LAB #8: THE SPEED OF SOUND AND SPECIFIC HEATS OF GASES

Introduction

This week you will measure the velocity of sound in a gas, and you will weigh the gas in order to find its density. These measurements will lead to a determination of C_P/C_V, the ratio of specific heats at constant pressure and constant volume, and thus should allow you to determine the molecular structure of the gas. It is a beautiful example of how measurements of macroscopic lengths, masses, and forces provide information about molecules having sizes and masses of 10^{-10} meters and 10^{-23} grams!

Let's quickly review how these quantities relate -- that is, how the speed of sound ultimately relates to degrees of freedom and specific heat. The speed c of longitudinal waves is given by (see Appendix A for a derivation)

$$c = \sqrt{B/\rho},$$

where ρ is the density of the medium, and B is the bulk modulus defined by the relationship $B = \dfrac{\Delta P}{\Delta\rho/\rho} = -\dfrac{\Delta P}{\Delta V/V}$ (noting that changes in density and volume have opposite signs). The bulk modulus is a property of a substance: it describes the fractional change in the volume V when the pressure is increased by an amount ΔP.

Now consider sound traveling in an ideal gas. If the temperature of the gas were unaffected by a sound wave, then the ideal gas law, $PV = NRT$, could be differentiated as $V\Delta P + P\Delta V = 0,$ such that $-\Delta P/\left(\Delta V/V\right) = P.$

However, it is not correct for a gas that $B = P$. Sound waves propagate as a series of compressions and expansions, which change the local kinetic energy of the gas. Since temperature is a measure of the internal energy of a system, temperature is not constant during a sound wave. But, the variations in density of the gas take place so rapidly that there is no time for heat transfer from one part of the medium to another. A process in which there is no heat flow is called adiabatic. For an adiabatic process involving a gas the quantity PV^γ remains constant, where γ is the ratio C_P / C_V, as shown in Appendix B. In this case, $V\Delta P + \gamma P\Delta V = 0,$ and $B = -\Delta P/\left(\Delta V/V\right) = \gamma P.$

From this it follows that the speed of sound is given by

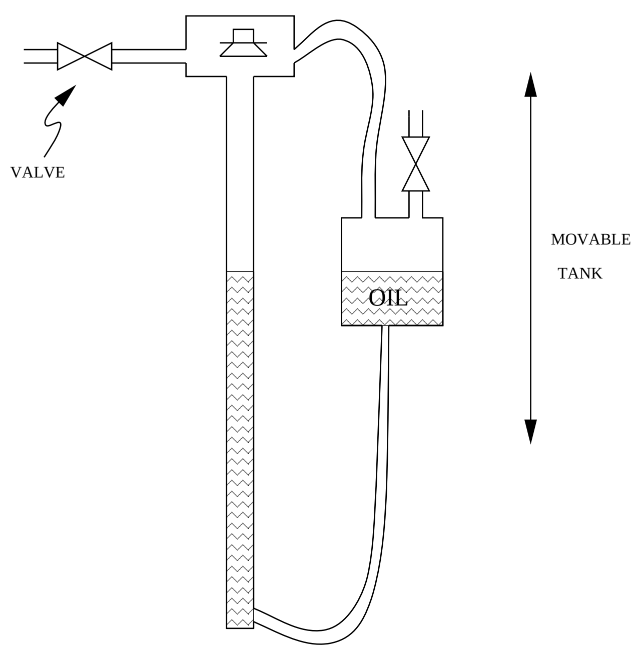
$$\boxed{c = \sqrt{\frac{\gamma P}{\rho}}.}$$

Gas Safety

Tanks of various gases will be available in the lab. Choose one of these for your measurement of the speed of sound and, of course, use the same gas for measurement of the density. The tanks contain gas at high pressure: *do not try to move the tanks*. Every tank has three valves: one on the throat of the bottle, which allows gas to enter a large reduction valve in the regulator, which bleeds the gas slowly out of a small needle valve to which the rubber tubing is attached. You need to use only the needle valve. The others have been adjusted: ***do not tamper with them***. The pressure has been set high enough to give a flow that will fill your flask in a reasonable time. If someone should increase the pressure it could cause a flask to blow up! When you open the needle valve you should feel a gentle breeze of gas flowing out of the rubber tube when held near your lips. Keep the needle valve closed when you are not taking gas, open it when you need some.

