PRINCETON UNIVERSITY   **PHYSICS 104 LAB**
Physics Department   **Week #4**

EXPERIMENT IV

FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD (*e/m* OF ELECTRON)

AND

FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD (μ_o)

This week you will perform two experiments concerning forces on moving charges in magnetic fields. The apparatus for the *e/m* experiment will be set up in one lab room and the apparatus for the μ_o experiment will be set up in the other lab room. Report to your regular lab room. You will spend the first half of the lab time doing one experiment, then you and your labmates will move to the other room to do the other experiment.

1. FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD (*e/m* OF ELECTRON)

In this experiment you use a beam of electrons in a nearly evacuated glass bulb into which has been put a small amount of hydrogen gas. The beam of electrons is formed and accelerated through a potential difference of *V* volts. The beam enters a region of magnetic field **B**, applied perpendicular to the electron beam causing it to move in a circular path. By measuring the energy of the electrons and the radius of curvature of the beam, you can determine the value of *e/m* for the electrons.

The theory of the orbit of a particle moving $\perp$ to a magnetic field **B** is given in Section 26-2 of Tipler. A quick review of orbit theory follows (the notation is slightly different from Tipler, but the physical quantities are the same):

You recall that if an electron with charge *e* and mass *m* is accelerated from rest by a potential difference *V*, the change in potential energy is *eV*, so it gains kinetic energy:

$$eV \;=\; \frac{1}{2}\,mv^2\,, \tag{1}$$

where *v* is the final speed of the electron.

If the electron then enters a uniform magnetic field **B** $\perp$ **v**, it travels in a circle with radius *r* satisfying:

$$F = evB = ma = \frac{mv^2}{r}. \tag{2}$$

Eliminating the speed *v* between eqs. (1) and (2) gives an expression for the charge-to-mass ratio of the electron, in Coulombs per kilogram:

$$\frac{e}{m} \;=\; \frac{2V}{(Br)^2} \tag{3}$$

(Compare eq. 26-12 in Tipler).

The apparatus you will use consists of a source of electrons inside an evacuated glass bulb (except for the small amount of hydrogen gas already referred to), as sketech below. The bulb is expensive and delicate and has a moderately short life, so be gentle when working with it.
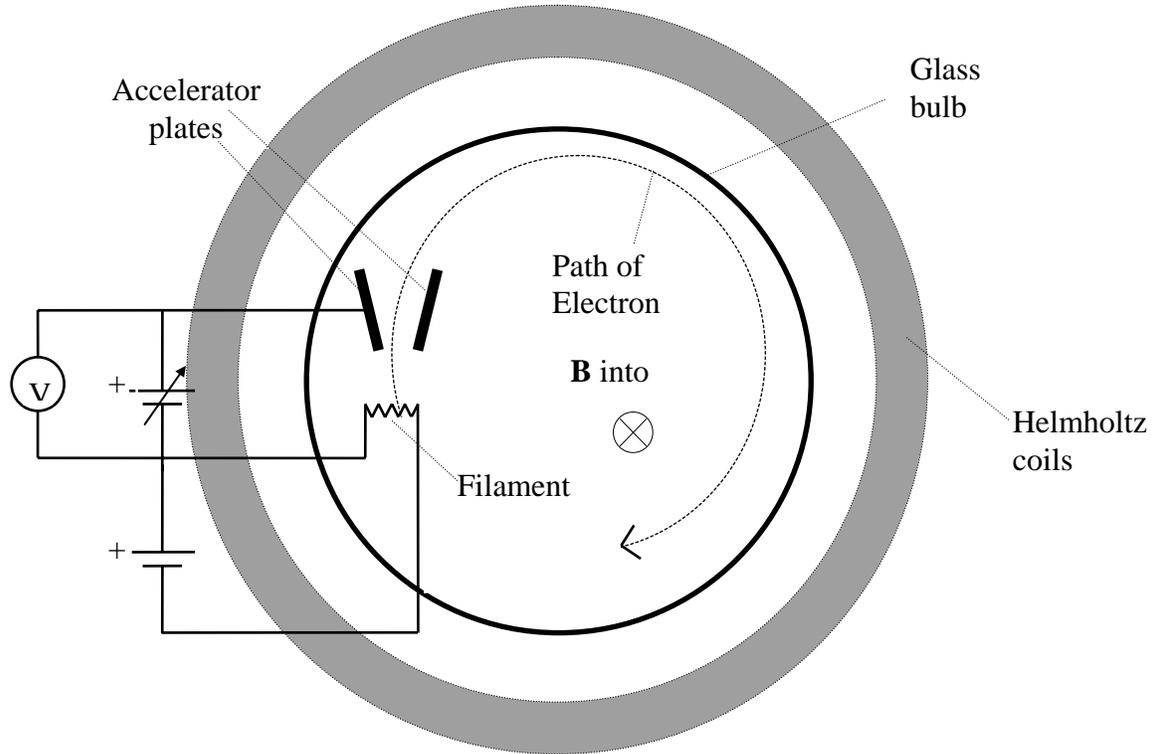
The electrons are emitted from a hot wire (filament) and accelerated by plates held at a voltage $+V$ above the filament by a power supply whose voltage can be varied. The other power supply provides the current to heat the filament. Not shown is the grid, which helps shape the beam.

