 3M$ | = | 0.00102 |
| E | = | $h / 2L$ | = | 0.01843 |
| F | = | $m / 2M$ | = | 0.00153 |

Overall mass and dimension correction factor:

$( 1 + A + B + C + D ) / ( 1 + E + F )$ = 1.01806

**Best Value of "g" (first term multiplied by overall correction factor):**

**g = 9.85807 m/sec$^2$**

*(e)* **Uncertainty in "g" -- Propagation of Errors:**

| Source of Error | | % Error (Original Estimates) | | Propagated Errors in Value of "g" Absolute | (Percent) |
|---|---|---|---|---|---|
| Wire length | L | 0.026% | +/- | 0.00248 m/sec$^2$ | 0.0251% |
| Bob mass | M | 0.004% | +/- | 0.00000 m/sec$^2$ | 0.0000% |
| Bob length | h | 0.694% | +/- | 0.00125 m/sec$^2$ | 0.0127% |
| Bob radius | r | 0.980% | +/- | 0.00001 m/sec$^2$ | 0.0001% |
| Wire mass | m | 2.703% | +/- | 0.00014 m/sec$^2$ | 0.0014% |
| Measured Period | T | 0.002% | +/- | 0.00035 m/sec$^2$ | 0.0035% |
| Amplitude | $\theta_0$ | 2.000% | +/- | 0.00333 m/sec$^2$ | 0.0344% |