Velocity of Sound

The apparatus for measuring the speed of sound is a vertical cylinder of gas. The length of the column of gas is adjusted by changing the height of a metal reservoir of oil. A loudspeaker is attached to the top of the cylinder, and it generates sound from an electrical audio oscillator. The loudspeaker also responds to sound vibrations (like a microphone) and thus the sound waves set up in the cylinder of gas affect it also. This action is detected by a meter. When the cylinder is in resonance you will get a significant change in the reading of the meter included in the speaker circuit.



As a preliminary step, turn on the oscillator at a convenient frequency (1000 cycles/sec) and let the oil descend from the top down about 60 cm. Watch the meter and listen to the sound. You should be able to find several positions of the oil level at which resonance occurs. You will want to adjust the output level of the oscillator so that the meter reading is near the end of the scale when the system is off resonance. When the system is at a resonance, the meter reading should change by about 10 or 15%. You might find that a different frequency (10-20%) will give a stronger indication of resonance. Explore a bit.

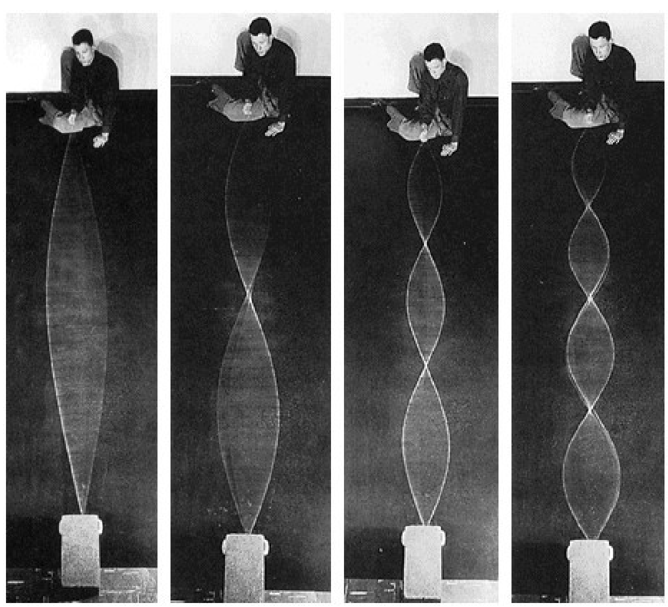
You are now ready to fill the cylinder with gas from one of the tanks. Note that you will need to flush out the air (or other gas) previously there. Raise the oil as high as possible in the transparent column to minimize the volume of air ``trapped at its top. Connect your flask of gas to the oil reservoir, so that as the gas enters the reservoir it creates a higher pressure there, which further raises the level of the oil in the column.

Be sure to open and close the valves in an order such the pressure in the system never gets too high, and causes the oil to overflow the top of the column.

Lower the oil level from the top and note the approximate location of the first three resonances. Then raise the oil slowly through each resonance. (Upward motion makes is easier to determine the level of oil, as you will discover on trying it both ways.) Find the position of resonance as accurately as possible.

Compute the speed of sound from the spacing between resonances and the frequency of the audio oscillator.

Figure from Tipler's 4th Edition (p. 492) – The first four harmonics of standing waves on a string fixed at both ends.



Density of the Gas

The density is found by weighing a known volume of the gas at atmospheric pressure. Two flasks of equal size are provided, one coated with plastic and the other clear.

Evacuate the coated flask with the vacuum pump (you can tell when this is achieved when a sharp clacking sound of the pump replaces the initial slurping sound). Weigh the evacuated flask (be careful not to drop it, the plastic coating has been put on to minimize hazard). Then fill the flask with your gas and reweigh.

Use the clear flask to find the volume by filling it with water to the proper level (remember the rubber stopper in the plastic coated flask). Measure the volume of water.