The magnetic field B is produced by a pair of Helmholtz coils (see Fig. 27-9 of Tipler). This is a special arrangement in which two identical coils are placed co-axially and separated by a distance equal to one radius R of the two identical coils. They produce a magnetic field that is, to a good approximation, uniform in the central region of the coils.

To see this, use Tipler's eq. (27.5) for a coil at $x = a$ and another at $x = -a$, each containing N turns of wire of radius R that carry current I. The magnetic field on the x-axis is given by:

$$B_x(x, r = 0) = \frac{\mu_0 R^2 NI}{2} \left(\frac{1}{\left[(x+a)^2 + R^2 \right]^{3/2}} + \frac{1}{\left[(x-a)^2 + R^2 \right]^{3/2}} \right). \tag{4}$$

Helmholtz' suggestion is to choose distance a so that the first, second and third derivatives of $B_x(x=0,r=0)$ are all equal to zero, which implies the magnetic field is very uniform near the origin.

The first derivative is

$$\frac{\partial B_x(x, r=0)}{\partial x} = -\frac{3\mu_0 R^2 N I}{2}\left(\frac{x+a}{\left[(x+a)^2 + R^2\right]^{5/2}} + \frac{x-a}{\left[(x-a)^2 + R^2\right]^{5/2}}\right), \tag{5}$$

which vanishes at $x = 0$ for any value of a. The second derivative is

$$\frac{\partial^2 B_x(x, r=0)}{\partial x^2} = -\frac{3\mu_0 R^2 N I}{2}\left(\frac{R^2 - 4(x+a)^2}{\left[(x+a)^2 + R^2\right]^{7/2}} + \frac{R^2 - 4(x-a)^2}{\left[(x-a)^2 + R^2\right]^{7/2}}\right), \tag{6}$$

which vanishes at $x = 0$ for $a = R/2$ as claimed by Helmholtz. You can "easily" verify that the third derivative (and any odd derivative) vanishes at $x = 0$ for any value of a. Substituting $a = R/2$ in eq. (4), the magnetic field at the origin is