You can check your measurement of the density by comparing to the density of an ideal gas of particular molar mass: $p\ V = n\ R\ T$ and so $M\ n\ /\ V = M\ p\ /\ R\ T$, where M is the molar mass. Consider such gases as Ar, N_2, and CO_2. What density does the ideal gas law predict?

From the measured density and speed of sound for your gas, and the pressure of the atmosphere in the lab, determine the value of γ for your gas.

Tips on Technique

- A good way find a resonance position is for one person to slowly raise the oil level while another records the meter readings. The resonance position can be found by graphing the meter reading versus position.
- You must measure the change in weight (few grams) of the evacuated and filled flask, which can be done by moving the rider on the balance. Be sure that the weights in the balance pan are not changed while you fill the flask with gas.
- To fill the flask, open the needle valve, repeat the lip test, and if a gentle flow is found attach the hose to your flask and open the stopcock. It will be filled when the hissing noise at the needle valve stops. Shut the stopcock. Shut the needle valve and remove the tubing. The pressure in the flask is slightly above atmospheric. Open the stopcock slightly and gas will hiss out allowing the pressure to drop to normal. Weigh the filled flask. After a minute, open the stopcock again briefly and close. Reweigh. Repeat after another minute. Reweigh. If the weight has not changed again you are finished. If it has, repeat again. Why does the weight change at all?

What Gas Did You Use?

The tanks will be labeled so that they can be distinguished, but the labels indicating what gases they contain will be hidden. Some time into the lab your AI will provide you with a list of gases which might have been in the tanks. Can you determine from your measurements of density and γ which gas you used? How confident are you? *I.e.*, what's your error, and how well can you distinguish between monatomic, diatomic, and polyatomic gases?

Appendix A: Wave Equation for Sound

Sound waves cause small changes in the density ρ and pressure P of the gas. We write $\rho = \rho_0 + \rho_1$ and $P = P_0 + P_1$ where the perturbations ρ_1 and P_1 are small compared to the equilibrium values ρ_0 and P_1. Recalling the definition of the bulk modulus,

$$B = \frac{\Delta P}{\Delta\rho / \rho} = -\frac{\Delta P}{\Delta V / V},$$

we see that the small changes in density and pressure are related by

$$P_1 = \Delta P = B\frac{\Delta\rho}{\rho} = B\frac{\rho_1}{\rho_0}.$$

For simplicity, we suppose that the sound wave propagates along the x-axis.

In the absence of the sound wave the average velocity of the gas molecules is zero. The sound wave imparts an average velocity v (in the x-direction) to the molecules in a small volume $dx\,dy\,dz$. The mass in this volume is

$$\rho\, dx\,dy\,dz \approx \rho_0\, dx\,dy\,dz,$$

and the force on this volume due to the pressure difference on its faces at x and $x + dx$ is

$$F = \left[P(x) - P(x+dx)\right]dy\,dz = -\frac{dP_1}{dx}dx\,dy\,dz.$$

Newton's equation of motion for this small mass of molecules is therefore,

$$F = ma \approx \rho_0 dx\,dy\,dz\frac{dv}{dt} \approx -\frac{dP_1}{dx}dx\,dy\,dz, \qquad \textit{i.e.}, \qquad \frac{dv}{dt} \approx -\frac{1}{\rho_0}\frac{dP_1}{dx} = -\frac{B}{\rho_0^2}\frac{d\rho_1}{dx}.$$

The density and the velocity are related in another way, because the density of the small volume can only change if the velocity is different at its two x-faces. In particular, the length of the small volume changes from dx to

$$dx' = dx + \left[v(x+dx) - v(x)\right]dt = dx + \frac{dv}{dx}dx\,dt = dx\left(1 + \frac{dv}{dx}dt\right)$$

during time dt. The resulting change in the density during time dt is

$$d\rho = d\rho_1 = \frac{m}{dx'\,dy\,dz} - \frac{m}{dx\,dy\,dz} = \frac{m}{dx\,dy\,dz}\left(\frac{1}{1 + \dfrac{dv}{dx}dt} - 1\right) \approx -\rho_0\frac{dv}{dx}dt,$$

and hence

$$\frac{d\rho_1}{dt} \approx -\rho_0\frac{dv}{dx} \qquad \text{(equation of “continuity”).}$$

Taking the time derivative of the equation of motion for dv/dt, we obtain the wave equation,

$$\frac{d^2v}{dt^2} \approx -\frac{B}{\rho_0^2}\frac{d\rho_1}{dx\,dt} \approx \frac{B}{\rho_0}\frac{d^2v}{dx^2} \qquad \text{(wave equation).}$$

For a travelling wave, $v = f(x - ct)$, we find that $c^2\dfrac{d^2v}{dt^2} = \dfrac{d^2v}{dx^2}$, so the speed of sound is given by $c = \sqrt{\dfrac{B}{\rho_0}}$ (Newton, 1687, with $B = P$; Laplace, 1816, noted that $B = \gamma P$).

Appendix B: Adiabats of an Ideal Gas

The First Law of Thermodynamics expresses conservation of energy for a gas as

$$\Delta U = Q - W,$$

where ΔU is the change in the internal energy U of the gas when energy Q flows **into** the gas in the form of heat, and the gas does work W **on** the external system. The gas only does work if its volume V changes, in which case

$$W = \int P\,dV,$$

where in general, the pressure P changes as the volume changes. If N moles of a gas are heated at constant volume, the change in internal energy is given by

$$\Delta U_V = Q = NC_V\Delta T,$$

Where C_V is the molar heat capacity at constant volume, and ΔT is the resulting change in temperature of the gas. The heat capacity of an ideal gas is independent of temperature, such that the internal energy of a gas at absolute temperature T is given by

$$U = NC_V T.$$

Of course, an ideal gas also obeys the ideal gas law,

$$PV = NRT,$$

where $R = 8.31$ J/(mole-K) is the ideal gas constant.

In an adiabatic process there is no heat flow, by definition. Hence, in an adiabatic process that makes only small changes in the parameters of the gas, the First Law can be written $NC_V\,dT = dU = -dW = -P\,dV,$

And the ideal gas law tells us that

$$N\,dT = \frac{P\,dV + V\,dP}{R}.$$

Combing these equations, we find that $0 = \dfrac{dP}{P} + \dfrac{C_V + R}{C_V}\dfrac{dV}{V} \equiv \dfrac{dP}{P} + \gamma\dfrac{dV}{V},$

where $\gamma = \dfrac{C_V + R}{C_V} = \dfrac{C_P}{C_V}$

is the ratio of specific heat at constant pressure to that at constant volume.
Integrating, we obtain $a = \ln P + \gamma \ln V,$ and hence

$$PV^\gamma = e^a = \text{constant}.$$

For completeness, we demonstrate that the molar heat capacity C_P at constant pressure is related by $C_P = C_V + R.$

If the gas is heated at constant pressure, then the heat required is $Q = NC_P\Delta T,$ the work done by the gas is $W = P\,\Delta V = NR\Delta T,$ and the change in the internal energy of the gas is $\Delta U = NC_V\Delta T.$ Inserting these relations in the First Law, we obtain

$$NC_V\Delta T = \Delta U = Q - W = NC_P\Delta T - NR\Delta T,$$

which confirms that $C_P = C_V + R.$

REFERENCE INFORMATION

The following pages contain general information which you may find useful. They include:

- A table from Tipler, on specific heats of gases. Values of γ may be derived from this data.
- A table of densities of various gases, directly relevant to your calculation of v.
- A Periodic Table of the Elements, giving the atomic weights of the various elements. This will be useful if you want to consider gas densities from the point of view of the ideal gas law, $PV = NRT$.

You may recall that 1 atm = 101.3 kPa = 76.00 cm Hg .