$$B = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 I\, N}{R}. \tag{7}$$

For the Helmholtz coils in this Lab, $R = 15$ cm and $N = 130$ turns.
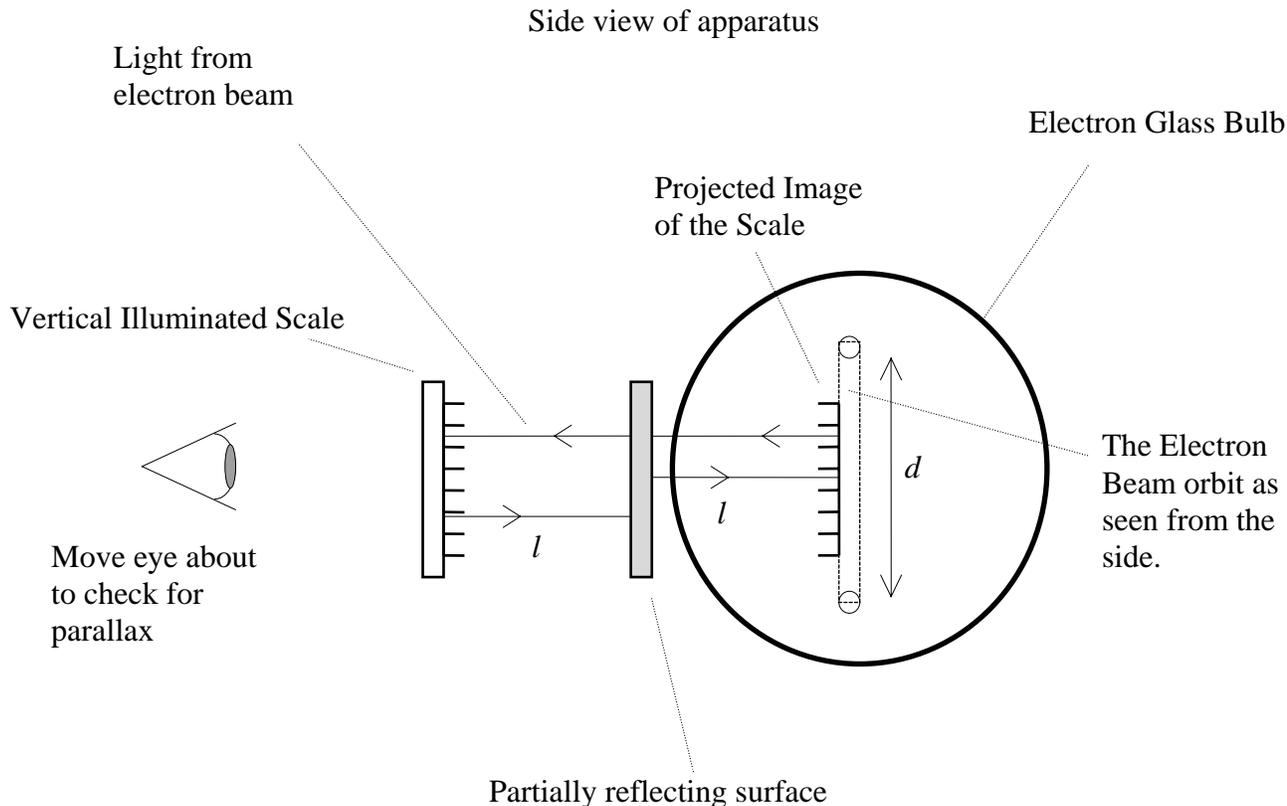
As usual, begin by getting familiar with the apparatus. Set the potential V to about 150 volts, and observe the electron beam. The beam is made visible when the hydrogen gas is ionized by electron collisions—the glow you see comes from the photons emitted when electrons are recaptured to form hydrogen again. The lights will have to be down, and you may need a black cloth cover over yourself and the apparatus.

Explore the effect on the electron beam of the **B** field of a permanent magnet as you bring it close to the glass bulb. Be careful not to hit the glass bulb with the permanent magnet.

Now turn on the Helmholtz coil current and study the orbits.

- Rotate the tube, observe and explain the electron orbits. The special cases are when v is $\perp$ to **B** and when v and **B** are parallel. What happens in each of these two cases and in between?

- Measure e/m for the electron, which is one of the most basic quantities in physics. The horizontal component of the Earth's **B** field in Princeton is about 0.1 Gauss. By what percentage does this field change the total magnetic field's magnitude? What percentage error is thereby introduced in the measurement of e/m?

Note: A clever optical system is used to avoid parallax when measuring the diameter of the beam orbit. To measure the diameter of the electron's orbit we project an image of an illuminated scale on the orbit, using a partially reflecting mirror as shown on the next page. If the illuminated scale is a distance l in front of the mirror as shown, its image will appear to be a distance l behind the mirror. The object is to have the image of the scale in the plane of the electron's orbit, the diameter of the orbit can then be measured without parallax. In order to achieve this, place the illuminated scale about as far in front of the mirror as the mirror is from the orbit. For a final adjustment move your head from side to side to check for parallax between the image of the scale and the orbit of the electron --- when they seem to move together, you have eliminated parallax. Make sure the scale is vertical, or nearly so.

Side view of apparatus

Light from
electron beam

Electron Glass Bulb

Projected Image
of the Scale

Vertical Illuminated Scale

The Electron
Beam orbit as
seen from the
side.

d

l

Move eye about
to check for
parallax

l

Partially reflecting surface

Describe your observations in your notebook (sketches are helpful). Calculate the charge-to-mass ratio $e/$m of the electron from your measured values, and estimate the error in your value for e/m. Each student should take his or her own measurements and determine a value of e/m. This (value $\pm$ error) should be written on the board in the place provided so that a classwide result, mean and standard deviation, can be computed.

2. FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD (μ_O)

In this experiment, the permeability of free space μ_o is measured using a current balance (see Figure 27-20 of Tipler), in which magnetic forces can be balanced with gravitational forces. A quick review of the basic principles is given here:

In the "see-saw" (current balance) setup used in this measurement, sketched in the figure below, a rectangular coil of N turns has one end, of length L, that is kept perpendicular to the uniform magnetic field B of a solenoid manget. The other end of the coil is mounted on the axis of the "see-saw".