Table 18-3 from Tipler (5th edition):

Molar Heat Capacities in J/mol·K of Various Gases at 25°C

Gas	c'_p	c'_v	c'_v/R	$c'_p - c'_v$	$(c'_p - c'_v)/R$
Monatomic					
He	20.79	12.52	1.51	8.27	0.99
Ne	20.79	12.68	1.52	8.11	0.98
Ar	20.79	12.45	1.50	8.34	1.00
Kr	20.79	12.45	1.50	8.34	1.00
Xe	20.79	12.52	1.51	8.27	0.99
Diatomic					
N_2	29.12	20.80	2.50	8.32	1.00
H_2	28.82	20.44	2.46	8.38	1.01
O_2	29.37	20.98	2.52	8.39	1.01
CO	29.04	20.74	2.49	8.30	1.00
Polyatomic					
CO_2	36.62	28.17	3.39	8.45	1.02
N_2O	36.90	28.39	3.41	8.51	1.02
H_2S	36.12	27.36	3.29	8.76	1.05

DENSITY AND SPECIFIC GRAVITY OF GASES AND VAPORS

Name	Formula	Density g/l 0°C 760 mm	Density lbs./ft.³ 32°F, 1 atm.	Specific gravity Air = 1	Specific gravity O_2 = 1
Acetylene	C_2H_2	1.173	0.07323	0.9073	0.8208
Air		1.2929	.08071	1.0000	0.9047
Ammonia	NH_3	0.7710	.04813	0.5963	0.5395
Argon	A	1.7837	.11135	1.3796	1.2482
Arsenic fluoride	AsF_5	7.71*	.481*	5.96*	5.40*
Arsenic hydride	AsH_3	3.484*	.2175*	2.695*	2.438*
Boron fluoride	BF_3	2.99*	.187*	2.31*	2.09*
Butane (*n*)	C_4H_{10}	2.5190†	.15725†	2.0854†	1.8868†
Butane, *iso*-	C_4H_{10}	2.673	.1669	2.067	1.870
Carbon dioxide	CO_2	1.9769	.12341	1.5290	1.3834
Carbon monoxide	CO	1.2504	.07806	0.9671	0.8750
Carbon oxysulfide	COS	2.72	.170	2.10	1.90
Chlorine	Cl_2	3.214	.2006	2.486	2.249
Chlorine dioxide	ClO_2	3.09[11]	.193[11]	2.39[11]	2.16[11]
Chlorine monoxide	Cl_2O	3.89	.243	3.01	2.72
Cyanogen	C_2N_2	2.335*	.1458*	1.806*	1.634*
Dimethylamine	$(CH_3)_2NH$	1.966[17]	.1227[17]	1.521[17]	1.376[17]
Ethane	C_2H_6	1.3566	.08469	1.0493	0.9493
Ethylene	C_2H_4	1.2604	.07868	0.9749	0.8820
Fluorine	F_2	1.696	.1059	1.312	1.187
Germanium hydride (di-germane)	Ge_2H_6	6.74[20]	.421[20]	5.21[20]	4.72[20]
Germanium tetrahydride	GeH_4	3.420	.2135	2.645	2.393
Helium	He	0.17847	.01114	0.13804	0.12489
Hydrogen	H_2	0.08988	.005611	0.06952	0.06290
Hydrogen bromide	HBr	3.6445	.2275	2.8189	2.5503
Hydrogen chloride	HCl	1.6392	.10233	1.2678	1.1471
Hydrogen iodide	HI	5.7891	.3614	4.4776	4.0510
Hydrogen selenide	H_2Se	3.670	.229	2.839	2.568
Hydrogen sulfide	H_2S	1.539	.09608	1.190	1.077
Hydrogen telluride	H_2Te	5.81	.363	4.49	4.07
Krypton	Kr	3.708	.2315	2.868	2.595
Methane	CH_4	0.7168	.04475	0.5544	0.5016
Methylamine	CH_3NH_2	1.396	.08715	1.080	0.9769
Methyl chloride	CH_3Cl	2.3076	.1441	1.7848	1.6148
Methyl ether	$(CH_3)_2O$	2.1098	.1317	1.6318	1.4764
Methyl fluoride	CH_3F	1.5452	.09646	1.1951	1.0813
Neon	Ne	0.90035	.05621	0.69638	0.63004
Nitric oxide	NO	1.3402	.08367	1.0366	0.9378
Nitrogen	N_2	1.25055	.07807	0.96724	0.87510
Nitrogen (atm.)		1.2568	.07846	0.9721	0.8795
Nitrosyl chloride	NOCl	2.992	.1868	2.314	2.094
Nitrosyl fluoride	NOF	2.176*	.1358*	1.683*	1.523*
Nitrous oxide	N_2O	1.9778	.1235	1.5297	1.3840
Nitroxyl chloride	NO_2Cl	2.57*	.160*	1.99*	1.798*
Nitroxyl fluoride	NO_2F	2.90	.181	2.24	2.03
Oxygen	O_2	1.42904	.08921	1.10527	1.0000
Ozone	O_3	2.144	.1338	1.658	1.500
Phosphine	PH_3	1.5294	.09548	1.1829	1.0702
Phosphorus fluoride	PF_3	3.907*	.2439*	3.022*	2.734*
Phosphorus oxyfluoride	POF_3	4.8	.30	3.7	3.4
Phosphorus pentafluoride	PF_5	5.81	.363	4.494	4.066
Propane	C_3H_8	2.0096	.1254	1.554	1.407