The (vector) force on N segments of wire of length L carrying current I in a uniform magnetic field B (See equation 26-4 of Tipler) is given by:

$$F = N\,I\,L \times B,\qquad(8)$$

And since vector L is perpendicular to vector B, the (scalar) force acting on the coil is

$$F_{\text{seesaw}} = N I_{\text{see-saw}} L B_{\text{solenoid}}, \tag{9}$$

where $N = 10$ for the see-saw coil.

The uniform magnetic field B of the solenoid is given by

$$B_{\text{solenoid}} = \mu_0 n f I_{\text{solenoid}}, \tag{10}$$

where I_{solenoid} is the current passing through the solenoid, n is the number of turns per unit length (turns/meter) and the coefficient $f = 0.90 \pm 0.03$ is a correction factor to account for the less-than-ideal solenoid with fringing fields (not infinite in length). Use this factor when calculating the field.
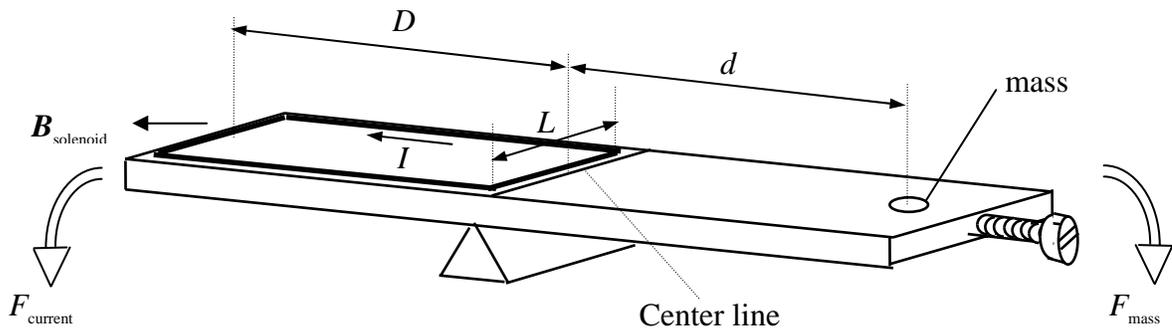
When the mass on the righthand side of the see-saw is adjusted so as to balance the magnetic force, then

$$F_{\text{current}} D = F_{\text{mass}} d. \tag{11}$$

Circuit diagram (looking down at a cutaway of the solenoid)
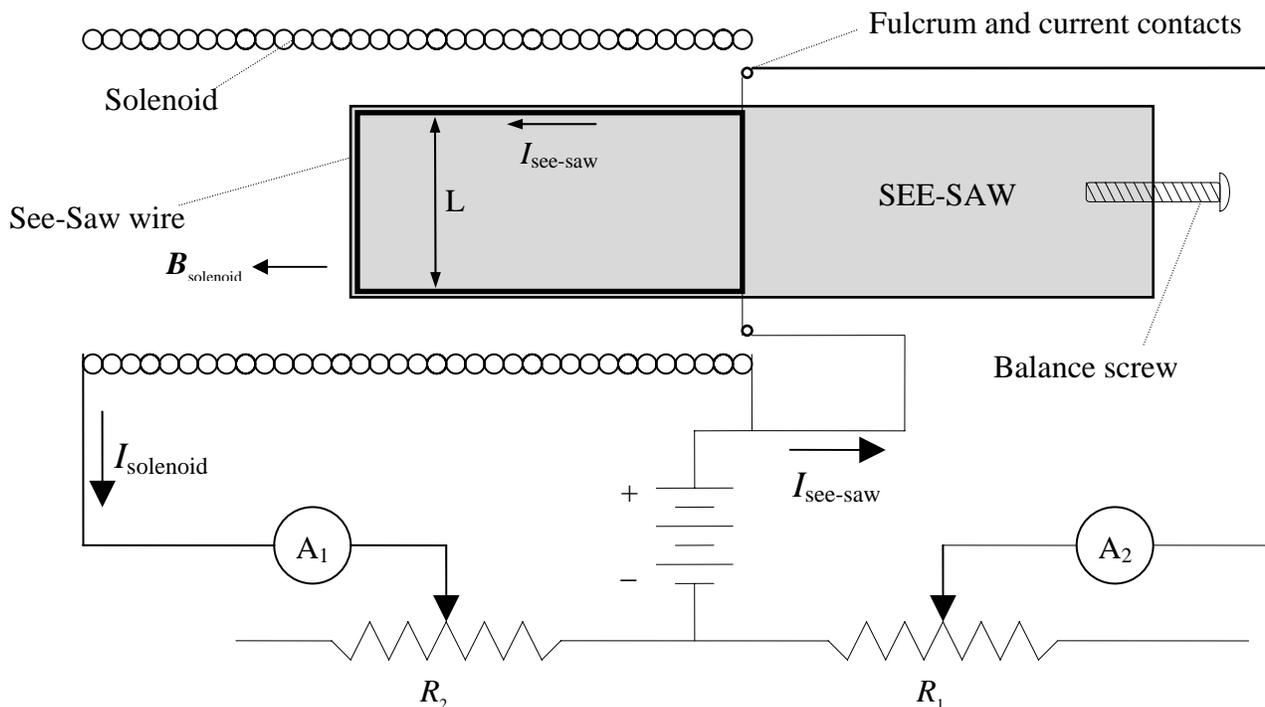
Build the circuit shown above. It provides current to both the solenoid and the see-saw coils, separately adjustable. Balance the see-saw while no current is flowing and without using weights. Accomplish this by turning the screw on the end of the see-saw, and then note the position of the see-saw on the scale provided (the cut-off ruler). Sketch the solenoid in your book, show the direction of current $I_{solenoid}$, and use the right-hand rule to show the direction of the magnetic field.