* Temperature not stated, probably 20°C.
† Both butane and air at 710 mm.

Handbook of Chemistry and Physics (32nd edn)
(Chemical Rubber Publishing Co.)

DENSITY AND SPECIFIC GRAVITY OF GASES AND VAPORS (Continued)

Name	Formula	Density g/l 0°C 760 mm	Density lbs./ft.³ 32°F, 1 atm.	Specific gravity Air = 1	Specific gravity O_2 = 1
Radon	Ra	9.73	.607	7.526	6.809
Silicane, chloro-	SiH_3Cl	3.03	.189	2.34	2.12
Silicane, chloromethyl	SiH_2ClCH_3	3.64	.227	2.82	2.55
Silicane, dichloromethyl	$SiHCl_2CH_3$	5.3	.33	4.1	3.7
Silicane, dimethyl	$SiH_2(CH_3)_2$	2.73	.170	2.11	1.91
Silicane, methyl	SiH_3CH_3	2.08	.130	1.61	1.46
Silicane, trifluoro-	$SiHF_3$	3.86	.241	2.99	2.70
Silicon fluoride	SiF_4	4.684	.2924	3.623	3.278
Silicon hexahydride	Si_2H_6	2.85	.178	2.204	1.994
Silicon tetrahydride	SiH_4	1.44	.0899	1.114	1.008
Stibine (15°C, 754 mm)	SbH_3	5.30	.331	4.10	3.71
Sulfur dioxide	SO_2	2.9269	.1827	2.2638	2.0482
Sulfur fluoride	SF_6	6.50*	.406*	5.03*	4.55*
Sulfuric oxyfluoride	SO_2F_2	3.72*	.232*	2.88*	2.60*
Trimethylamine	$(CH_3)_3N$	2.580	.1611	1.996	1.805
Trimethyl boron	$(CH_3)_3B$	2.52	.157	1.95	1.76
Tungsten fluoride	WF_6	12.9	.805	9.98	9.03
Xenon	Xe	5.851	.3653	4.525	4.094

* Temperature not stated, probably 20°C.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** 1.0079																	2 **He** 4.0026
3 **Li** 6.941	4 **Be** 9.0122											5 **B** 10.811	6 **C** 12.011	7 **N** 14.007	8 **O** 15.999	9 **F** 18.998	10 **Ne** 20.180
11 **Na** 22.990	12 **Mg** 24.305											13 **Al** 26.982	14 **Si** 28.086	15 **P** 30.974	16 **S** 32.065	17 **Cl** 35.453	18 **Ar** 39.948
19 **K** 39.098	20 **Ca** 40.078	21 **Sc** 44.956	22 **Ti** 47.867	23 **V** 50.942	24 **Cr** 51.996	25 **Mn** 54.938	26 **Fe** 55.845	27 **Co** 58.933	28 **Ni** 58.693	29 **Cu** 63.546	30 **Zn** 65.409	31 **Ga** 69.723	32 **Ge** 72.64	33 **As** 74.922	34 **Se** 78.96	35 **Br** 79.904	36 **Kr** 83.798
37 **Rb** 85.468	38 **Sr** 87.62	39 **Y** 88.906	40 **Zr** 91.224	41 **Nb** 92.906	42 **Mo** 95.94	43 **Tc** (98)	44 **Ru** 101.07	45 **Rh** 102.91	46 **Pd** 106.42	47 **Ag** 107.87	48 **Cd** 112.41	49 **In** 114.82	50 **Sn** 118.71	51 **Sb** 121.76	52 **Te** 127.60	53 **I** 126.90	54 **Xe** 131.29
55 **Cs** 132.91	56 **Ba** 137.33	57-71 *	72 **Hf** 178.49	73 **Ta** 180.95	74 **W** 183.84	75 **Re** 186.21	76 **Os** 190.23	77 **Ir** 192.22	78 **Pt** 195.08	79 **Au** 196.97	80 **Hg** 200.59	81 **Tl** 204.38	82 **Pb** 207.2	83 **Bi** 208.98	84 **Po** (209)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** (226)	89-103 #	104 **Rf** (261)	105 **Db** (262)	106 **Sg** (266)	107 **Bh** (264)	108 **Hs** (277)	109 **Mt** (268)	110 **Ds** (281)	111 **Uuu** (272)	112 **Uub** (285)		114 **Uuq** (289)				

* Lanthanide series	57 **La** 138.91	58 **Ce** 140.12	59 **Pr** 140.91	60 **Nd** 144.24	61 **Pm** (145)	62 **Sm** 150.36	63 **Eu** 151.96	64 **Gd** 157.25	65 **Tb** 158.93	66 **Dy** 162.50	67 **Ho** 164.93	68 **Er** 167.26	69 **Tm** 168.93	70 **Yb** 173.04	71 **Lu** 174.97
# Actinide series	89 **Ac** (227)	90 **Th** 232.04	91 **Pa** 231.04	92 **U** 238.03	93 **Np** (237)	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259)	103 **Lr** (262)

PRELAB Problems for Lab #8; The Speed of Sound and Specific Heats of Gases

1. In this experiment, you will vary the height of the oil in a column of air, producing an air column with varying length. At certain lengths, acoustic resonances will occur for a given frequency of sound, f. Given two different (consecutive) heights h_1 and h_2, of the oil in the column that produce resonance, derive an expression for the speed of sound, v, in terms of f, h_1 and h_2.

2. What are the resonant wavelengths, λ_n, for standing sound waves in a small-diameter pipe of length L having one end open and the other end closed ? Assume that there is a displacement node at the closed end of the pipe, and a pressure node at the open end, and write down a few words as to why you think these assumptions are reasonable. (Tipler's pages 517 and 518 are relevant to this problem.)

3. Assume that various traveling waves can exist in the same space, described by the equations

$$\psi_1(x,t) = A \sin(\ 7\ \mathrm{m}^{-1} \cdot x\ -\ 520\ \mathrm{sec}^{-1} \cdot t\ ) \quad ,$$

$$\psi_2(x,t) = A \sin(\ 7\ \mathrm{m}^{-1} \cdot x\ +\ 520\ \mathrm{sec}^{-1} \cdot t\ ) \quad ,$$

$$\psi_3(x,t) = A \sin(\ -7\ \mathrm{m}^{-1} \cdot x\ -\ 520\ \mathrm{sec}^{-1} \cdot t\ ) \quad , \qquad \text{and}$$

$$\psi_4(x,t) = A \sin(\ -7\ \mathrm{m}^{-1} \cdot x\ +\ 520\ \mathrm{sec}^{-1} \cdot t\ ) \quad .$$

   (a) Which waves are traveling in the direction of the positive x axis, and which in the negative x direction?

   (b) What are the frequencies f and wavelengths λ of the waves? What are their speeds of propagation?

   (c) What are the angular frequencies (ω) and wave numbers (k) of the waves?

   (d) Are any of the four wave functions identical with each other? Explain why or why not. (A is the same in each case.) You may want to consider the behavior of the four functions in the vicinity of $x = 0$, $t = 0$.

Note: It is irrelevant to answering these questions whether the ψ's (and A) describe sound waves, the electric fields of a radio or light wave, or a quantum-mechanical wave function. Superposition, and the conditions leading to standing waves and resonance, are important aspects of any wave phenomenon.