Also sketch the see-saw in your notebook and indicate magnetic field supplied by the solenoid. Show the direction of the current $I_{see-saw}$ (which is not necessarily in the direction shown in the figure on p. 4), such that there will be a downward force on the part of the see-saw in the solenoid. Thus the see-saw becomes a current balance. The rectangular coil on the current balance has four sides and 10 turns. Explain why you can ignore three sides as contributing nothing to the magnetic force.

When current flows in both the solenoid and the coil attached to the see-saw, the see-saw tips. Restore the see-saw to its original position by adding weight to the opposite end. When the mechanical balance is restored, the condition of static equilibrium is that the magnetic and gravitational torques are equal, as stated in eq. (11).

You may have to use several different masses in combination to find the correct balance. Use the hollow in the end of the see-saw to help keep all masses together at the same distance d. Always handle the masses with tweezers to reduce any damage to them. Once you are close, it is easy to adjust the see-saw current to achieve the final balance.

When the current is turned on and the required mass M is added to re-establish balance, you should have about 1 ampere in the see-saw wires and around 8 amperes in the solenoid. Don't leave the current on too long since there is some heating of these circuits.

Calculate μ_o and state your estimated error. Given that you have achieved balance, you will know the values of all the variables in equations 2 and 3 except μ_o --- which is to be found. Each group should report the measured value of ($\mu_o \pm$ error) on the board so that we may obtain a class value, mean and standard deviation.

NOTE: The balancing of the see-saw depends on the knife edge contacts remaining in the small grooves in the support, and also remaining reasonably sharp. Please handle these parts with care. Always wait for the balance and current to settle before taking measurements.

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Name:_____   **Date/Time of Lab:**_____

EXPERIMENT IV   PRELAB PROBLEM SET

1) A particle of mass m and charge e is moving with kinetic energy K. It is in a region of uniform magnetic field of strength $\boldsymbol{B}$; its motion is perpendicular to the field, along the arc of a circle of radius R_o.

If the kinetic energy were *2K* instead of *K,* what would be the new radius R_1, assuming m, e, and $\boldsymbol{B}$ remain unchanged?

Given the expected value of *e/m* (*i.e.,* the one you can find in Tipler) and supposing that the electrons are accelerated with a potential difference of 150 Volts, what magnetic field strength will be required to hold the electrons in orbits with 4 cm radius?

What current must flow in the Helmholtz coils to produce this magnetic field? Our coils are described in the lab write-up.

Continued on reverse

2) A current-carrying conductor passes through a magnetic field **B**; both are in the plane of the paper, as shown. It experiences a force. In what direction is the force? What would happen to the force if the current doubled?

I

B
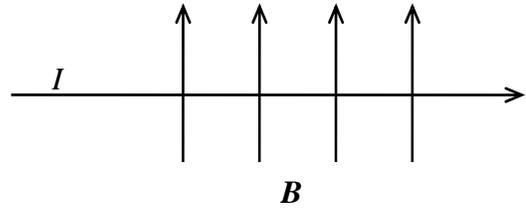
If the field **B** were large (say, 1 Tesla, as in the Cyclotron magnet, which you will visit next week), and the current *I* were also large (say, 1 Ampere), and the wire was 20 cm long, what would be the magnitude of the force? Would you be able to feel this force if you held the wire in your hand? Would you be able to feel it if there were 400 such wires bundled together?